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Lecture: (6)

Subject: Statistics and probability
Class: Second
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probability theory

a branch of mathematics concerned with the analysis of random phenomena. The outcome of a random event cannot be determined before it occurs, but it may be any one of several possible outcomes. The actual outcome is considered to be determined by chance.

- The word *probability* has several meanings in ordinary conversation.
- One of them is the interpretation of probabilities as relative frequencies, for which simple games involving coins, cards, dice, and roulette wheels provide examples.

In probability theory, random experiment means a repeatable process that yields a result or an observation.

- Tossing a coin, rolling a die, extracting a ball from a box are random experiments.

When tossing a coin, we get one of the following elementary results:

(heads)*;* (tails)*:*

When throwing a die, if we denote by (1) the appearance of the face with one dot, with (2) the appearance of the face with two dots, etc., then we get the following elementary results:

(1)*;* (2)*;* (3)*;* (4)*;* (5)*;* (6)*:*

Sample Space

In the study of statistics, we are concerned basically with the presentation and interpretation of **chance outcomes** that occur in a planned study or scientific investigation.

We shall refer to any recording of information, whether it be numerical or categorical, as an observation.

Statisticians use the word **experiment** to describe any process that generates a set of data.

- The set of all possible outcomes of a statistical experiment is called the **sample space** and is represented by the symbol *S*.

Each outcome in a sample space is called an **element** or a **member** of the sample space, or simply a **sample point**. If the sample space has a finite number of elements, we may *list* the members separated by commas and enclosed in braces.
Thus, the sample space *S*, of possible outcomes when a coin is flipped, may be written

$$S = \{H, T\},$$

where *H* and *T* correspond to heads and tails, respectively.

Events

- For any given experiment, we may be interested in the occurrence of certain **events** rather than in the occurrence of a specific element in the sample space.
- To each event we assign a collection of sample points, which constitute a subset of the sample space.
- That subset represents all of the elements for which the event is true.
- **An event** is a subset of a sample space.

