جامعة المستقبل
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قسم هندسة تقنيات الاجهزة الطبية
Course: Digital Signal Processing

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Lecture 19 & 20

Design of FIR Filter

Design of FIR Filter

19th&20th modular units

1/ Overview

1 / A –Target population :-

For students of third class

Department of Medical Instrumentation Eng. Techniques

1 / B –Rationale :-

This unit introduces principles of the finite impulse response (FIR) filter design and investigates the design methods such as the Fourier transform method and window method.

1 / C –Central Idea :-

In this unit, we describe techniques of designing finite impulse response (FIR) filters. The major topics discussed in this unit are included in the following outline.

- Finite Impulse Response (FIR) Filter Format
- Fourier Transform Design
- Window Method

2/ Performance Objectives :-

After studying the 10th modular unit , the student will be able to:-

1. Define FIR filter .
2. Know the methods of Designing FIR filter.
3. Analyze the frequency response of FIR filter .

3/ Pre test :-

Circle the correct answer :-

1. **Digital filter design is the process of:-**
   a- Deriving Impulse Response.
   b- Deriving step response.
   c- Deriving Transfer function.
   d- Deriving frequency response.

2. **The required magnitude in the pass band is approximately:-**
   a- Unity   b- Zero
   c- Between zero & unity   d- Random

3. **The pass band and stop band edge frequencies, in most applications are specified in:-**

a- Per unit   b- Hertz

c- Desible   d- Sampling rate

4. **Optimum FIR filter design is the process of:-**

a- Determining optimum coefficients with minimum no. of taps.

b- Determining edge frequencies.

c- Determining pass band magnitude.

d- Determining stop band magnitude.

4/ the text :-

Finite Impulse Response (FIR) Filter Format

In this section, we describe techniques of designing finite impulse response (FIR) filters. An FIR filter is completely specified by the following input-output relationship:

$$y[n] = \sum_{i=0}^{N} b_i x[n-i]$$
$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \cdots + b_N x[n-N]$$

where b_i represents FIR filter coefficients and $N+1$ denotes the FIR filter length. Applying the z-transform on both sides of the above Equation leads to

$$Y(Z) = b_0 X(Z) + b_1 Z^{-1} X(Z) + b_2 Z^{-2} X(Z) + \cdots + b_N Z^{-N} X(Z)$$

$$H(Z) = \frac{Y(Z)}{X(Z)} = b_0 + b_1 Z^{-1} + \cdots + b_N Z^{-N}$$

Example

Given the following FIR filter:

$$y[n] = 0.1x[n] + 0.25x[n-1] + 0.2x[n-2]$$

Determine the transfer function, filter length, nonzero coefficients, and impulse response.

Solution:

Applying z-transform on both sides of the difference equation yields

$$Y(z) = 0.1X(z) + 0.25X(z)z^{-1} + 0.2X(z)z^{-2}.$$

Then the transfer function is found to be

$$H(z) = \frac{Y(z)}{X(z)} = 0.1 + 0.25z^{-1} + 0.2z^{-2}.$$

The filter length is $N + 1 = 3$, and the identified coefficients are

$$b_o = 0.1, \quad b_1 = 0.25, \quad b_2 = 0.2$$

Taking the inverse z-transform of the transfer function, we have

$$h[n] = 0.1\delta[n] + 0.25\delta[n-1] + 0.2\delta[n-2]$$

Fourier Transform Design

We begin with an ideal low pass filter with a normalized cutoff frequency Ω_c, whose magnitude frequency response in terms of the normalized digital frequency Ω is plotted in the Figure below and is characterized by
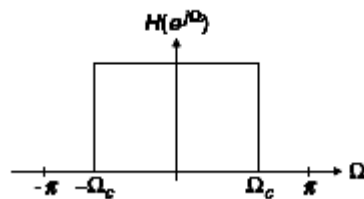
$$H(e^{j\Omega}) = \begin{cases} 1, & 0 \le |\Omega| \le \Omega_c \\ 0, & \Omega_c \le |\Omega| \le \pi. \end{cases}$$

