



3.4 Equilibrium of Coplanar General Force system

There are three equilibrium conditions that can be used for non-concurrent, non-parallel force system.

To achieve the equilibrium:

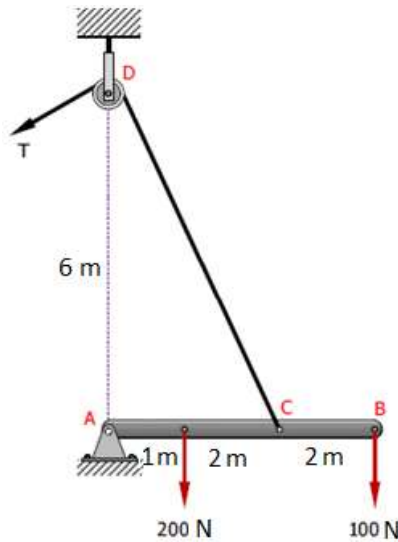
$$\sum F_x = 0 \dots\dots\dots (1)$$

$$\sum F_y = 0 \dots\dots\dots (2)$$

$$\sum M = 0 \dots\dots\dots (3)$$

Three unknowns can be determined.

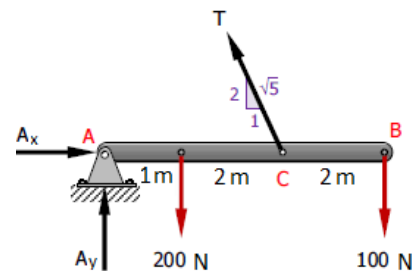
Example No. 1: A beam AB is supported in a horizontal position by a hinge A and a cable at C as shown in Figure. Compute the tension T in the cable and the reaction force at A.



Solution:

Draw F.B.D. for all beams

$$\sum M_A = 0$$



$$200 \times 1 + 100 \times 5 - T \times \frac{2}{\sqrt{5}} \times 3 = 0$$

$$T = 260.87 \text{ N} \quad \text{answer}$$

$$\uparrow^+ \sum F_y = 0$$

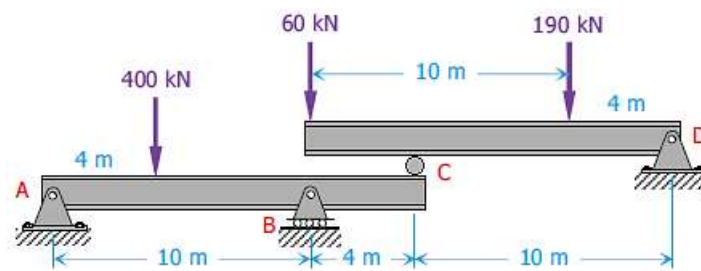
$$260.87 \times \frac{2}{\sqrt{5}} + A_y - 200 - 100 = 0$$

$$A_y = 66.67 \text{ N} \quad \uparrow \quad \text{answer}$$

$$\rightarrow^+ \sum F_x = 0$$

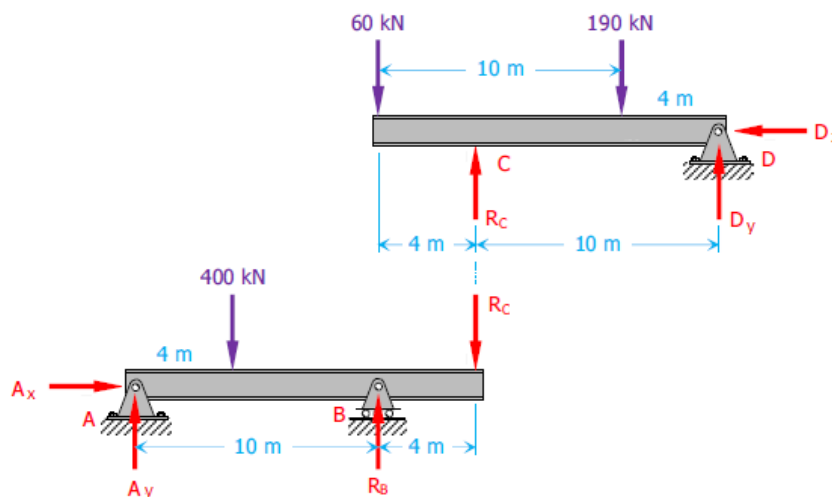
$$A_x - 260.87 \times \frac{1}{\sqrt{5}} = 0 \quad \Rightarrow \quad A_x = 116.66 \text{ N} \quad \rightarrow \quad \text{answer}$$

Example No. 2: Determine the values of the reactions at A, B, C, and D.



