**Al-Mustaqbal university**

**Engineering technical college**

**Department of Building &Construction Engineering**

***Mathematics***

***First class***

***Lecture No.7***

***Assist. Lecture***

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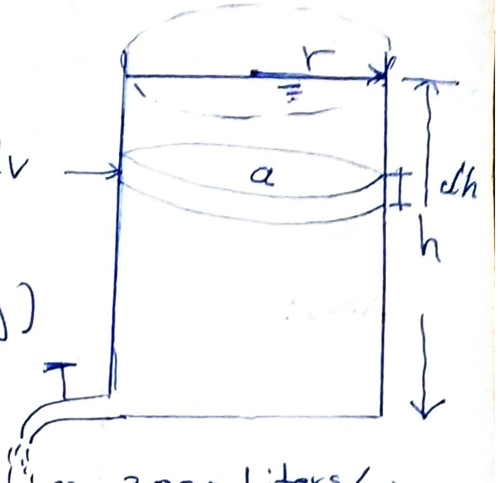
# Applications

1) **pumping out tank**

a) Determine the rate of height (h) drop.

b) the height (h) after $t = 100$ min

c) determine $t$ when $h = 0$ (tank empty)




$Q = 3000$ Liters/min $= 3\ m^3/min$

$r = 5\ m$

$h = 10\ m$

Ans:

$dv = a\,dh$ (a: tank area) $a = \pi r^2$

$dv = \pi r^2 dh$

$V$ = volume of the tank

$$\frac{dv}{dt} = \pi r^2 \frac{dh}{dt}$$

$\left(\text{but } \frac{dV}{dt} = Q = 3000 \frac{\text{Liters}}{\text{min}} = 3\ m^3/\text{min}\right)$

$$3 \frac{m^3}{min} = \pi r^2 \frac{dh}{dt}$$

a) $\dfrac{dh}{dt} = -\dfrac{3}{\pi r^2}$ (m/min) [height drop rate]

b) $dh = -\dfrac{3}{\pi r^2}\,dt$

$h = -\dfrac{3}{\pi r^2} t + c$ [at $h = 10$, $t = 0$]

$10 = c$

$$\boxed{h = -\frac{3}{\pi r^2} t + 10}$$

after $t = 100$ min

$$h = -\frac{3}{\pi (5)^2} * 100 + 10 =$$

$= -3.82 + 10 = 6.12$ m

c) $0 = -\dfrac{3}{\pi 5^2} t + 10$

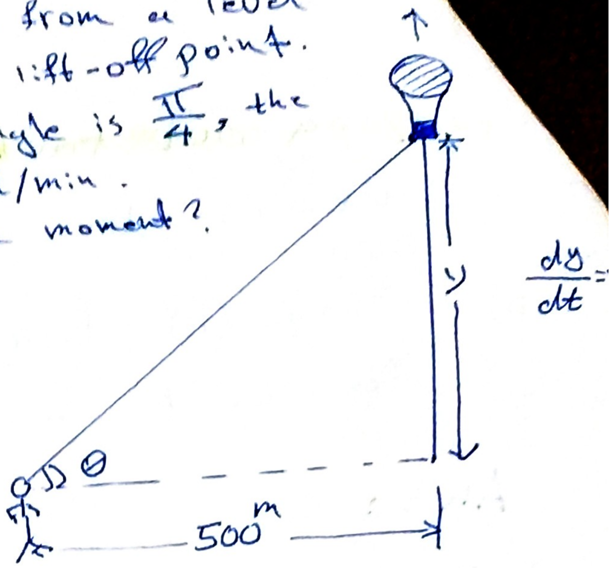
$\dfrac{3}{\pi 25} t = 10$

$t = \dfrac{250\pi}{3} = 261.8$ min

علي قاسم
كرار جسم

**A rising ballon**

EX: A hot-air ballon rising straight up from a level feild is tracked by a ranger 500 m from lift-off point. At the moment the ranger elevation angle is $\frac{\pi}{4}$, the angle is increasing at rate of 0.14 rad/min. How fast is the balloon rising at the moment?




$\frac{dy}{dt} = ?$

Ans:

$\frac{d\theta}{dt} = 0.14$ rad/min , $\theta = \frac{\pi}{4}$

$\tan\theta = \frac{y}{500}$

$y = 500 \tan\theta$

$\frac{dy}{d\theta} = 500 \sec^2\theta \Rightarrow dy = 500 \sec^2\theta \, d\theta$