Since the frequency response is periodic with a period of $\Omega = 2\pi$ radians, we can extend the frequency response of the ideal filter $H(e^{j\Omega})$, as shown in the Figure below.
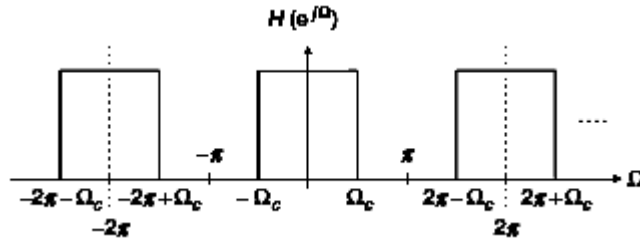
The desired impulse response of the ideal filter is

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} \quad for \quad -\infty < n < \infty$$

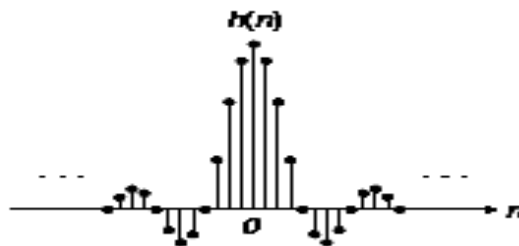The desired impulse response approximation of the ideal lowpass filter is solved as

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega \times 0} d\Omega \qquad for\ n = 0$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\Omega}) e^{j\Omega n} d\Omega \qquad for\ n \neq 0$$

$$= \frac{1}{2\pi} \int_{-\Omega_c}^{\Omega_c} H(e^{j\Omega}) e^{j\Omega n} d\Omega = \frac{e^{j\Omega n}}{2\pi jn}\bigg|_{-\Omega_c}^{\Omega_c} = \frac{1}{\pi n} \frac{e^{jn\Omega_c} - e^{-jn\Omega_c}}{2j}$$

$$= \frac{\sin(\Omega_c n)}{\pi n}$$

The desired impulse response h(n) is plotted versus the sample number n in the Figure below

Now, let us look at the possible z -transfer function. If we substitute $e^{j\Omega} = Z$, we yield a z-transfer function in the following format:

$$H(Z) = \sum_{n=-\infty}^{\infty} h[n] Z^{-n}$$

$$= \cdots + h[-2]Z^2 + h[-1]Z + h[0] + h[1]Z^{-1} + h[2]Z^{-2} + \cdots$$

The obtained filter is a non causal z-transfer function of the FIR filter, since the filter transfer function contains terms with positive powers of z, which in turn means that the filter output depends on the future filter inputs. To remedy the noncausal z-transfer function, we delay the truncated impulse response $h[n]$ by N samples to yield the following causal FIR filter:

$$H(Z) = b_0 + b_1 Z^{-1} + \cdots + b_{2N} Z^{-2N}$$

Where

$$b_n = h[n - N] \qquad for \ n = 0, 1, \ldots, 2N$$

Similarly, we can obtain the design equations for other types of FIR filters, such as highpass, bandpass, and bandstop, according to the following table

Table 6.1

Filter Type	Ideal Impulse response $h[n]$(non causal FIR coefficients)	
Low Pass	$h[n] = \begin{cases} \dfrac{\Omega_c}{\pi} & n = 0 \\ \dfrac{\sin(\Omega_c n)}{\pi n} & -N \le n \le N \end{cases}$	
High Pass	$h[n] = \begin{cases} \dfrac{\pi - \Omega_c}{\pi} & n = 0 \\ -\dfrac{\sin(\Omega_c n)}{\pi n} & -N \le n \le N \end{cases}$	
Band Pass	$h[n] = \begin{cases} \dfrac{\Omega_H - \Omega_L}{\pi} & n = 0 \\ \dfrac{\sin(\Omega_H n)}{\pi n} - \dfrac{\sin(\Omega_L n)}{\pi n} & -N \le n \le N \end{cases}$	
Band Stop	$h[n] = \begin{cases} \dfrac{\pi - \Omega_H + \Omega_L}{\pi} & n = 0 \\ -\dfrac{\sin(\Omega_H n)}{\pi n} + \dfrac{\sin(\Omega_L n)}{\pi n} & -N \le n \le N \end{cases}$	

