

# كلية العلـوم قـســـم الانـظــمـة الـطبية الـذكـيــة 

## Lecture: ( 4 )

Subject: Statistics and probability
Class: Second
Lecturer: Asst. Lecturer Nabaa Ali

## CHAPTER THREE

## Data Description

3-1 Measures of Central Tendency

## 3-2 Measures of Variation

Measures of Central Tendency: Summarize data, using measures of central tendency, such as the mean, median, mode, and midrange.

Measures of Variation: Describe data, using measures of variation, such as the range, variance, and standard deviation.

| Measures of Central Tendency |  |  |
| :--- | :--- | :---: |
| A statistic is a characteristic or | A parameter is a characteristic <br> measure obtained by using the <br> or measure obtained by using <br> data values from a sample. |  |
| all the data values for a a <br> specific population. |  |  |

## Mean

- The mean is the quotient of the sum of the values and the total number of values.
- The symbol $\bar{X}$ is used for sample mean.

$$
\bar{X}=\frac{X_{1}+X_{2}+X_{3}+\cdots+X_{n}}{n}=\frac{\sum X}{n}
$$

- For a population, the Greek letter $\mu(\mathrm{mu})$ is used for the mean.

$$
\mu=\frac{X_{1}+X_{2}+X_{3}+\cdots+X_{N}}{N}=\frac{\sum X}{N}
$$

- The mean is the sum of the values, divided by the total number of values.

| $\bar{c} \bar{x}$ | $\mu$ |
| :--- | :--- |
| The symbol for the <br> sample mean: | The symbol for the <br> population mean: |
| $\bar{x}=\frac{\sum x}{n}=\frac{X_{1}+X_{2}+X_{3}+\ldots \ldots \ldots+X_{n}}{n}$ | $\mu=\frac{\sum x}{N}=\frac{X_{1}+X_{2}+X_{3}+\ldots \ldots+X_{n}}{N}$ |
| Where n: no. of val. In <br> sample. | Where N: no. of val. In <br> population. |

## Examples

## Days off per Year

The data represent the number of days off per year for a sample of individuals selected from nine different countries. Find the mean.

$$
\begin{aligned}
& 20,26,40,36,23,42,35,24,30 \\
& \bar{X}=\frac{X_{1}+X_{2}+X_{3}+\cdots+X_{n}}{n}=\frac{\sum X}{n} \\
& \bar{X}=\frac{20+26+40+36+23+42+35+24+30}{9}=\frac{276}{9}=30.7
\end{aligned}
$$

The mean number of days off is 30.7 years.

## Police Incidents

The number of calls that a local police department responded to for a sample of 9 months is shown. Find the mean.

$$
\begin{aligned}
& 475,447,440,761,993,1052,783,671,621 \\
& \begin{aligned}
\bar{X} & =\frac{\sum x}{n}=\frac{475+447+440+761+993+1052+783+671+621}{9} \\
& =\frac{6243}{9} \approx 693.7
\end{aligned}
\end{aligned}
$$

## Finding the Mean for Grouped Data

Step 1 Make a table as shown

| A | B | C | D |
| :---: | :---: | :---: | :---: |
| Class | Frequency $f$ | Midpoint $X_{m}$ | $f \cdot X_{m}$ |

Step 2 Find the midpoints of each class and place them in column C.
Step 3 multiply the frequency by the midpoint for each class, and place the product in column D.

Step 4 Find the sum of column D.
Step 5 Divide the sum obtained in column D by the sum of frequencies obtained in column B.

The formula for the mean is

$$
\bar{X}=\frac{\sum f \cdot X_{m}}{n}
$$

## Miles Run per Week

Using the frequency distribution for Example below, find the mean. The data represent the number of miles run during one week for a sample of 20 runners.

## Solution

The procedure for finding the mean for grouped data is given here.
Step 1 Make a table as shown.

| A <br> Class | B <br> Frequency $\boldsymbol{f}$ | C <br> Midpoint $\boldsymbol{X}_{\boldsymbol{m}}$ | $\boldsymbol{f} \cdot \boldsymbol{X}_{\boldsymbol{m}}$ |
| :---: | :---: | :---: | :---: |
| $5.5-10.5$ | 1 |  |  |
| $10.5-15.5$ | 2 |  |  |
| $15.5-20.5$ | 3 |  |  |
| $20.5-25.5$ | 5 |  |  |
| $25.5-30.5$ | 4 |  |  |
| $30.5-35.5$ | 3 |  |  |
| $35.5-40.5$ | $n=\underline{2}$ |  |  |
|  | $n=20$ |  |  |