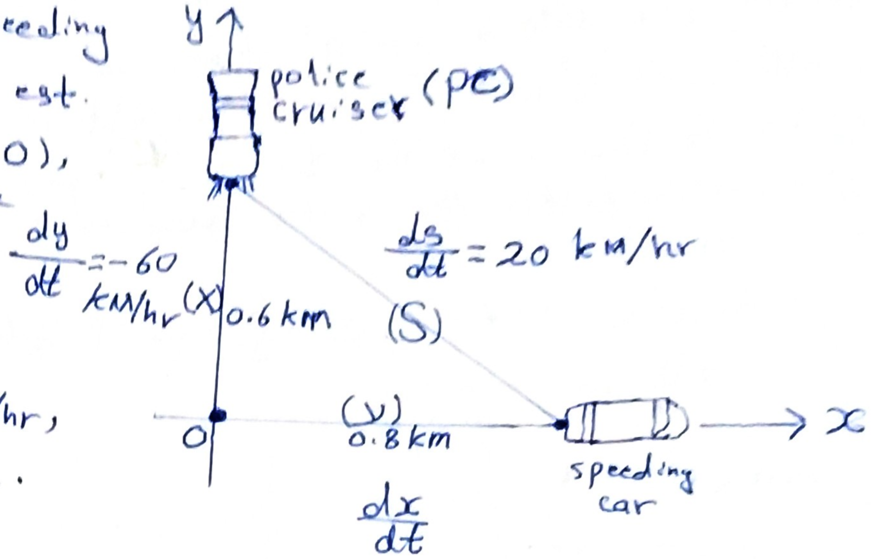
$\frac{dy}{dt} = 500 \sec^2\theta \frac{d\theta}{dt}$ $\quad \left[\frac{d\theta}{dt} = 0.14 \frac{rad}{min}, \text{ when } \theta = \frac{\pi}{4}\right]$

$= 500^m (\sqrt{2})^2 * 0.14$ rad/min $\quad \left[\sec\frac{\pi}{4} = \sqrt{2}\right]$

$= 140$ m/min

# Highway chase مطاردة الطريق السريع

polic cruiser is chasing a speeding car which is now moving straight est. When PC at 0.6 KM north the (O), the police radar determine the distance betwen them and the car is increasing 20 KM/hr. If the cruiser is moving at 60 KM/hr, What the speed of the Care.




Ans:

By Pythagoras" theorem

$$S^2 = x^2 + y^2$$ , derive with respect to (t), t: time

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \quad — (1)$$

at the moment of chasing

$\frac{ds}{dt} = 20$ km/hr (distance increasing)

$\frac{dy}{dt} = -60$ KM/hr (cruiser speed in negative direction)

$x = 0.6$ KM (cruiser axis above or north the intersection ●)

$y = 0.6$ km (car axis east ●)

$S = \sqrt{0.8^2 + 0.6^2} = 1$ km

sub in (1)

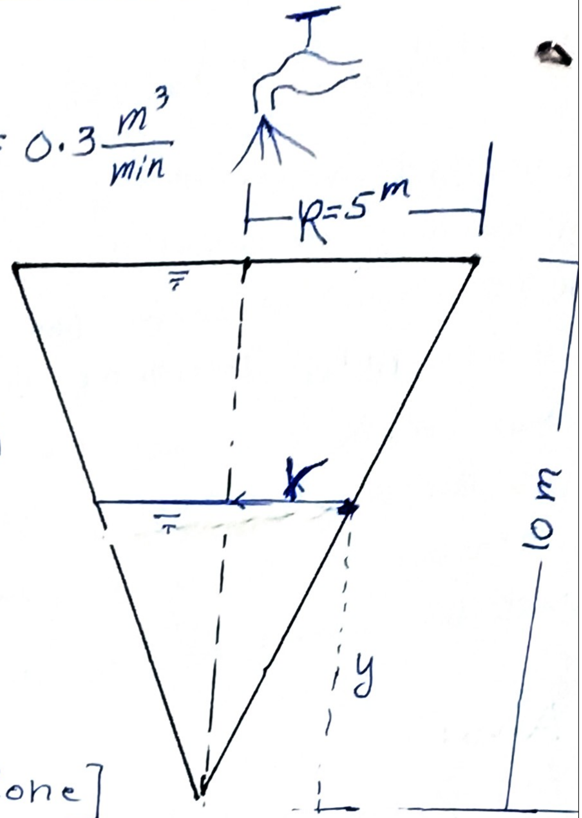
$$2 * 1 * 20\frac{KM}{hr} = 2 * 0.8 * \frac{dx}{dt} + 2 * 0.6 * (-60\frac{KM}{hr})$$

$$40 = 1.6\frac{dx}{dt} - 72$$

$$\therefore \frac{dx}{dt} = \frac{112}{1.6} = 70 \text{ KM}$$

# EX: Water tank

$$\frac{dv}{dt} = Q = 0.3 \frac{m^3}{min}$$




water run into a conial tank at a rate $0.3\ m^3/hr$. The tank height is 10 m. The radius of the tank base is 5m. 1- How fast is the water level rising up

2- what is the rising speed at height $y = 6\,m$

Ans:

1) $V = \frac{\pi}{3} r^2 y$ —① [Volume of Cone]

but $\frac{r}{R} = \frac{y}{10} \Rightarrow \frac{r}{5} = \frac{y}{10}$

$\therefore r = \frac{y}{2}$ sub in ①

$$V = \frac{\pi}{3} \left(\frac{y}{2}\right)^2 y$$

$$V = \frac{\pi}{12} y^3$$

derive with respect to time

$$\frac{dV}{dt} = \frac{\pi}{12} * 3y^2 \frac{dy}{dt}$$

but $\frac{dV}{dt} = Q = 0.3 \frac{m^3}{min}$

$$\therefore 0.3 = \frac{\pi}{4} y^2 \frac{dy}{dt}$$

$$\therefore \frac{dy}{dt} = \frac{1.2}{\pi y^2}$$

2) $\frac{dy}{dt} = \frac{1.2}{\pi(6)^2} = 0.001137\ m/min = 1.137\ mm/min = 68.2\ mm/hr$

$= 6.82\ cm/hr$

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