AL-MUSTAQBAL UNIVERSITY
Babylon–Iraq

Department of Biomedical Engineering

- Subject : Physics
- Grade: 1th Class
- Lecture : 4 Fluids in Statics
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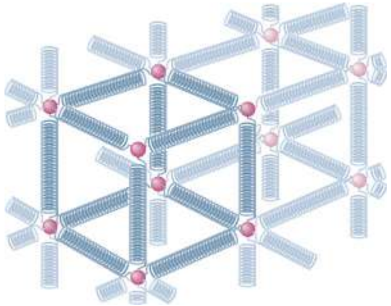
What is a fluid?

Solids: strong intermolecular forces

- definite volume and shape
- rigid crystal lattices, as if atoms on stiff springs
- deforms elastically (strain) due to moderate stress (pressure) in any direction



Fluids: substances that can "flow"

- **no definite shape**
- **molecules are randomly arranged, held by weak cohesive intermolecular forces and by the walls of a container**
- **liquids and gases are both fluids**

Liquids: definite volume but no definite shape

- **often almost incompressible under pressure (from all sides)**
- **can not resist tension or shearing (crosswise) stress**
- **no long range ordering but near neighbor molecules can be held weakly together**

Gases: neither volume nor shape are fixed

- **molecules move independently of each other**
- **comparatively easy to compress: density depends on temperature and pressure**

Mass and Density

- **Density is mass per unit volume at a point**:

$$\rho \equiv \frac{\Delta m}{\Delta V} \quad \text{or} \quad \rho \equiv \frac{m}{V}$$

- scalar
- units are kg/m^3, gm/cm^3..
- ρ_{water}= 1000 kg/m^3= 1.0 gm/cm^3

- Volume and density vary with temperature - slightly in liquids
- The average molecular spacing in gases is much greater than in liquids.

Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)

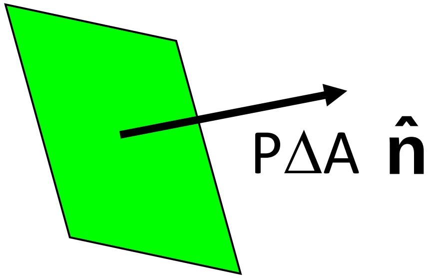
Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Air	1.29	Ice	0.917×10^3
Aluminum	2.70×10^3	Iron	7.86×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Copper	8.92×10^3	Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3	Oak	0.710×10^3
Fresh water	1.00×10^3	Oxygen gas	1.43
Glycerin	1.26×10^3	Pine	0.373×10^3
Gold	19.3×10^3	Platinum	21.4×10^3
Helium gas	1.79×10^{-1}	Seawater	1.03×10^3
Hydrogen gas	8.99×10^{-2}	Silver	10.5×10^3

Force & Pressure

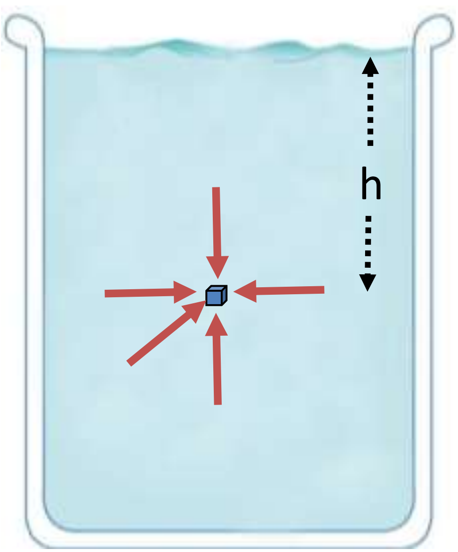
- The pressure P on a “small” area ΔA is the ratio of the magnitude of net force to the area

$$P = \frac{\Delta F}{\Delta A} \quad \text{or} \quad P = F / A$$

$$\Delta\vec{F} = P\Delta\vec{A} = P\Delta A\hat{n}$$



- Pressure is a scalar while force is a vector
- The direction of the force producing a pressure is perpendicular to some area of interest



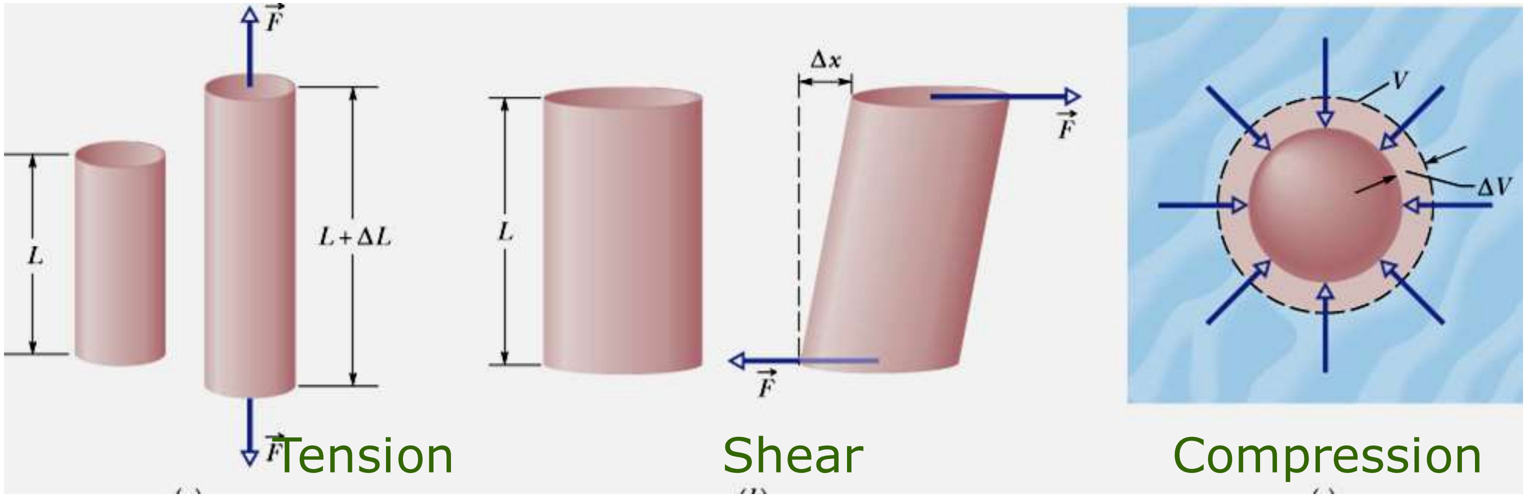
- At a point in a fluid (in mechanical equilibrium) the pressure is the same in any direction

Pressure units:

- 1 Pascal (Pa) = 1 Newton/m^2 (SI)
- 1 PSI (Pound/sq. in) = 6894 Pa.
- 1 milli-bar = 100 Pa.