Solution:

Draw F.B.D. for all beams



At beam DC as F.B.D:

$$\sum M_D = 0$$

$$R_C \times 10 - 60 \times 14 - 190 \times 4 = 0$$

$$R_C = 160 \text{ kN} \quad \uparrow \text{ answer}$$

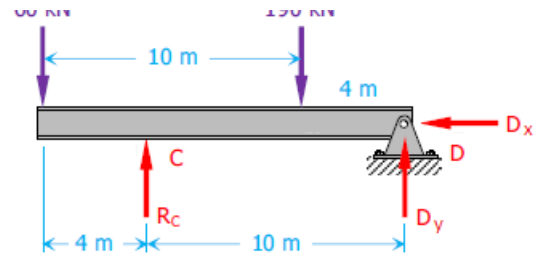
$$\sum F_x = 0$$

$$D_x = 0 \quad \text{answer}$$

$$\sum F_y = 0$$

$$160 + D_y - 60 - 190 = 0$$

$$D_y = 90 \text{ kN} \quad \uparrow \text{ answer}$$



At beam AB as F.B.D:

$$\sum M_A = 0$$

$$-R_B \times 10 + 400 \times 4 + 160 \times 14 = 0$$

$$R_B = 384 \text{ kN} \quad \uparrow \text{ answer}$$

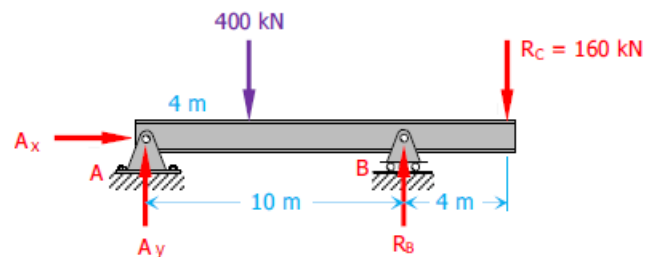
$$\sum F_x = 0$$

$$A_x = 0 \quad \text{answer}$$

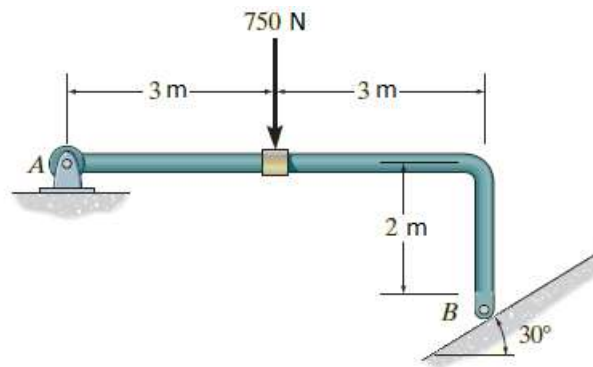
$$\sum F_y = 0$$

$$384 + A_y - 160 - 400 = 0$$

$$A_y = 176 \text{ kN} \quad \uparrow \text{ answer}$$



Example No. 3: Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at B in Figure.



