Chapter Four: Analysis of Structures

4.1 1 Analysis of Frames

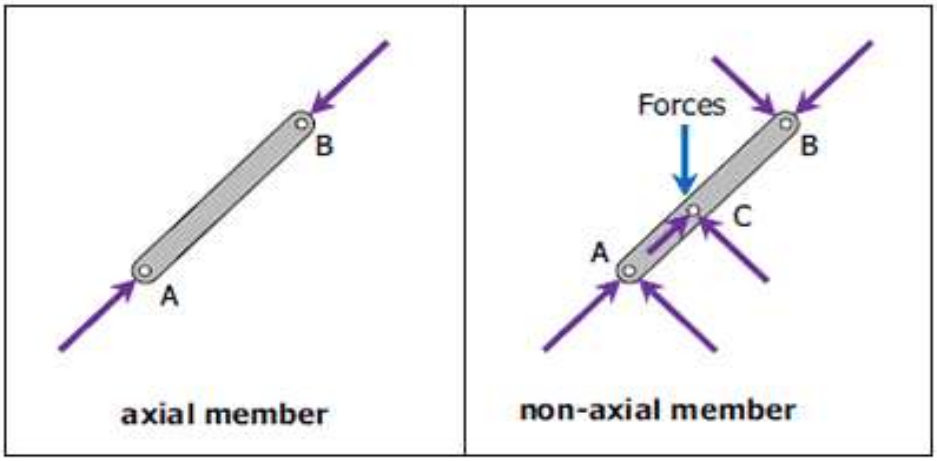
To analyze a frame, we can disconnect the member from the structure and draw the free-body diagram of the member. This approach is called the method of members. In this method, three equilibrium equations can be used:

$$\sum F_x = 0 \dots\dots\dots\dots\dots (1)$$

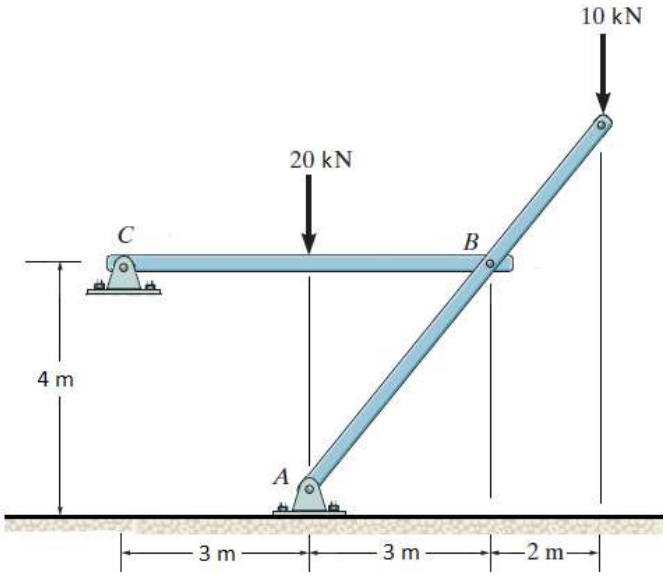
$$\sum F_y = 0 \dots\dots\dots\dots\dots (2)$$

$$\sum M = 0 \dots\dots\dots\dots\dots (3)$$

Below is a figure that shows the difference between axial and non-axial members.