Example

a) Calculate the filter coefficients for a 3-tap FIR **low pass** filter with a cutoff frequency of 800 Hz and a sampling rate of 8,000 Hz using the Fourier transform method.
b) Determine the transfer function and difference equation of the designed FIR system.
c) Compute and plot the magnitude frequency response for :
$$\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, and \ \pi \ radians$$

Solution:

a. Calculating the normalized cutoff frequency leads to
$$\Omega_c = 2\pi f_c T_s = 2\pi \times \frac{800}{8000} = 0.2\pi \ rad$$

Since $2N + 1 = 3$ in this case, using the equation in Table 6.1

$$h[n] = \begin{cases} \dfrac{\Omega_c}{\pi} & n = 0 \\ \dfrac{\sin(\Omega_c n)}{\pi n} & -N \le n \le N \end{cases}$$

The computed filter coefficients via the previous expression are listed as:

$$h[0] = \frac{0.2\pi}{\pi} = 0.2$$
$$h[1] = \frac{\sin(0.2\pi)}{\pi} = 0.1871$$

Using the symmetry leads to
$$h[-1] = h[1] = 0.1871$$
Thus delaying h(n) by $N = 1$ sample
$$b_0 = h[0 - 1] = h[-1] = 0.1871$$
$$b_1 = h[1 - 1] = h[0] = 0.2$$
$$b_2 = h[2 - 1] = h[1] = 0.1871$$

b. The transfer function is achieved as
$$H(z) = 0.1871 + 0.2z^{-1} + 0.1871z^{-2}.$$

$$\frac{Y(z)}{X(z)} = H(z) = 0.1871 + 0.2z^{-1} + 0.1817z^{-2}.$$

$$Y(z) = 0.1871X(z) + 0.2z^{-1}X(z) + 0.1871z^{-2}X(z).$$

Applying the inverse z-transform on both sides, the difference equation is yielded as
$$y[n] = 0.1871x[n] + 0.2x[n-1] + 0.1871x[n-2]$$

c. The magnitude frequency response and phase response can be obtained by substituting $Z = e^{j\Omega}$ into $H(Z)$

$$H(e^{j\Omega}) = 0.1871 + 0.2e^{-j\Omega} + 0.1871e^{-j2\Omega}.$$

Factoring the term $e^{-j\Omega}$ and using the Euler formula $e^{jx} + e^{-jx} = 2\cos(x)$, we achieve

$$H(e^{j\Omega}) = e^{-j\Omega}\left(0.1871e^{j\Omega} + 0.2 + 0.1871e^{-j\Omega}\right)$$

$$= e^{-j\Omega}(0.2 + 0.3742\cos(\Omega))$$

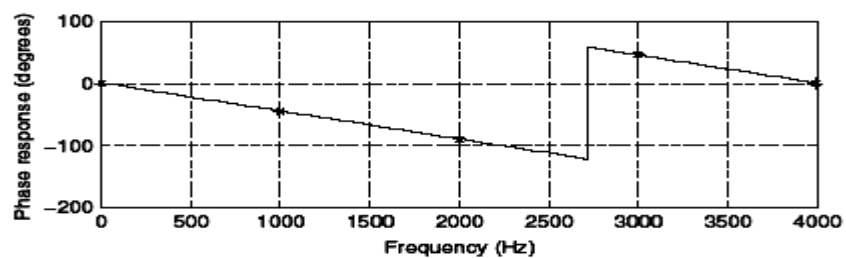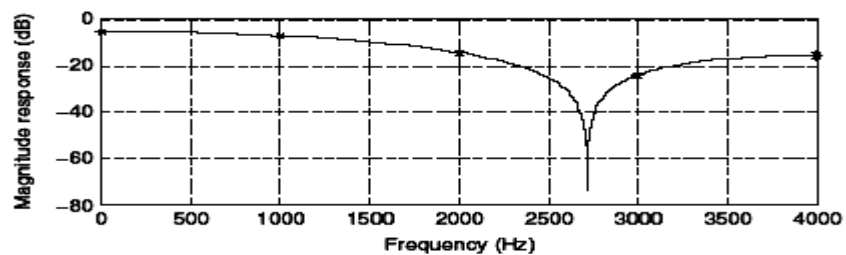Then the magnitude frequency response and phase response are found to be

$$\left|H(e^{j\Omega})\right| = |0.2 + 0.3472\cos\Omega|$$

and $\angle H(e^{j\Omega}) = \begin{cases} -\Omega & \text{if } 0.2 + 0.3472\cos\Omega > 0 \\ -\Omega + \pi & \text{if } 0.2 + 0.3472\cos\Omega < 0. \end{cases}$

Details of the magnitude calculations for several typical normalized frequencies are listed in Table 6.2.