Step 2 Find the midpoints of each class and enter them in column C

$$
X_{m}=\frac{5.5+10.5}{2}=8 \quad \frac{10.5+15.5}{2}=13 \quad \text { etc. }
$$

Step 3 For each class, multiply the frequency by the midpoint, as shown, and place the product in column D.
$1 * 8=8 \quad 2 * 13=26 \quad$ etc. The completed table is shown here.

| A <br> Class | B <br> Frequency $\boldsymbol{f}$ | $\mathbf{C}$ <br> Midpoint $\boldsymbol{X}_{\boldsymbol{m}}$ | $\mathbf{D}$ <br> $\boldsymbol{f} \cdot \boldsymbol{X}_{\boldsymbol{m}}$ |
| :---: | :---: | :---: | :---: |
| $5.5-10.5$ | 1 | 8 | 8 |
| $10.5-15.5$ | 2 | 13 | 26 |
| $15.5-20.5$ | 3 | 18 | 54 |
| $20.5-25.5$ | 5 | 23 | 115 |
| $25.5-30.5$ | 4 | 28 | 112 |
| $30.5-35.5$ | 3 | 33 | 99 |
| $35.5-40.5$ | $n=\underline{2}$ | 38 |  |
|  | $n=$ |  | $\Sigma f \cdot X_{m}=\underline{76}$ |

Step 4 Find the sum of column D.
Step 5 Divide the sum by n to get the mean.

$$
\bar{X}=\frac{\sum f \cdot X_{m}}{n}=\frac{490}{20}=24.5 \mathrm{miles}
$$

## MEDIAN

The median is the midpoint of the data array. The symbol for the median is MD.

## Finding the median

Step 1 Arrange the data values in ascending order.
Step 2 determine the number of values in the data set.
Step 3 a. If $n$ is odd, select the middle data value as the median.
b. If $n$ is even, find the mean of the two middle values. That is, add them and divide the sum by 2 .

## Examples

## Police Officers Killed

The number of police officers killed in the line of duty over the last 11 years is shown. Find the median.

177153122141189155162165149157240
Sort in ascending order
$122,141,149,153,155,157,162,165,177,189,240$
Select the middle value.
MD $=157$

$$
\text { The median is } 157 \text { rooms. }
$$

## Tornadoes in the U.S.

The number of tornadoes that have occurred in the United States over an 8-year period follows. Find the median.

684, 764, 656, 702, 856, 1133, 1132, 1303
Find the average of the two middle values.
$656,684,702,764,856,1132,1133,1303$

$$
\mathrm{MD}=\frac{764+856}{2}=\frac{1620}{2}=810
$$

## The median number of tornadoes is 810 .

## The Mode

- The mode is the value that occurs most often in a data set.
- It is sometimes said to be the most typical case.
- There may be no mode, one mode (unimodal), two modes (bimodal), or many modes (multimodal).


## Example

NFL Signing Bonuses
Find the mode of the signing bonuses of eight NFL players for a specific year. The bonuses in millions of dollars are
$18.0,14.0,34.5,10,11.3,10,12.4,10$
You may find it easier to sort first.
$10,10,10,11.3,12.4,14.0,18.0,34.5$
Select the value that occurs the most.
The mode is 10 million dollars.

## Licensed Nuclear Reactors

The data show the number of licensed nuclear reactors in the United States for a recent 15 -year period. Find the mode.

$$
\begin{aligned}
& 104104104104104107109109109110 \\
& 109111112111109
\end{aligned}
$$

104 and 109 both occur the most. The data set is said to be bimodal.
The modes are 104 and 109.

## Miles Run per Week

Find the modal class for the frequency distribution of miles that 20 runners ran in one week.

| Class | Frequency |
| :---: | :---: |
| $5.5-10.5$ | 1 |
| $10.5-15.5$ | 2 |
| $15.5-20.5$ | 3 |
| $20.5-25.5$ | 5 |
| $25.5-30.5$ | 4 |
| $30.5-35.5$ | 3 |
| $35.5-40.5$ | 2 |

The modal class is 20.5-25.5.

The mode, the midpoint of the modal class, is 23 miles per week.

## Area Boat Registrations

The data show the number of boats registered for six counties in southwestern Pennsylvania. Find the mode.

| Westmoreland | 11,008 |
| :--- | ---: |
| Butler | 9,002 |
| Washington | 6,843 |
| Beaver | 6,367 |
| Fayette | 4,208 |
| Armstrong | 3,782 |

Since the category with the highest frequency is Westmoreland, the most typical case is Westmoreland. Hence, the mode is Westmoreland.