Forces/Stresses in Fluids

- **Fluids do not allow shearing stresses or tensile stresses.**



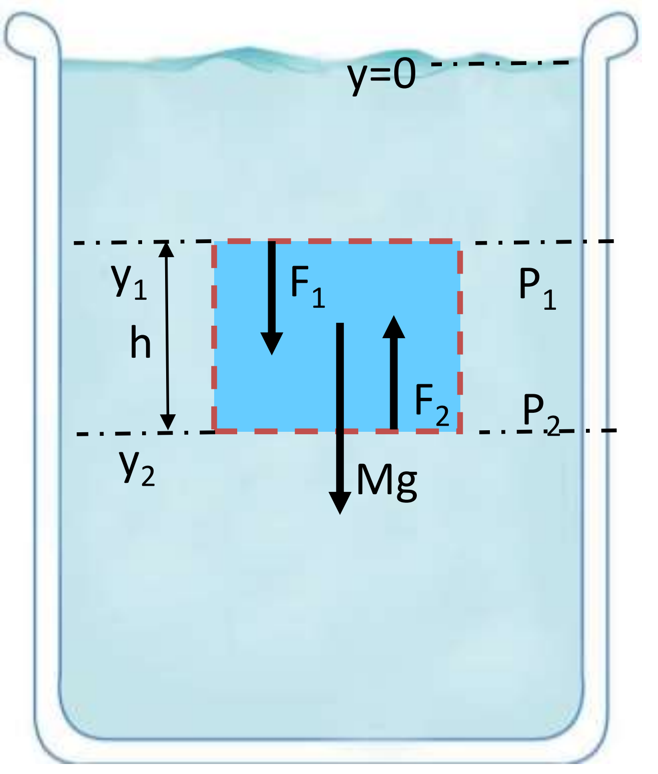
- The only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides
- The force exerted by a static fluid on an object is always perpendicular to the surfaces of the object

Pressure in a fluid varies with depth

Fluid is in static equilibrium
The net force on the shaded volume = 0

- Incompressible liquid - constant density ρ
- Horizontal surface areas = A
- Forces on the shaded region:
  - Weight of shaded fluid: Mg
  - Downward force on top: $F_1 = P_1A$
  - Upward force on bottom: $F_2 = P_2A$

$$\sum F_y = 0 = P_2A - P_1A - Mg$$



- In terms of density, the mass of the shaded fluid is:

$$M = \rho\Delta V = \rho Ah$$

$$\therefore\ P_2A \equiv P_1A + \rho ghA$$

The extra pressure at extra depth h is:

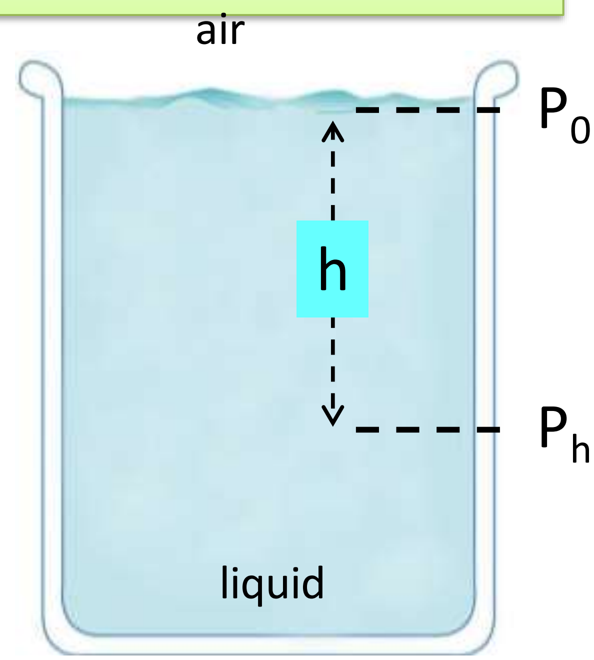
$$\Delta P = P_2 - P_1 = \rho gh$$

$$h \equiv y_1 - y_2$$

Pressure relative to the surface of a liquid

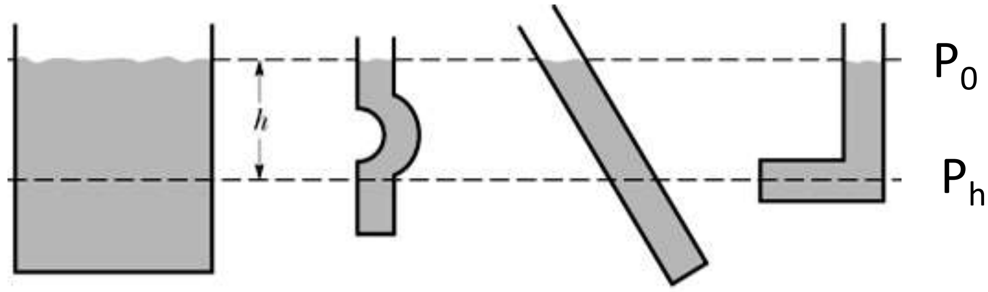
Example: **The pressure at depth h is:**

$$\boxed{P_h = P_0 + \rho g h}$$



- P_0 is the local atmospheric (or ambient) pressure
- P_h is the absolute pressure at depth h
- The difference is called the gauge pressure
- All points at the same depth are at the same pressure; otherwise, the fluid could not be in equilibrium
- The pressure at depth h does not depend on the shape of the container holding the fluid

Preceding equations also hold approximately for gases such as air if the density does not vary much across h



Atmospheric pressure and units conversions

- P_0 is the *atmospheric pressure i*f the liquid is open to the atmosphere.
- Atmospheric pressure varies locally due to altitude, temperature, motion of air masses, other factors.
- *Sea level atmospheric pressure* P_0 = 1.00 atm
  = 1.01325 x 10^5 Pa = 101.325 kPa = 1013.25 mb (millibars)
  = 29.9213” Hg = 760.00 mmHg ~ 760.00 Torr
  = 14.696 psi (pounds per square inch)