Table 6.2

Ω radians	$f = \Omega f_s (2\pi)$ Hz	$0.2 + 0.3742\cos\Omega$	$\left|H(e^{j\Omega})\right|$	$\left|H(e^{j\Omega})\right|_{dB}$ dB	$\angle H(e^{j\Omega})$ degree
0	0	0.5742	0.5742	−4.82	0
$\pi/4$	1000	0.4646	0.4646	−6.66	−45
$\pi/2$	2000	0.2	0.2	−14.0	−90
$3\pi/4$	3000	−0.0646	0.0646	−23.8	45
π	4000	−0.1742	0.1742	−15.2	0

Next, we see that the 3-tap FIR filter does not give an acceptable magnitude frequency response. To explore this response further, Figure below displays the magnitude and phase responses of 3-tap ($N = 1$) and 17-tap ($N = 8$) FIR lowpass filters with a normalized cutoff frequency of ($\omega_c = 0.2\pi$) radian.

The calculated coefficients for the 17-tap FIR lowpass filter are listed in Table 6.3.
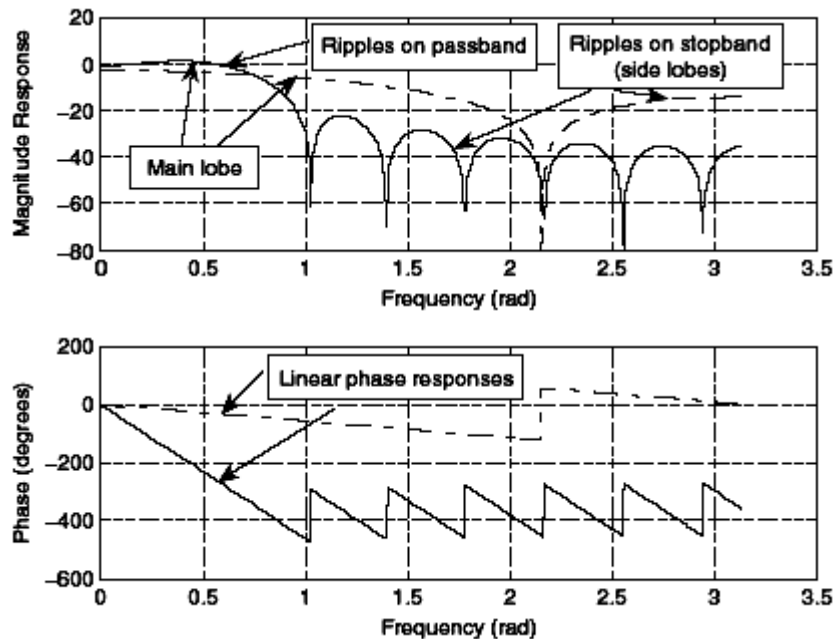
Table 6.3

$b_0 = b_{16} = -0.0378$	$b_1 = b_{15} = -0.0432$
$b_2 = b_{14} = -0.0312$	$b_3 = b_{13} = 0.0000$
$b_4 = b_{12} = 0.0468$	$b_5 = b_{11} = 0.1009$
$b_6 = b_{10} = 0.1514$	$b_7 = b_9 = 0.1871$ $b_8 = 0.200$

H.W

a. Calculate the filter coefficients for a 5-tap FIR bandpass filter with a lower cutoff frequency of 2,000 Hz and an upper cutoff frequency of 2,400 Hz at a sampling rate of 8,000 Hz using the Fourier transform method.

b. Determine the transfer function and difference equation of the designed FIR system.

c. Compute and plot the magnitude frequency response for :
$$\Omega = 0, \ \frac{\pi}{4} \ , \ \frac{\pi}{2} \ , \ \frac{3\pi}{4} \ , and \ \pi \ radians$$