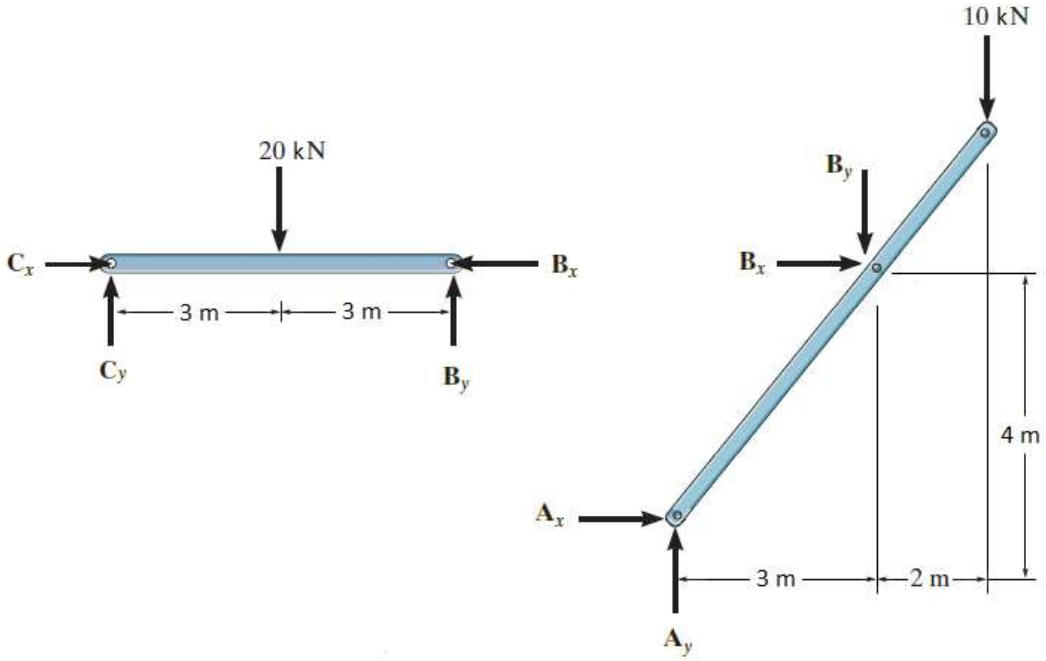


Example No. 1: For structure shown in Figure, Find the horizontal and vertical components of the hinge force at B, C, and A.

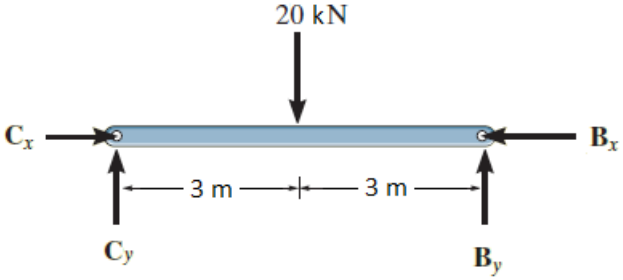


Solution:

Draw F.B.D. for all member



At member CB as F.B.D:



$$\circlearrowleft + \sum M_C = 0$$

$$20 \times 3 - B_y \times 6 = 0$$

$B_y = 10\ kN\ \uparrow$ *answer*

$\rightarrow^+ \sum F_x = 0$

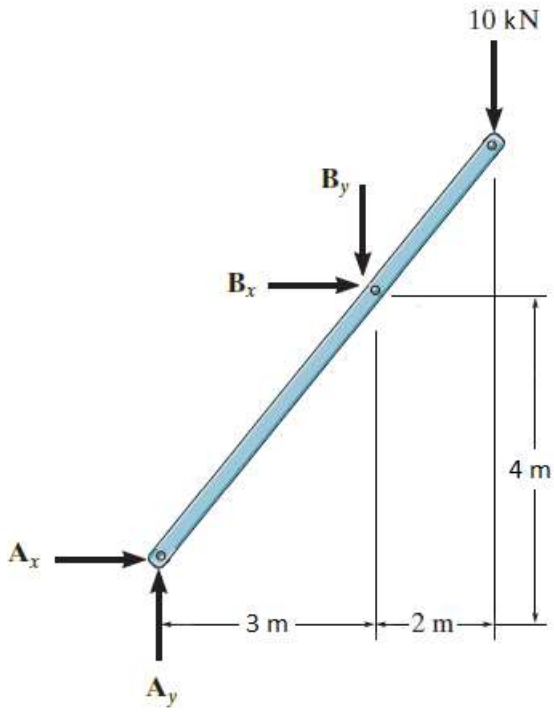
$C_x - B_x = 0$

$C_x = B_x \quad \ldots\ldots\ldots\ldots (1)$

$\uparrow^+ \sum F_y = 0$

$C_y - 20 + 10 = 0 \quad \Longrightarrow \quad C_y = 10\ kN\ \uparrow$ *answer*

At member AB as F.B.D:



$\circlearrowleft^+ \sum M_A = 0$

$10 \times 5 + 10 \times 3 + B_x \times 4 = 0$

$B_x = -20\ kN = 20\ kN\ \leftarrow$ *answer*

sub B_x in eq. (1): (in member CB)

