1.10 Differentiation and Integration of inverse trigonometric functions

$$\frac{d}{dx}\sin^{-1}u = \frac{\frac{du}{dx}}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}cos^{-1}u = \frac{-\frac{du}{dx}}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}tan^{-1}u = \frac{\frac{du}{dx}}{1+u^2}$$

$$\frac{d}{dx}cot^{-1}u = \frac{-\frac{du}{dx}}{1+u^2}$$

$$\frac{d}{dx}sec^{-1}u = \frac{\frac{du}{dx}}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}csc^{-1}u = \frac{-\frac{du}{dx}}{|u|\sqrt{u^2-1}}$$

Example 1: find $\bar{y}$. y=tan$^{-1}(x\ \sin x)$

Solution//

$$\bar{y} = \frac{1}{1+(x\,sin\,x)^2} * [x * cos\,x + sin\,x * 1]$$

$$\bar{y} = \frac{x * cos\,x + sin\,x}{1+(x\ \ sin\,x)^2}$$

Example 2: find $\bar{y}$. $y = \sin^{-1}\sqrt{x}\ \ sec^{-1}\ (sin\ x)$

Solution //

$$\bar{y} = \sin^{-1}\sqrt{x} * (\frac{cos\ x}{|sin\ x|\sqrt{sin^2 x - 1}}$$

$$+ sec^{-1}\ (sin\ x) * \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1 - (\sqrt{x})^2}}$$

Integration of inverse

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + c$$

$$\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + c$$

$$\int \frac{-du}{a^2 + u^2} = \frac{1}{a}\cot^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + c$$

$$\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\csc^{-1}\frac{u}{a} + c$$

Example 1: $\boldsymbol{Find\ \int \frac{dx}{\sqrt{9-x^2}}}$

Solution //

$$\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{3^2-x^2}} = \sin^{-1}\frac{x}{3} + c$$

Example 2: $\boldsymbol{Find\ \int \frac{-dx}{\sqrt{4-(x-2)^2}}}$

Solution //

$$\int \frac{-dx}{\sqrt{4-(x-2)^2}} = \int \frac{-dx}{\sqrt{2^2-(x-2)^2}} = \cos^{-1}\frac{x-2}{2} + c$$

1.11 Differentiation and Integration of hyperbolic functions

hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

<u>Example:</u> prove that $\sinh 2x$=2 $\sinh x \cosh x$

<u>Solution //</u>

$$2\ \sinh x \cosh x = 2 * \frac{e^x - e^{-x}}{2} * \frac{e^x + e^{-x}}{2}$$

$$= \frac{1}{2}(e^x - e^{-x}).(e^x + e^{-x})$$

$$= \frac{1}{2}(e^{2x} + e^0 - e^0 - e^{-2x}) = \frac{e^{2x} - e^{-2x}}{2} = \sinh 2x$$

Exercises 1: prove that $\cosh^2 x$- $\sinh^2 x$ =1

2: prove that $\cosh x + \sinh x = e^x$

Differentiation of hyperbolic functions

$$\frac{d}{dx}(\sinh ax) = a\cosh ax$$

$$\frac{d}{dx}(\cosh ax) = a\sinh ax$$

$$\frac{d}{dx}(\tanh ax) = a\,\mathrm{sech}^2\,ax$$

$$\frac{d}{dx}(\mathrm{csch}\,ax) = -a\,\mathrm{csch}\,ax\coth ax$$

$$\frac{d}{dx}(\mathrm{sech}\,ax) = -a\,\mathrm{sech}\,ax\tanh ax$$

$$\frac{d}{dx}(\coth ax) = -a\,\mathrm{csch}^2\,ax$$

Integration of hyperbolic functions

$$\int \sinh x\,dx = \cosh x + c$$

$$\int \cosh x\,dx = \sinh x + c$$

$$\int \tanh x\,dx = \ln(\cosh x) + c$$

$$\int \coth x\,dx = \ln|\sinh x| + c$$

$$\int \operatorname{sech} x\,dx = \tan^{-1}|\sinh x| + c$$

$$\int \operatorname{csch} x\,dx = \ln\left|\tanh\frac{x}{2}\right| + c$$