	Pascal (Pa)	bar (bar)	atmosphere (atm)	torr (Torr)	pound-force per square inch (psi)
1 Pa	≡ 1 N/m^2	10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	100,000	≡ 10^6 dyn/cm^2	0.98692	750.06	14.5037744
1 atm	101,325	1.01325	≡ 1 atm	760	14.696
1 torr	133.322	$1.3332\times10{-3}$	$1.3158\times10{-3}$	≡ 1 Torr; ≈ 1 mmHg	$19.337\times10{-3}$
1 psi	6.894×10^3	68.948×10^{-3}	68.046×10^{-3}	51.715	≡ 1 lbf/in^2

Measurement of Pressure

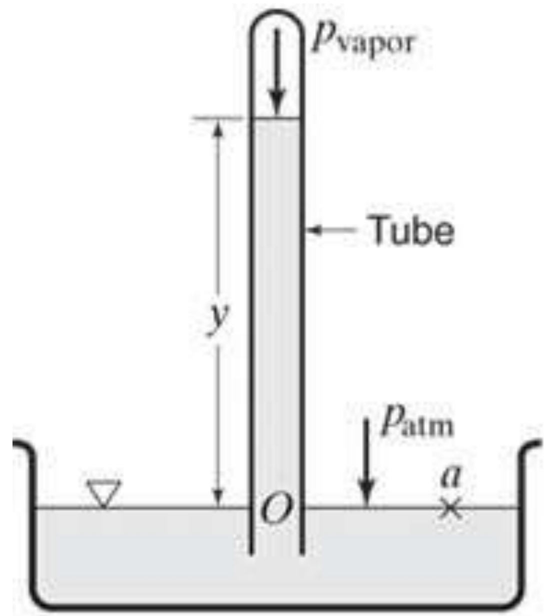
- There are many ways to measure pressure in a fluid. Some are discussed here:

1. Barometers
2. Bourdon gauge
3. Pressure transducers
4. Piezometer Column
5. Simple Manometers
6. Differential Manometers

1. Barometers:

- To measure the atmospheric pressure.
- Procedure:

1. Immerse the open end of tube in a liquid which is open to atmosphere.
2. The liquid will rise in the tube if we exhaust air from the tube.
3. If all the air is removed and the tube is long enough, than only pressure on the surface is the vapour pressure and liquid will reach its max. possible height (y).



(a) Mercury barometer

$$p_O = p_a = p_{atm} = \gamma y + p_{vapour}$$

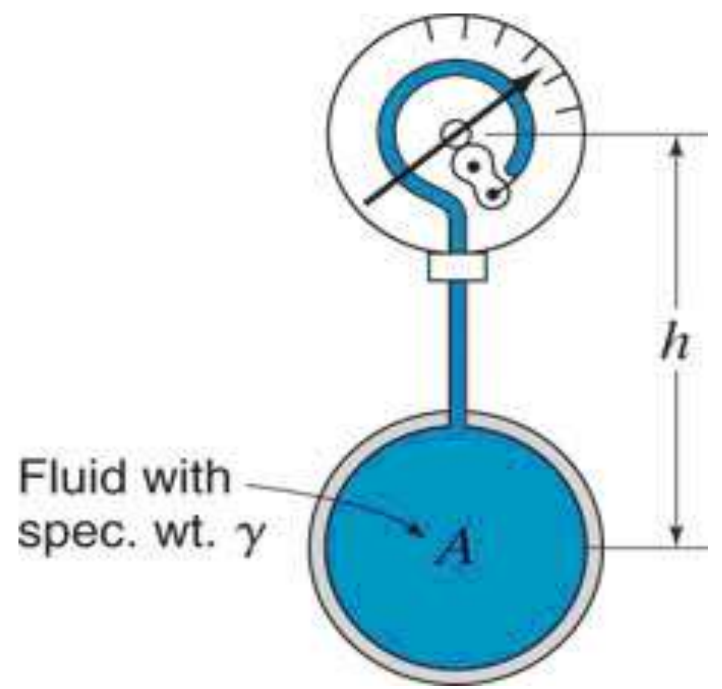
If the vapour pressure on liquid surface in tube is negligible than :

$$p_{atm} = \gamma y$$

2. Bourdon Gauge:

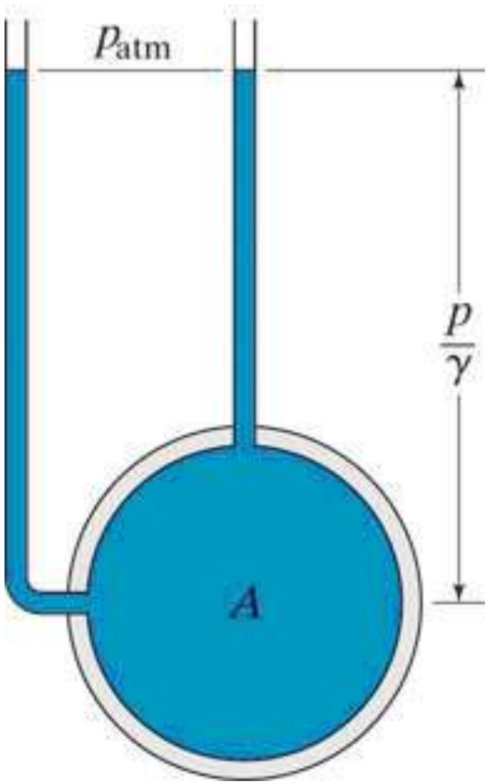
- The pressure, above or below the atmospheric pressure, may be easily measured with the help of a bourdon's tube pressure gauge.
- It consists on an elliptical tube: bent into an arc of a circle. This bent up tube is called Bourdon's tube.

- Tube changes its curvature with change in pressure inside the tube. Higher pressure tends to "straighten" it.
- The moving end of tube rotates needle on a dial through a linkage system.



3. Piezometer Column/Tube:

- A piezometer tube is the simplest form of instrument, used for measuring, moderate pressure.
- It consists of long tube in which the liquid can freely rise without overflowing.
- The height of the liquid in the tube will give the pressure head (p/γ) directly.
- To reduce capillary error, the tube error should be at least 0.5 in.