$C_x = B_x = -20\ kN = 20\ kN\ \leftarrow$ *answer*

$\rightarrow^+ \sum F_x = 0$

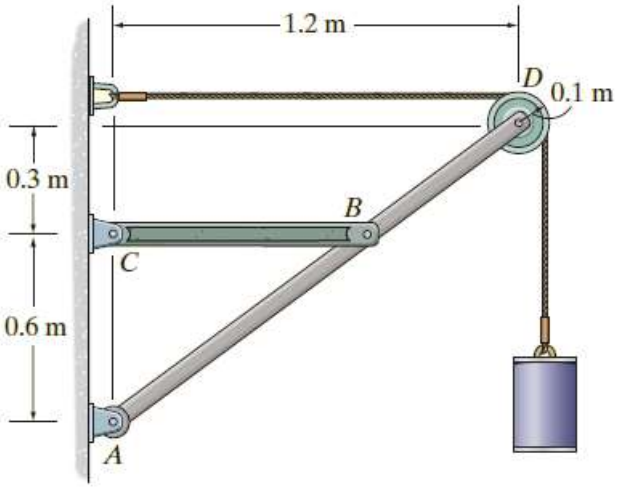
$A_x - 20 = 0 \quad \Longrightarrow \quad A_x = 20\ kN\ \rightarrow$ *answer*

$\uparrow^+ \sum F_y = 0$

$A_y - 10 - 10 = 0$

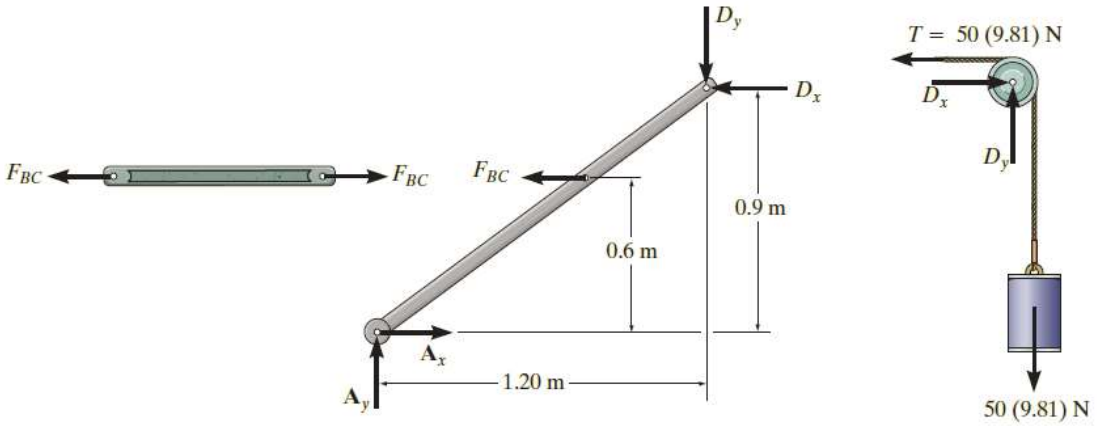
$A_y = 20\ kN\ \uparrow$ *answer*

Example No. 2: The frame in Figure supports the 50-kg cylinder. Determine the horizontal and vertical components of reaction at *A* and the force at *C*.



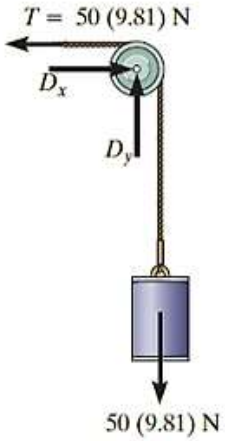
Solution:

Draw F.B.D. for pulley *D* and all member:



Note: member BC is axial member

At pulleys D as F.B.D:



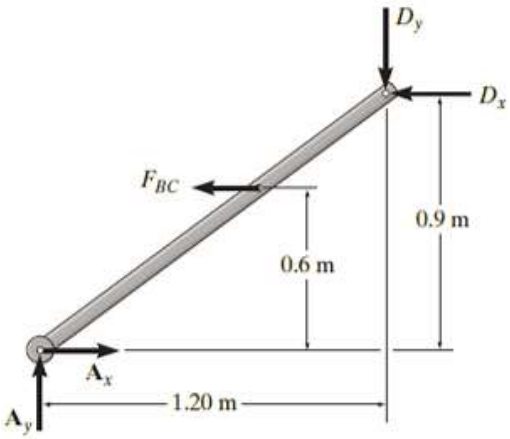
$$\rightarrow^{+} \sum F_x = 0$$

$$D_x - 50 \times 9.81 = 0 \quad \Longrightarrow \quad D_x = 490.5\ N \rightarrow$$