Solution:

Draw F.B.D. for all beams

$$\sum M_A = 0$$

$$750 \times 3 + N_B \times \sin 30 \times 2 - N_B \times \cos 30 \times 6 = 0$$

$$N_B = 536.21 \text{ N} \quad \text{answer}$$

$$\sum F_x = 0$$

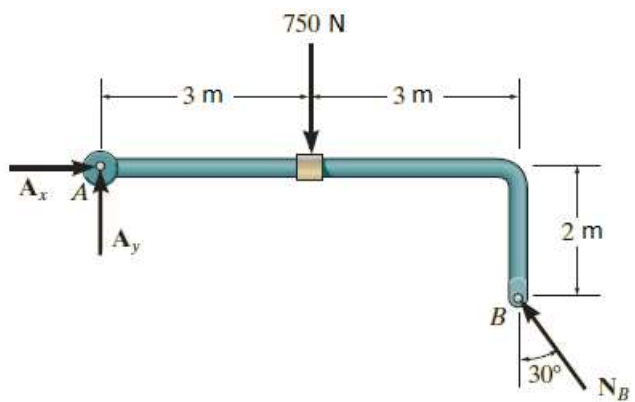
$$A_x - 536.21 \times \sin 30 = 0$$

$$A_x = 268.1 \text{ N} \quad \rightarrow \text{answer}$$

$$\sum F_y = 0$$

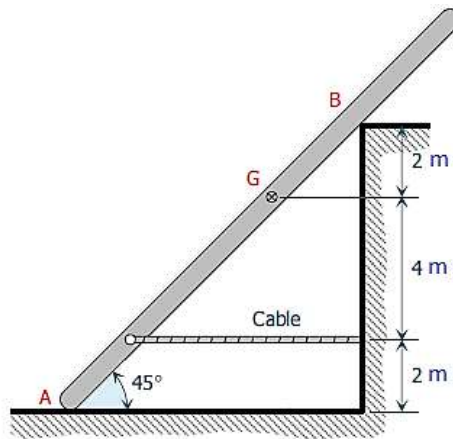
$$A_y - 750 + 536.21 \times \cos 30 = 0$$

$$A_y = 285.63 \text{ N} \quad \uparrow \text{answer}$$



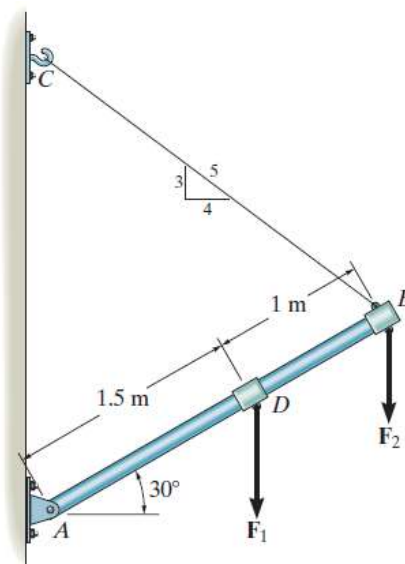
Problem:

1. The uniform member in Figure weighs 420 N and has its center of gravity at G. Determine the tension in the cable and the reactions at the smooth surfaces at A and B.



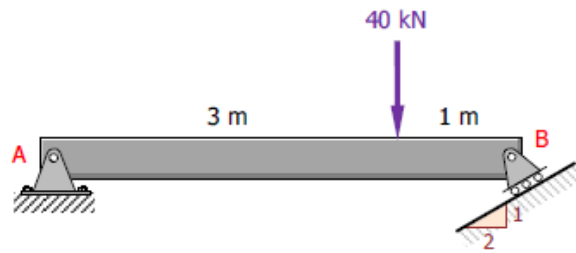
Answer: $R_A = 420\text{ N } \uparrow$, $T = 420\text{ N } \rightarrow$, $R_B = 420\text{ N } \leftarrow$

2. If the cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A.



Answer: $F_2 = 724\text{ N}$, $F_1 = 1450\text{ N}$, $A_x = 1200\text{ N } \rightarrow$, $A_y = 1274\text{ N } \uparrow$

3. The beam shown in Figure is supported by a hinge at A and a roller on a 1 to 2 slope at B. Determine the resultant reactions at A and B.



Answer: $R_B = 33.54 \text{ kN}$, $A_x = 15 \text{ kN} \rightarrow$, $A_y = 10 \text{ kN} \uparrow$