Window Method

In this section, the window method (Fourier transform design with window functions) is developed to remedy the undesirable Gibbs oscillations in the passband and stopband of the designed FIR filter. Recall that Gibbs oscillations originate from the abrupt truncation of the infinite-length coefficient sequence. Then it is natural to seek a window function, which is symmetrical and can gradually weight the designed FIR coefficients down to zeros at both ends for the range of $-N \leq n \leq N$. Applying the window sequence to the filter coefficients gives

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$$h_w[n] = h[n].w[n]$$

where $w[n]$ designates the window function. Common window functions used in the FIR filter design are as follows:

1. Rectangular window:
$$w_{rec}[n] = 1 \qquad\qquad -N \leq n \leq N$$

2. Triangular (Bartlett) window:
$$w_{tri}[n] = 1 - \frac{n}{N} \qquad -N \leq n \leq N$$

3. Hanning window:
$$w_{han}[n] = 0.5 + 0.5\cos\left(\frac{n\pi}{N}\right) \qquad -N \leq n \leq N$$

4. Hamming window:
$$w_{ham}[n] = 0.54 + 0.46\cos\left(\frac{n\pi}{N}\right) \qquad -N \leq n \leq N$$

5. Blackman window:
$$w_{black}[n] = 0.42 + 0.5\cos\left(\frac{n\pi}{N}\right) + 0.08\cos\left(\frac{2n\pi}{N}\right) \qquad -N \leq n \leq N$$

The design procedure of the FIR filter via windowing is summarized as follows:

1. Obtain the FIR filter coefficients h(n) via the Fourier transform method (Table 6.1).
2. Multiply the generated FIR filter coefficients by the selected window sequence
$$h_w[n] = h[n]w[n], \qquad\qquad n = -N, ... , 0,1, ... , N$$
where w(n) is chosen to be one of the window functions listed in the above equations.

3. Delay the windowed impulse sequence $h_w[n]$ *by N* samples to get the windowed FIR filter coefficients:
$$b_n = h_w[n - N], \qquad\qquad n = 0,1, ... , 2N$$

Example
Design a 3-tap FIR lowpass filter with a cutoff frequency of 800 Hz and a sampling rate of 8,000 Hz using the Hamming window function.

Solution:

FIR coefficients obtained previously:

$$h[0] = \frac{0.2\pi}{\pi} = 0.2$$

$$h[-1] = h[1] = 0.1871$$

Applying the Hamming window function, we have

$$w_{ham}[0] = 0.54 + 0.46\cos\left(\frac{0\pi}{1}\right) = 1$$

$$w_{ham}[1] = 0.54 + 0.46\cos\left(\frac{1 \times \pi}{1}\right) = 0.08$$

Using the symmetry of the window function gives:

$$w_{ham}[-1] = w_{ham}[1] = 0.08$$

The window impulse response is calculated as

$$h_w[0] = h[0]w_{ham}[0] = 0.2 \times 1 = 0.2$$

$$h_w[1] = h[1]w_{ham}[1] = 0.1871 \times 0.08 = 0.01497$$

$$h_w[-1] = h[-1]w_{ham}[-1] = 0.1871 \times 0.08 = 0.01497$$

5/ Post test :-

Circle the correct answer:-

1- Gibbs oscillations originate from the:

a- Truncation of the infinite-length.

b- Sampling process.

c- Sharp cut – off frequency.

d- Realization process.

2- The number of common window function is:

a- Two.

b- Four.

c- Six.

d- Infinite.

3- The basic process in window design method is:

a- Subtracting h(n) from w(n).

b- Multiplying h(n) by w(n).

c- Adding h(n) by w(n).

d- Dividing w(n) by h(n).

6/ key answer :-

1- Pre test :-
1. c
2. a
3. b
4. a

2- Post test :-
1. a
2. c
3. b

7/References :-

1. Schaum's Outline of Theory and Problems of Digital Signal processing.

2. Digital signal processing, principles, algorithms, and applications by John G. Proakis and Dimitris G. Manolakis.

3. Signal and systems, Alan Oppenheim.