4. Manometer:

- Manometer is an improved form of a piezometer tube. With its help we can measure comparatively high pressures and negative pressure also. Following are few types of manometers.

1. Simple Manometer
2. Micro-manometer
3. Differential manometer
4. Inverted differential manometer

Simple Manometer:

2. **The horizontal surface, at which the heavy and light liquid meet in the left limb, is known as datum line.**

Let $h1$ = height of light liquid in the left limb above datum.

$h2$ = height of heavy liquid in the right limb above datum.

h= Pressure in the pipe, expressed in terms of head of water.

$s1$=Sp. Gravity of light liquid.

$s2$=Sp. Gravity of heavy liquid.

3. **Pressure in left limb above datum = $h + s1h1$**
4. **Pressure in right limb above datum = $s2h2$**
5. **Since the pressure is both limbs is equal So,**

$h + s1h1 = s2h2$

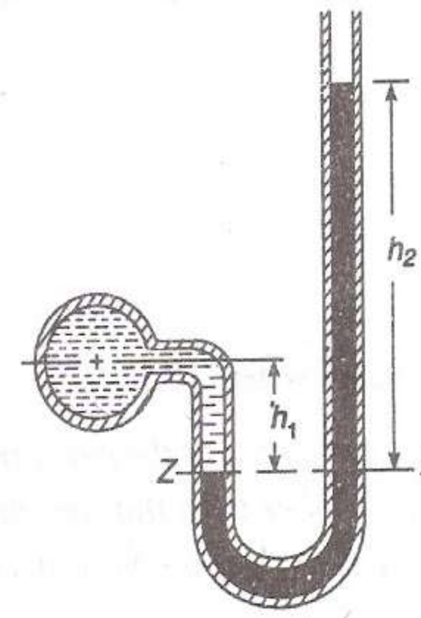
$h = (s2h2 - s1h1)$

Simple Manometer:

- It consists of a tube bent in U-Shape, one end of which is attached to the gauge point and the other is open to the atmosphere.
- Mercury is used in the bent tube which is 13.6 times heavier than water. Therefore it is suitable for measuring high pressure as well.

Procedure:

1. Consider a simple Manometer connected to a pipe containing a light liquid under high pressure. The high pressure in the pipe will force the mercury in the left limb of U-tube to move downward, corresponding the rise of mercury in the right limb.



(a) Positive pressure

2. Consider the vessel to be completely filled with water.

As a result, let the mercury level goes down by x meters in the right limb, and the mercury level go up by the same amount in the left limb.

Therefore total height of water in the right limb

$$= x + h + 3 = x + 2.72 + 3 = x + 5.72$$

Pressure head in the right limb $= 1(x + 5.72) = x + 5.72$

We know that manometer reading in this case:

$$= 0.2 + 2x$$

Pressure head in the left limb

$$= 13.6\ (0.2 + 2x) = 2.72 + 27.2x$$

Equating the pressures:

$$x + 5.72 = 2.72 + 27.2x$$

$$x = 0.115\text{m}$$

and manometer reading $= 0.2 + (2x0.115) = 0.43\text{m} = 430\ \text{mm}$

Differential Manometer:

- It is a device used for measuring the difference of pressures, between the two points in a pipe, on in two different pipes.
- It consists of U-tube containing a heavy liquid (mercury) whose ends are connected to the points, for which the pressure is to be found out.

Procedure:

- ❑ Let us take the horizontal surface Z-Z, at which heavy liquid and light liquid meet in the left limb, as datum line.
- ❑ Let, h=Difference of levels (also known as differential manomter reading)

  ha, hb= Pressure head in pipe A and B, respectively.

  s1, s2= Sp. Gravity of light and heavy liquid respectively.

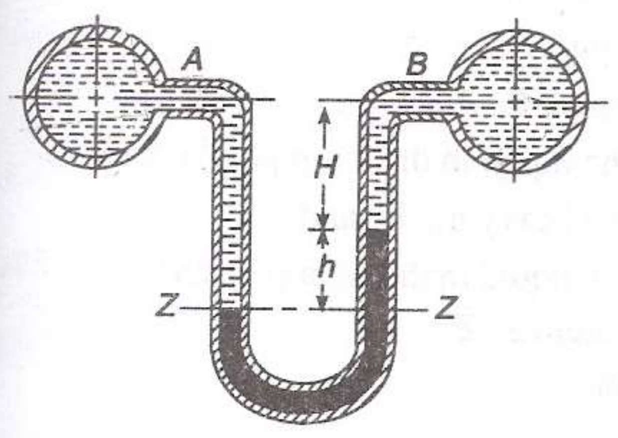
Differential Manometer:

1. Consider figure (a):
2. Pressure head in the left limb above Z-Z = ha+s1(H+h)= ha+s1H+s1h

3. Pressure head in the right limb above Z-Z = hb+s1H+s2h
4. Equating we get,

   ha+s1H+s1h = hb+s1H+s2h

   ha-hb=s2h-s1h = h(s2-s1)



(a) A and B at the same level and containing same liquid.

Differential Manometer:

Two pipes at different levels:

1. Pressure head in the left limb above Z-Z = ha+s1h1
2. Pressure head in the right limb above Z-Z = s2h2+s3h3+hb
3. Equating we get,

   ha+s1h1 = s2h2+s3h3+hb

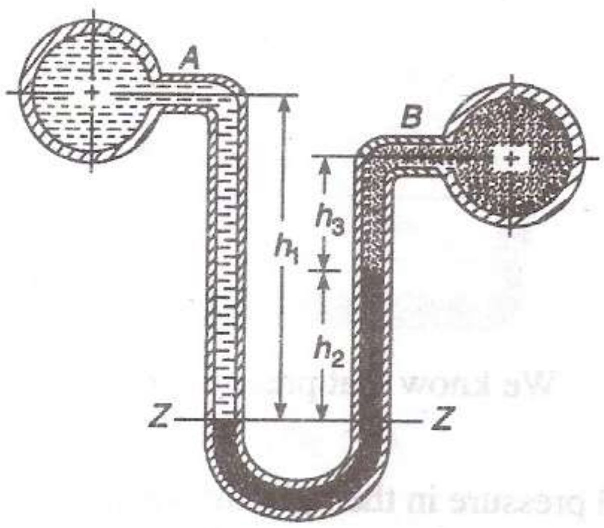
Where;

h1= Height of liquid in left limb

h2= Difference of levels of the heavy liquid in the right and left limb (reading of differential manometer).

h3= Height of liquid in right limb

s1,s2,s3 = Sp. Gravity of left pipe liquid, heavy liquid, right pipe liquid, respectively.