$$\uparrow^{+} \sum F_y = 0$$

$$D_y - 50 \times 9.81 = 0 \quad \Longrightarrow \quad D_y = 490.5\ N \uparrow$$

At member ABD as F.B.D:



$$\circlearrowleft^{+} \sum M_A = 0$$

$$490.5 \times 1.2 - 490.5 \times 0.9 - F_{BC} \times 0.6 = 0$$

$$F_{BC} = 245.25\ N \ \leftarrow$$

$$\rightarrow^{+} \sum F_x = 0$$

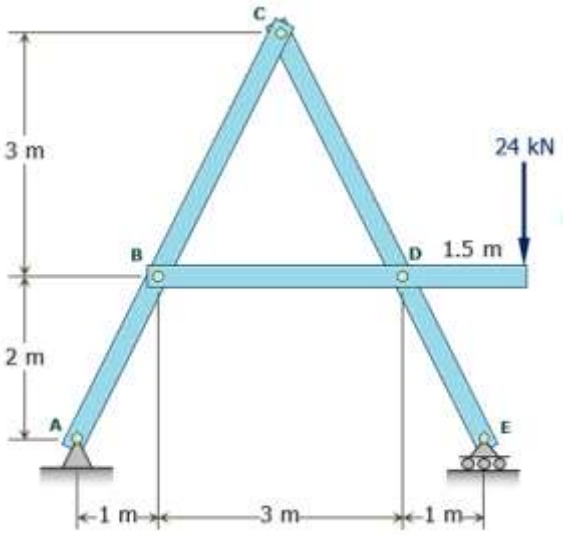
$$-490.5 - 245.25 + A_x = 0$$

$$A_x = 735.75\ N \ \rightarrow$$

$$\uparrow^{+} \sum F_y = 0$$

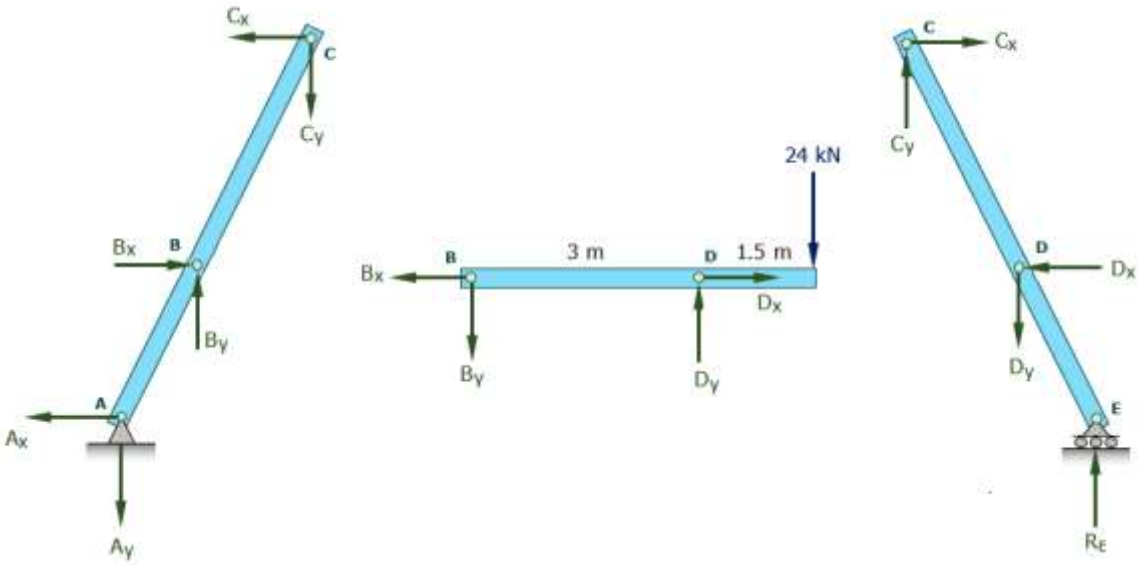
$$A_y - 490.5 = 0 \quad \Longrightarrow \quad A_y = 490.5\ N \uparrow$$

Example No. 3: The frame shown in Figure is supported by a hinge at A and a roller at E. Compute the horizontal and vertical components of the hinge forces at B and C as they act upon member ABC.



