جامـــــعة المـــــستـقبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـــوم
قـســـــــم الانـــظـــمـــة الـــطـبـيـة
الـــذكـــــيـــة

Lecture: (7)

Applications of Integrals

Subject: mathematics
Class: First
Lecturer: Dr. Mustafa Talal

Chapter seven

Application of integrals

7-1- Definite integrals:

If $f(x)$ is continuous in the interval $a \le x \le b$ and it is integrable in the interval then the area under the curve:-

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F(x)$ is any function such that $F'(x) = f(x)$ in the interval.

Some of the more useful properties of the definite integral are:-

1) $\int_a^b c\, f(x)\, dx = c\int_a^b f(x)\, dx$, *where c is constant.*

2) $\int_a^b \left(f(x) \mp g(x)\right)dx = \int_a^b f(x)dx \mp \int_a^b g(x)dx$

3) $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

4) *Let* $a < c < b$ *then* $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

5) $\int_a^a f(x)\, dx = 0$

6) *If* $f(x) \ge 0$ *for* $a \le x \le b$ *then* $\int_a^b f(x)\, dx \ge 0$

7) *If* $f(x) \le g(x)$ *for* $a \le x \le b$ *then* $\int_a^b f(x)\, dx \le \int_a^b g(x)\, dx$

<u>EX-1</u> – Evaluate the following definite integrals:

$$1)\ \int_{2}^{6}\frac{dx}{x+2} \qquad 2)\ \int_{\pi/2}^{3\pi/2}\cos x\,dx$$

$$3)\ \int_{-\sqrt{3}}^{\sqrt{3}}\frac{dx}{1+x^2} \qquad 4)\ \int_{0}^{\sqrt{3}/2}\frac{dx}{\sqrt{1-x^2}}$$

$$5)\ \int_{-2}^{4}e^{-\frac{x}{2}}\,dx \qquad 6)\ \int_{0}^{\pi}(\pi-x)\cdot\cos x\,dx$$

<u>Sol.</u> –

$$1)\ \int_{2}^{6}\frac{dx}{x+2}=\ln(x+2)\Big|_{2}^{6}=\ln(6+2)-\ln(2+2)=\ln 8-\ln 4=3\ln 2-2\ln 2=\ln 2$$

$$2)\ \int_{\pi/2}^{3\pi/2}\cos x\,dx=\sin x\Big|_{\pi/2}^{3\pi/2}=\sin\left(\frac{3}{2}\pi\right)-\sin\left(\frac{\pi}{2}\right)=-1-1=-2$$

$$3)\ \int_{-\sqrt{3}}^{\sqrt{3}}\frac{dx}{1+x^2}=\tan^{-1}\Big|_{-\sqrt{3}}^{\sqrt{3}}=\tan^{-1}\sqrt{3}-\tan^{-1}(-\sqrt{3})=\frac{\pi}{3}-\left(-\frac{\pi}{3}\right)=\frac{2}{3}\pi$$

$$4)\ \int_{0}^{\sqrt{3}/2}\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x\Big|_{0}^{\sqrt{3}/2}=\sin^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}0=\frac{\pi}{3}-0=\frac{\pi}{3}$$

$$5)\ \int_{-2}^{4}e^{-\frac{x}{2}}\,dx=-2e^{-\frac{x}{2}}\Big|_{-2}^{4}=-2(e^{-2}-e)=2(e-e^{-2})$$

$$6)\ \text{Let}\quad u=\pi-x\ \Rightarrow\ du=-dx\quad\&\quad dv=\cos x\,dx\ \Rightarrow\ v=\sin x$$

$$\int_{0}^{\pi}(\pi-x)\cdot\cos x\,dx=(\pi-x)\sin x\Big|_{0}^{\pi}+\int_{0}^{\pi}\sin x\,dx=(\pi-x)\sin x-\cos x\Big|_{0}^{\pi}$$

$$=(\pi-\pi)\sin\pi-\cos\pi-\big((\pi-0)\sin 0-\cos 0\big)=0-(-1)-(0-1)=2$$