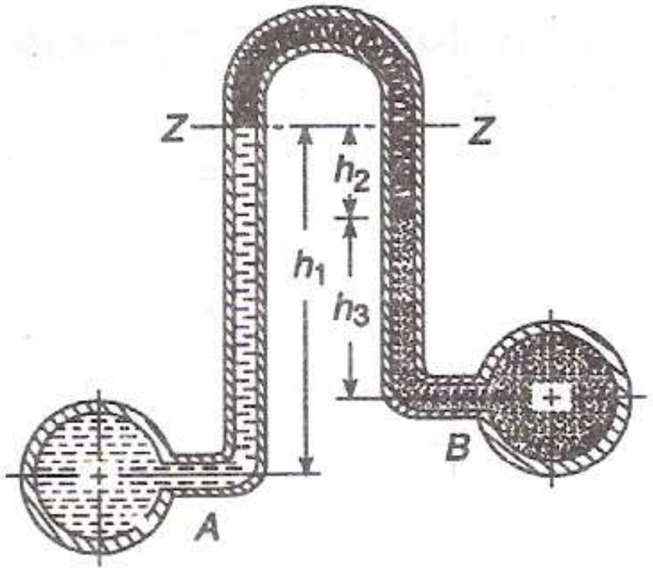
(b) A and B at different levels and containing different liquids.

Inverted Differential Manometer:

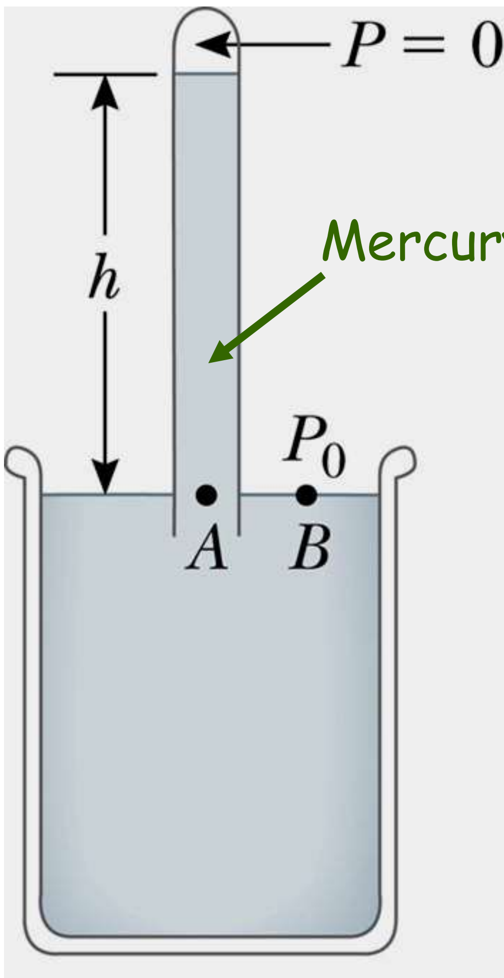
- Type of differential manometer in which an inverted U-tube is used.
- Used for measuring difference of low pressure.

1. Pressure head in the left limb above Z-Z = ha-s1h1
2. Pressure head in the right limb above Z-Z = hb-s2h2-s3h3
3. Equating we get, ha-s1h1 = hb-s2h2-s3h3

(Where; ha, hb are Pressure in pipes A and B expressed in terms of head of liquid, respectively)



Pressure Measurement: Barometer



- Invented by Torricelli (1608-47)
- Measures atmospheric pressure P_0 as it varies with the weather
- The closed end is nearly a vacuum (P = 0)
- One standard atm = 1.013×10^5 Pa.

$$P_0 = \rho_{Hg} g h$$

How high is the Mercury column?

$$h = \frac{P_0}{\rho_{Hg} g} = \frac{1.013 \times 10^5 \text{ Pa}}{(13.6 \times 10^3 \text{kg/m}^3)(9.80 \text{ m/s}^2)} = 0.760 \text{ m}$$

One 1 atm = 760 mm of Hg
= 29.92 inches of Hg

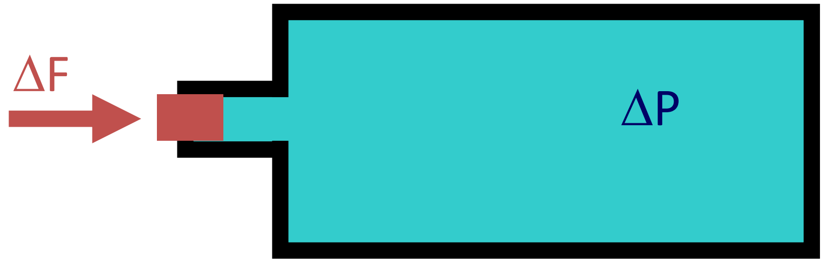
How high would a water column be?

$$h = \frac{P_0}{\rho_{water} g} = \frac{1.013 \times 10^5 \text{ Pa}}{(1.0 \times 10^3 \text{kg/m}^3)(9.80 \text{ m/s}^2)} = 10.34 \text{ m}$$

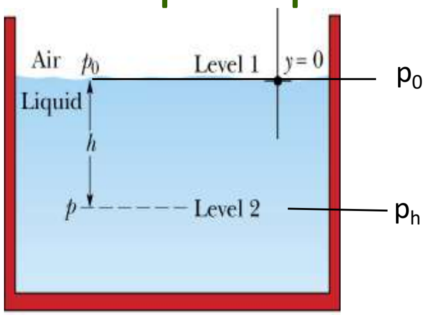
Height limit for a suction pump

Pascal's Principle

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

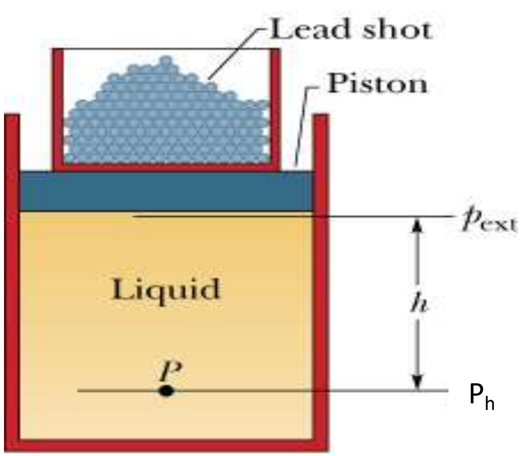


Example: open container



- The pressure in a fluid depends on depth h and on the value of P_0 at the surface
- All points at the same depth have the same pressure.