Solution:

Draw F.B.D. for all member

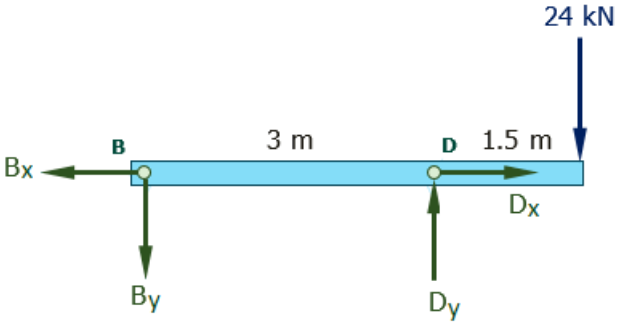


At member BD as F.B.D:

$$\circlearrowleft + \sum M_D = 0$$

$$24 \times 1.5 - B_y \times 3 = 0$$

$$B_y = 12\ kN\ \downarrow \qquad \textit{answer}$$



From the FBD of the whole system

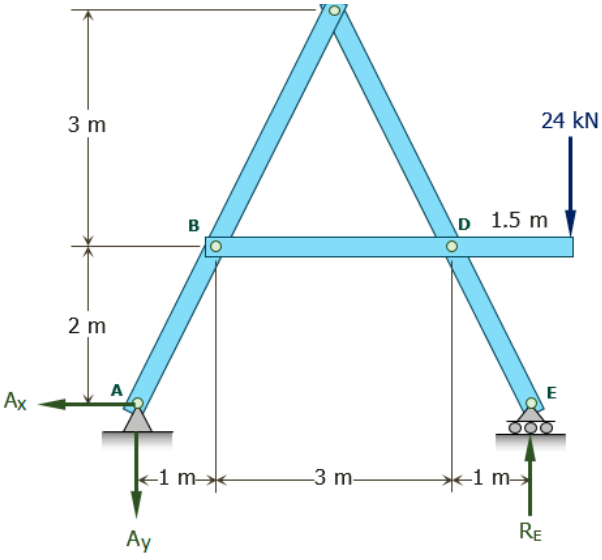
$$\circlearrowleft + \sum M_E = 0$$

$$24 \times 0.5 - A_y \times 5 = 0$$

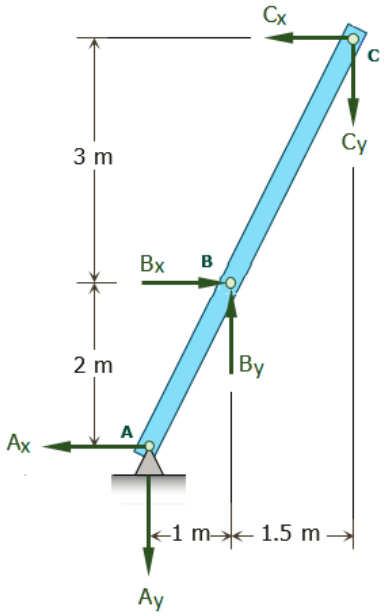
$$A_y = 2.4\ kN\ \downarrow$$

$$\rightarrow^{+} \sum F_x = 0 \qquad \Longrightarrow$$

$$A_x = 0$$



At member ABC as F.B.D:



$$\circlearrowleft^{+} \sum M_C = 0$$

$$-A_y \times 2.5 + A_x \times 5 + B_y \times 1.5 - B_x \times 3 = 0$$

$$-2.4 \times 2.5 + 0 + 12 \times 1.5 - B_x \times 3 = 0$$

$$B_x = 4\ kN \ \rightarrow \quad \textit{answer}$$

$$\rightarrow^{+} \sum F_x = 0$$

$$0 + 4 - C_x = 0$$

$$C_x = 4\ kN \ \leftarrow \quad \textit{answer}$$

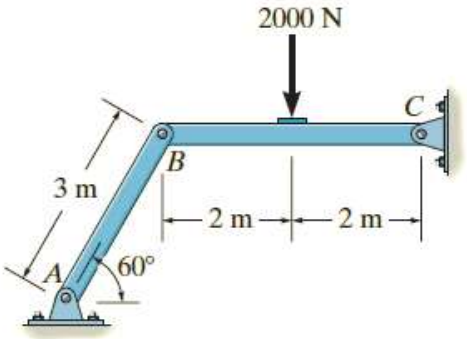
$$\uparrow^{+} \sum F_y = 0$$

$$-2.4 + 12 - C_y = 0$$

$$C_y = 9.6\ kN \ \downarrow \quad \textit{answer}$$

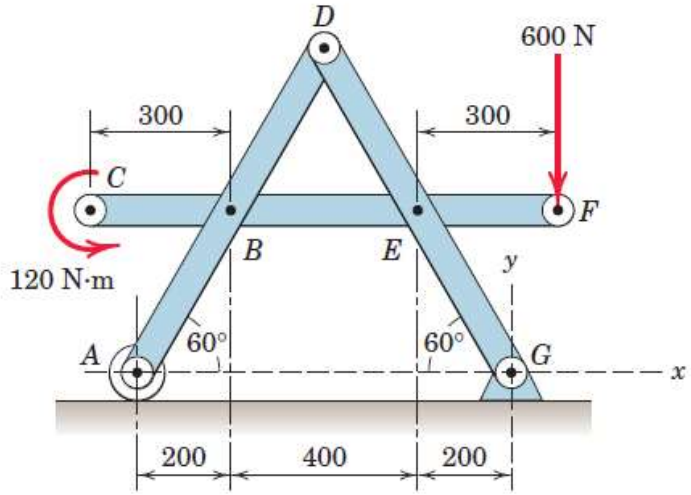
Problem:

1. Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame in Figure.



Answer: $C_x = 577.35\ N \ \leftarrow, \quad C_y = 1000\ N \ \uparrow$

2. Calculate the *x*- and *y*-components of all forces acting on each member of the loaded frame.

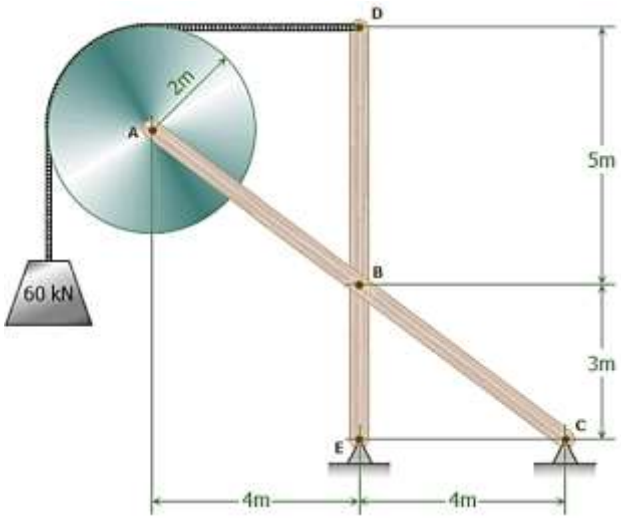


Dimensions in millimeters

Answer: $A_y = 75\ N\ \uparrow,\ \ B_y = 150\ N,\ \ D_y = 225\ N,\ \ B_x = D_x = E_x = 173.2\ N,$

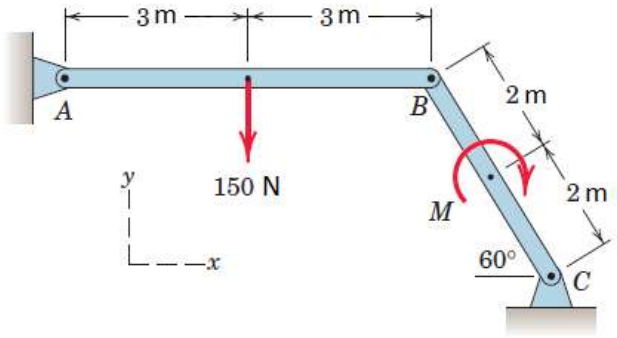
$E_y = 750\ N,\ \ G_x = 0,\ \ G_y = 525\ N\ \uparrow$

3. For the frame shown in Figure, determine the horizontal and vertical components of the hinge force at B as it acts upon member AC.



Answer: $B_x = 160\ kN\ \leftarrow,\ \ B_y = 150\ kN\ \uparrow$

4. For what value M of the clockwise couple will make the horizontal component of the pin reaction at A be zero.



Answer: $M = 150\ N.m$