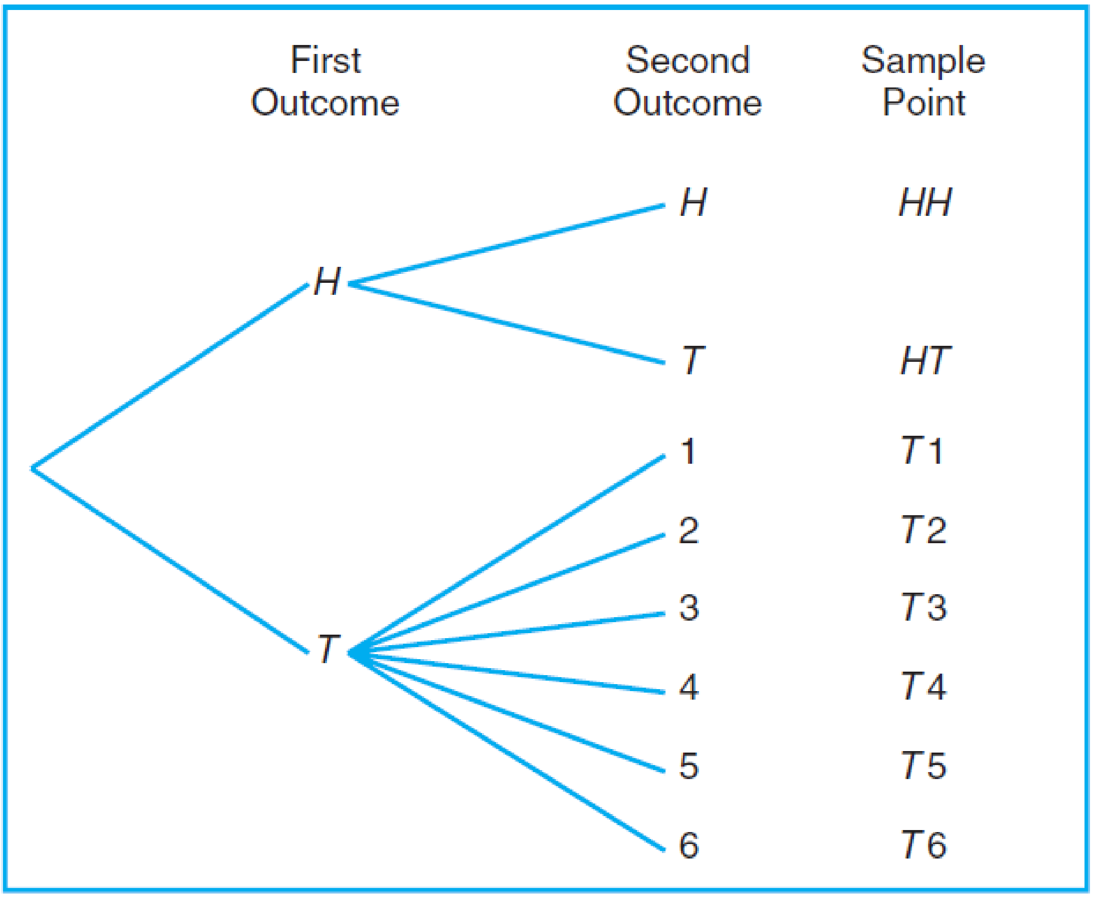
$$\textbf{NO. of outcomes} = N^n$$

Example 1: find the sample space for random experiment (flipping) a coin of two times?

$$S=\{HH,HT,TH,TT\}$$

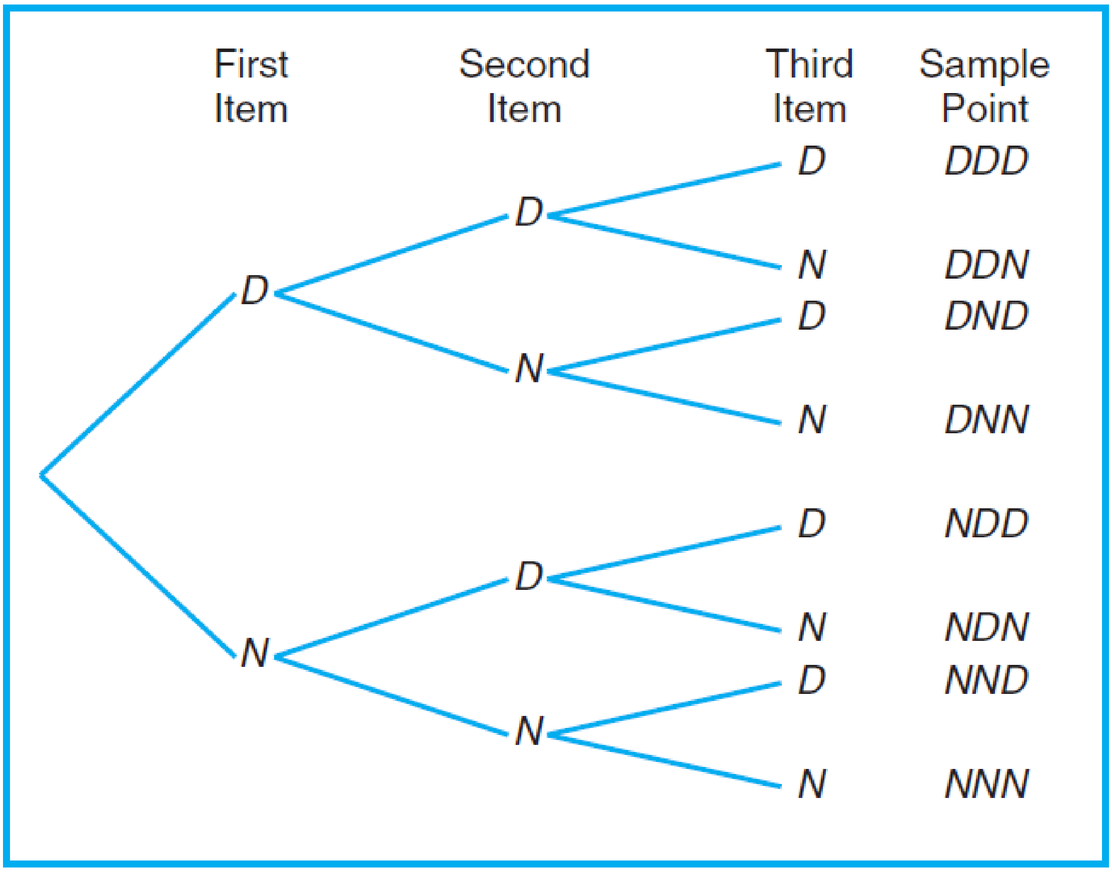
Example 2: An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once.

$$S = \{HH, HT, T1, T2, T3, T4, T5, T6\}.$$



Example 3: Suppose that three items are selected at random from a manufacturing process. Each item is inspected and classified defective, *D*, or non-defective, *N*. list the elements of the sample space providing the most information.

***S* = {DDD, DDN, DND, DNN, NDD, NDN, NND, NNN}.**



Example 4: Consider the experiment of tossing a die. If we are interested in the number that shows on the top face, the sample space is

$$S1 = \{1, 2, 3, 4, 5, 6\}.$$

If we are interested only in whether the number is even or odd, the sample space is simply

$$E1 = \{2,4,6\}.$$
$$E2 = \{1,3,5\}.$$

Example 5: a dice is rolled twice , what is the event that the sum of the faces greater than 7 , given that the first outcome was 4 ?