$$\boxed{P_h = P_0 + \rho g h}$$



- Add piston of area A with lead balls on it & weight W. Pressure at surface increases by $\Delta P = W/A$

$$P_{ext} = P_0 + \Delta P$$

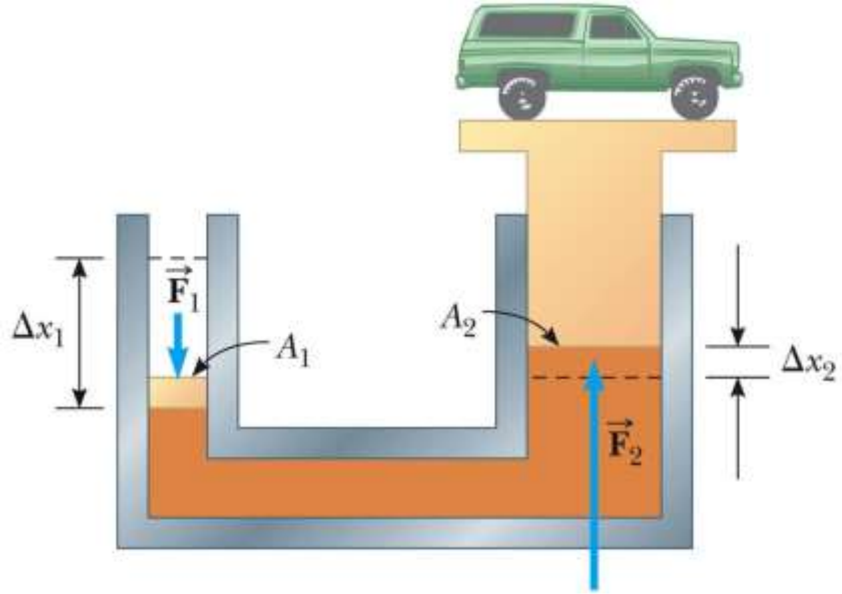
- Pressure at every other point in the fluid (Pascal's law), increases by the same amount, including all locations at depth h.

$$P_h = P_{ext} + \rho g h$$

Pascal's Law Device - Hydraulic press

A small input force generates a large output force

- Assume the working fluid is incompressible
- Neglect the (small here) effect of height on pressure



- The volume of liquid pushed down on the left equals the volume pushed up on the right, so:

$$A_1\Delta x_1 = A_2\Delta x_2$$

$$\therefore \frac{\Delta x_2}{\Delta x_1} = \frac{A_1}{A_2}$$

- Assume no loss of energy in the fluid, no friction, etc.

$$\text{Work}_1 = F_1\Delta x_1 = \text{Work}_2 = F_2\Delta x_2$$

mechanical advantage

$$\boxed{\therefore \frac{F_2}{F_1} = \frac{\Delta x_1}{\Delta x_2} = \frac{A_2}{A_1}}$$

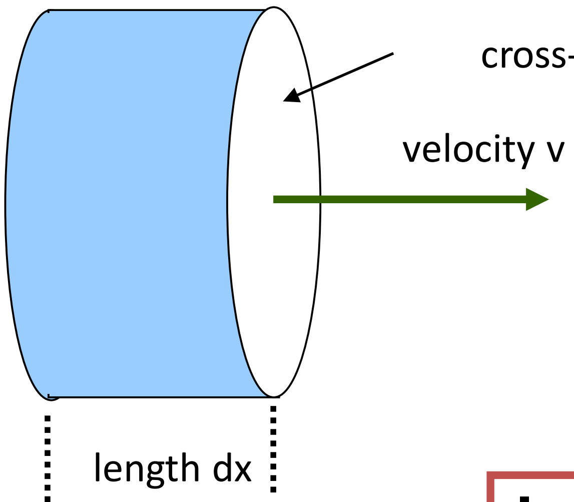
Other hydraulic lever devices using Pascal's Law:

- Squeezing a toothpaste tube
- Hydraulic brakes
- Hydraulic jacks
- Forklifts, backhoes

Flow of an ideal fluid through a short section of pipe

Constant density and velocity within volume element dV
Incompressible fluid means $d\rho/dt = 0$

Mass flow rate = amount of mass crossing area A per unit time
= a "current" sometimes called a "mass flux"



$$\text{volume of fluid in cylinder} = dV = A\,dx$$

$$I_{mass} = \text{mass flow rate} = \frac{dM}{dt} = \frac{d}{dt}(\rho V)$$

$$= \rho A \frac{dx}{dt} = \rho A v$$

$$\boxed{I_{mass} \equiv \text{mass flow rate} = \rho A v}$$

$$\boxed{I_{vol} \equiv \text{volume flow rate} = A v}$$

$$J_{mass} \equiv \text{mass flow/unit area} = \rho v$$

Equation of Continuity: conservation of mass

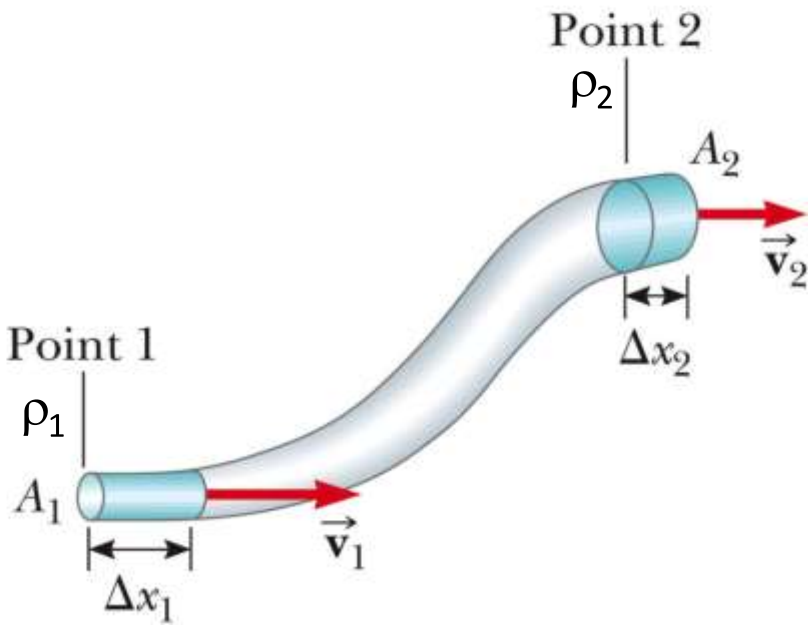
- An ideal fluid is moving through a pipe of nonuniform diameter
- The particles move along streamlines in steady-state flow
- The mass entering at point 1 cannot disappear or collect in the pipe
- The mass that crosses A_1 in some time interval is the same as the mass that crosses A_2 in the same time interval.

$$\text{mass flow in} = \rho_1 A_1 v_1 = \text{mass flow out} = \rho_2 A_2 v_2$$

The fluid is incompressible so:

$$\rho_1 = \rho_2 = \text{a constant}$$

$$\boxed{\therefore\ A_1 v_1 = A_2 v_2}$$



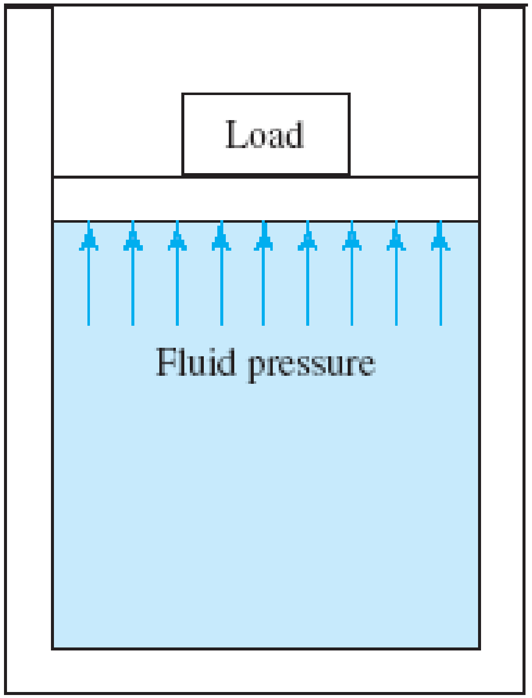
- This is called the equation of continuity for an incompressible fluid
- The product of the area and the fluid speed (volume flux) at all points along a pipe is constant.

The rate of fluid volume entering one end equals the volume leaving at the other end
Where the pipe narrows (constriction), the fluid moves faster, and vice versa

Example

A load of 200 pounds (lb) is exerted on a piston confining oil in a circular cylinder with an inside diameter of 2.50 inches (in). Compute the pressure in the oil at the piston.

Solution:



$$A = \pi D^2/4 = \pi(2.50\text{ in})^2/4 = 4.91\text{ in}^2$$

$$p = \frac{F}{A} = \frac{200\text{ lb}}{4.91\text{ in}^2} = 40.7\text{ lb/in}^2$$

Example

A simple manometer containing mercury is used to measure the pressure of water flowing in a pipeline. The mercury level in the open tube is 60mm higher than that on the left tube. If the height of water in the left tube is 50mm, determine the pressure in the pipe in terms of head of water.

Solution:

Pressure head in the left limb above Z - Z

$$= h + s_1 h_1 = h + (1x50)$$

$$= h + 50 \text{ mm}$$

Pressure head in the right limb above Z - Z

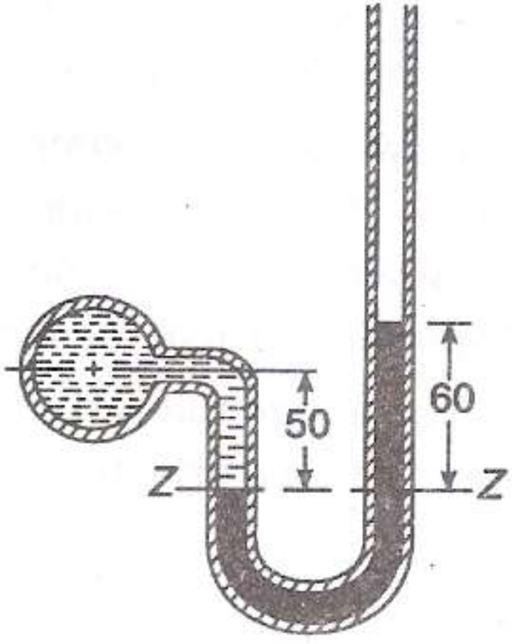
$$= s_2 h_2 = 13.6x60$$

$$= 816 \text{ mm}$$

Equating;

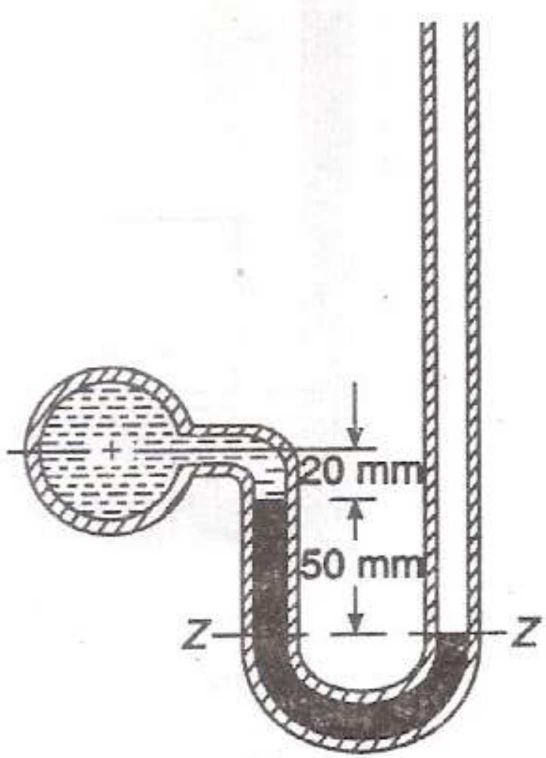
$$h + 50 = 816$$

$$h = 766 \text{ mm}$$



Example

A simple manometer containing mercury was used to find the negative pressure in pipe containing water. The right limb of the manometer was open to atmosphere. Find the negative pipe.