S={11,12,13,14,15,16,

21,22,23,24,25,26,

31,32,33,34,35,36,

41,42,43,44,45,46,

51,52,53,54,55,56,

61,62,63,64,65,66}

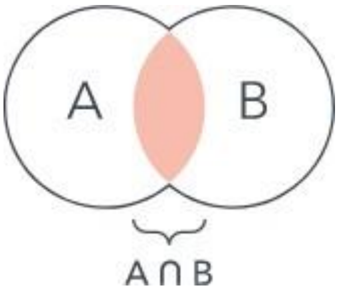
E={44,45,46}

Set operations on events

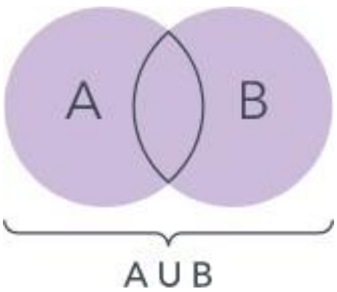
Rule 1(complement): The complement of an event *A* with respect to *S* is the subset of all elements of *S* that are not in *A*. We denote the complement of *A* by the symbol *A′*.

Rule 2(mutually exclusive): that is A and B have no outcomes in common

Rule 3 (intersection): The intersection of two events *A* and *B*, denoted by the symbol $A \cap B$, is the event containing all elements that are common to *A* and *B*.



Rule 4 (union): The union of the two events *A* and *B*, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to *A* or *B* or both.



Example 6: Consider an experiment where the smoking habits of the employees of a manufacturing firm are recorded. A possible sample space might classify an individual as a nonsmoker, a light smoker, a moderate

smoker, or a heavy smoker. Let the subset of smokers be some event. Then all the nonsmokers correspond to a different event, also a subset of S, which is called the **complement** of the set of smokers.

Example 7: Let R be the event that a red card is selected from an ordinary deck of 52 playing cards, and let S be the entire deck. Then $R\_$ is the event that the card selected from the deck is not a red card but a black card.

Example 8: Consider the sample space
$S = \{$book, cell phone, mp3, paper, stationery, laptop$\}$.
Let $A = \{$book, stationery, laptop, paper$\}$. Then the complement of A is $A' = \{$cell phone, mp3$\}$.

Example 9: Let $V = \{a, e, i, o, u\}$ and $C = \{l, r, s, t\}$; then it follows that $V \cap C = \varphi$. That is, V and C have no elements in common and, therefore, cannot both simultaneously occur. The events A and B are then said to be **mutually exclusive**.

Example 10: Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$; then

$A \cup B = \{a, b, c, d, e\}$.

Example 11:in the tossing of die , we might let A be the event that an even number occurs and B the event that an number greater than 3 shows.

S={1,2,3,4,5,6}

A={2,4,6}   B={4,5,6}

$A \cap B=\{4,6\}$

$A \cup B=\{2,4,5,6\}$

$A'=\{1,3,5\}$

$B'=\{1,2,3\}$

What is a Venn Diagram?

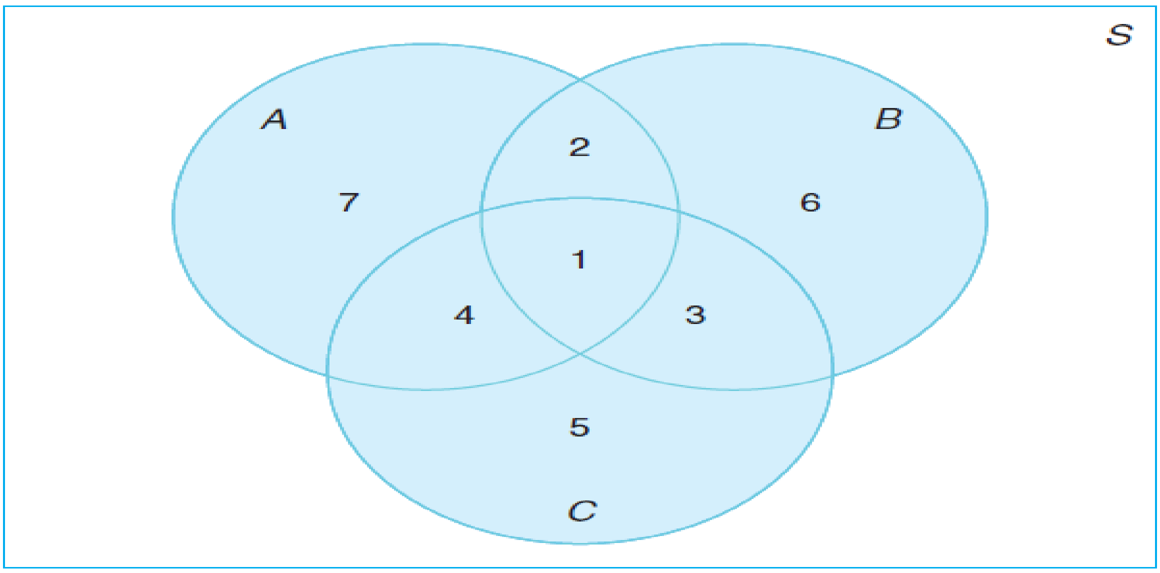
A Venn diagram uses overlapping circles or other shapes to illustrate the logical relationships between two or more sets of items. Often, they serve to graphically organize things, highlighting how the items are similar and different.