Solution:

$$\text{Pressure head in the left limb above Z - Z}$$

$$= h + s_1 h_1 + s_2 h_2 = h + (1x50) + (13.6x50)$$

$$= h + 700 \text{ mm}$$

$$\text{Pressure head in the right limb above Z - Z}$$

$$= 0$$

$$\text{Equating;}$$

$$h + 700 = 0$$

$$h = -700 \text{ mm} = -7\text{m}$$

$$\text{Gauge pressure in the pipe} = p = \gamma h$$

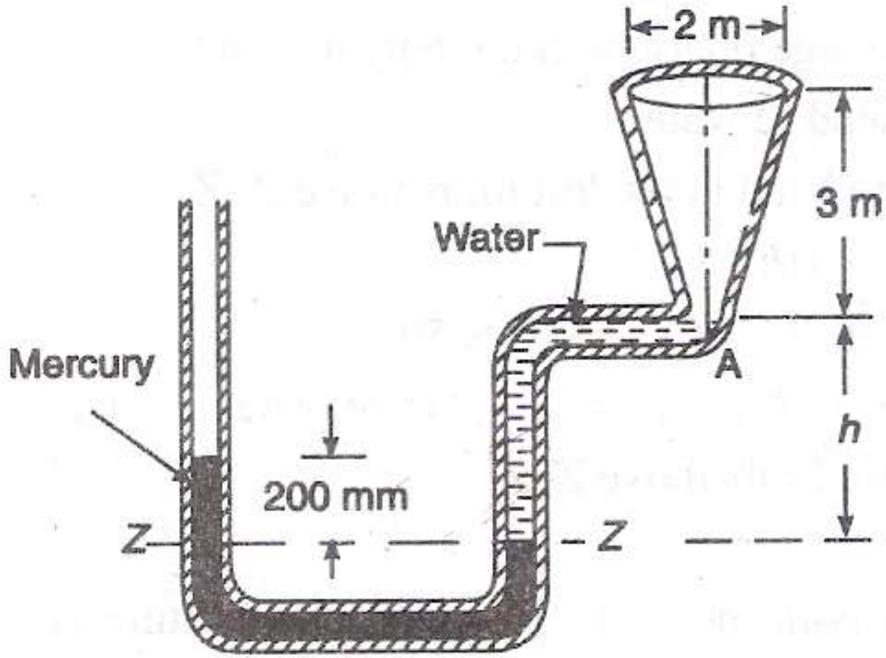
$$9.81x(-7) = -68.67 \text{kN/m}^2$$

$$= -68.67 \text{kPa}$$

$$= 68.67 \text{kPa} \quad (\text{Vacuum})$$

Example

Figure shows a conical vessel having its outlet at A to which U tube manometer is connected. The reading of the manometer given in figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.



Solution:

$h_2 = 200\,mm = 0.2m$

$s_1 = 1$ and $s_2 = 13.6$

Let h = Pressure head of mercury in terms on head of water.

1. Let us consider the vessel is to be empty and Z - Z be the datum line.

Pressure head in the right limb above Z - Z

$= s_1 h_1 = 1xh = h$

Pressure head in the left limb above Z - Z

$= s_2 h_2 = 13.6x0.2 = 2.72m$

Equating; $h = 2.72m$

Example

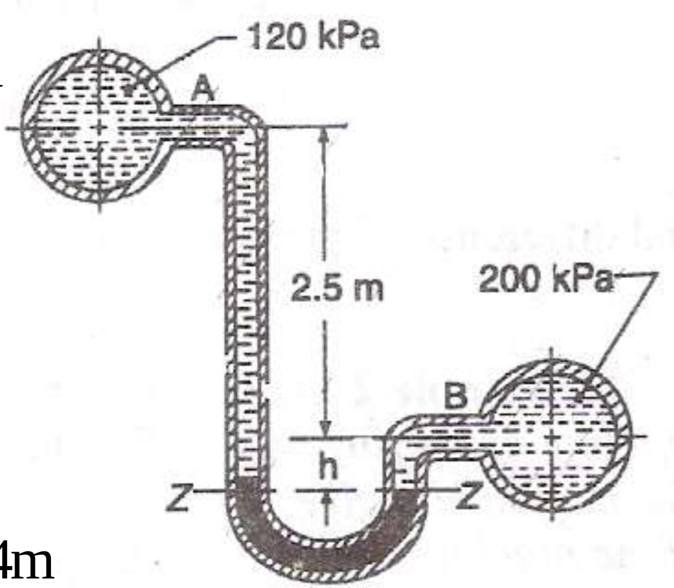
A U-tube differential manometer connects two pressure pipes A and B. The pipe A contains carbon Tetrachloride having a Sp. Gravity 1.6 under a pressure of 120 kPa. The pipe B contains oil of Sp. Gravity 0.8 under a pressure of 200 kPa. The pipe A lies 2.5m above pipe B. Find the difference of pressures measured by mercury as fluid filling U-tube.

Solution:

$$\text{Given}: s_a = 1.6,\ p_a = 120\text{kPa};\ s_b = 0.8,\ p_b = 200\text{kPa};$$

$$h_1 = 2.5\text{m and } s = 13.6$$

Let h = Differnce of pressure measured by mercury in terms of head of water.



We know that pressure head in pipe A,

$$\frac{p_a}{\gamma} = \frac{120}{9.81} = 12.2\text{m}$$

Pressure head in pipe B, $\dfrac{p_b}{\gamma} = \dfrac{200}{9.81} = 20.4\text{m}$

We also know that pressure head in Pipe A above Z - Z

$$= 12.2 + (s_a . h_1) + s.h$$

$$= 12.2 + (1.6 \text{ x } 2.5) + 13.6 \text{ x h}$$

$$= 16.2 + 13.6 \text{ h}$$

Pressure head in Pipe B above Z - Z

$$= 20.4 + s_b h = 20.4 + (0.8 \text{ x h})$$

Equating;

$$16.2 + 13.6 \text{ h} = 20.4 + (0.8 \text{ x h})$$

$$\text{h} = 0.328 \text{ m} = 328 \text{ mm}$$