They are used to think through and depict how items relate to each within a particular "universe" or segment. Venn diagrams allow users to visualize data in clear, powerful ways, and therefore are commonly used in presentations and reports. Venn diagrams show relationships even if a set is empty.

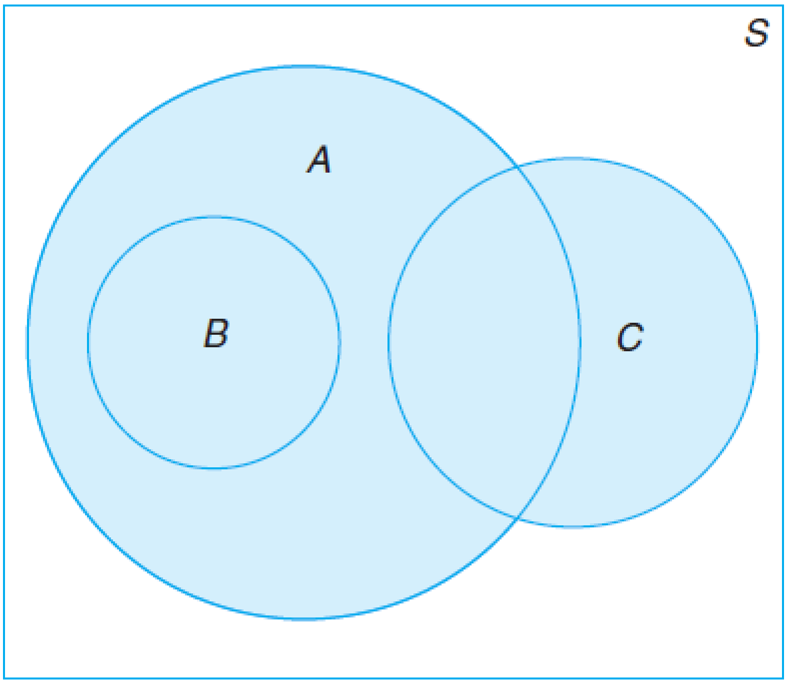
Example 12: The relationship between events and the corresponding sample space can be illustrated graphically by means of **Venn diagrams**. In a Venn diagram we let the sample space be a rectangle and represent events by circles drawn inside the rectangle. Thus, in Figure 2.3, we see that

$$A \cap B = \text{regions 1 and 2},$$

$$B \cap C = \text{regions 1 and 3}$$



$$\begin{aligned} A \cup C &= \text{regions 1, 2, 3, 4, 5, and 7,} \\ B' \cap A &= \text{regions 4 and 7,} \\ A \cap B \cap C &= \text{region 1,} \\ (A \cup B) \cap C' &= \text{regions 2, 6, and 7,} \end{aligned}$$



we see that events A, B, and C are all subsets of the sample space S. It is also clear that event B is a subset of event A; event $B \cap C$ has no elements and hence B and C are mutually exclusive; event $A \cap C$ has at least one element; and event A □ $B = A$, depict a situation where we select a card at random from an ordinary deck of 52 playing cards and observe whether the following events occur:

A: the card is red,

B: the card is the jack, queen, or king of diamonds,

C: the card is an ace.

Clearly, the event $A \cap C$ consists of only the two red aces.

Several results that follow from the foregoing definitions, which may easily be verified by means of Venn diagrams, are as follows:

1. $A \cap \phi = \phi$.
2. $A \cup \phi = A$.
3. $A \cap A' = \phi$.
4. $A \cup A' = S$.
5. $S' = \phi$.
6. $\phi' = S$.
7. $(A')' = A$.
8. $(A \cap B)' = A' \cup B'$.
9. $(A \cup B)' = A' \cap B'$.