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Graph of Functions (Graph of Curves) رسم الدوال

To graph the curve of a function, we can follow the following steps:

1. Find the domain and range of the function.
2. Check the symmetry of the function
3. Find (if any found) points of intersection with *x*-axis and *y*-axis.
4. Choose some another points on the curve.
5. Draw s smooth line through the above points.

Example: Sketch the graph of the curve $y = f(x) = x^2 - 1$

Sol.:

Step 1: Find Df, Rf of the function?

Df = (-∞,∞);

To find Rf : we must convert the function from y= f(x) into x=f(y).

$$y = x^2 - 1$$

$y = x^2 - 1 \quad \rightarrow \quad x^2 = y + 1$

$$x = \pm\sqrt{y+1}$$

So y +1 ≥ 0 ⇒ y ≥ -1 ⇒ Rf = (-1,∞)

Step 2: Find x and y intercept:

To find x-intercept put y=0 → $x^2 - 1$ =0 → $X = \pm 1$

So x-intercept are (-1,0) and (+1,0).

To find y-intercept put x=0 → y = 0-1 → y = -1

So y-intercept is (0,-1).

Step 3: check the symmetry:

$x^2 - y - 1 = 0$

$f(x, -y) = x^2 + y - 1 \neq f(x, y)$

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$\mathbf{f(-x, y) = x^2 - y - 1 = f(x, y)}$ **so that the function is symmetry about y.**

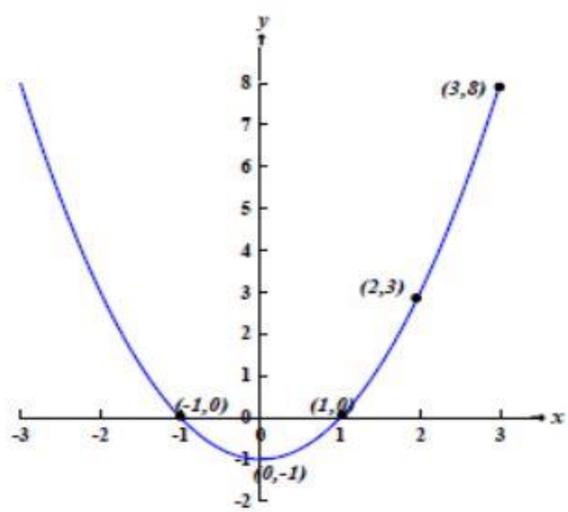
$\mathbf{f(-x, -y) = x^2 + y - 1 \neq f(x, y)}$

Step 4: **Choose some another point on the curve.**

x	y
2	3
3	8

(2,3) ,(3,8)

Step 5: Draw smooth line through the above points



H.W

1- $\boldsymbol{y = 3x^2 - 2}$

2- $\boldsymbol{y^2 = 4x - 1}$

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symmetry of the function (تماثل الدالة)

If f(x,y) = 0 is any function then:

1. Symmetry about x-axis: If f(x,-y) = f(x,y)
2. Symmetry about y-axis: If f(-x,y) = f(x,y) It is called an **even function.**
3. Symmetry about the origin: If f(-x,-y) = f(x,y) It is called an **odd function**

Examples : Check the symmetry of the following curves:

1) $y = x^2$

Sol \ $f(x,y) = x^2 - y = 0$

$f(x,-y) = x^2 - (-y) = x^2 + y$ ⇒ $f(x,-y) \neq f(x,y)$ NOT OK

$f(-x,y) = (-x)^2 - (y) = x^2 - y$ ⇒ $f(-x,y) = f(x,y)$ OK

$f(-x,-y) = (-x)^2 - (-y) = x^2 + y$ ⇒ $f(-x,-y) \neq f(x,y)$ NOT OK

So the function has symmetry only about y-axis. It is called an even function.

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2) $y = x^3$

Sol \ $f(x, y) = x^3 - y = 0$

$f(x, -y) = x^3 - (-y) = x^3 + y$ ⇨ $f(x, -y) \neq f(x, y)$ NOT OK

$f(-x, y) = (-x)^3 - (y) = -x^3 - y$ ⇨ $f(-x, y) \neq f(x, y)$ NOT OK

$f(-x, -y) = (-x)^3 - (-y) = -x^3 + y = x^3 - y$ ⇨
$f(-x, -y) = f(x, y)$ OK

So the function has symmetry only about origin. It is called an odd function.

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3) $x^2 = y^2 + 4$

Sol \ $f(x, y) = y^2 - x^2 + 4 = 0$

$f(x, -y) = (-y)^2 - x^2 + 4 = y^2 - x^2 + 4$ ⇨ $f(x, -y) = f(x, y)$ OK

$f(-x, y) = y^2 - (-x)^2 + 4 = y^2 - x^2 + 4$ ⇨ $f(-x, y) = f(x, y)$ OK

$f(-x, -y) = (-y)^2 - (-x)^2 + 4 = y^2 - x^2 + 4$ ⇨ $f(-x, y) = f(x, y)$ OK

So the function has symmetry about x-axis, y-axis and the origin.

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H.W:

1) $y = 3x^2 + 2.$
2) $x^2 + y^2 = 1$

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(LIMITS الغاية)

Properties of limits

1- If f(x)=k then $\lim_{x\to a} f(x) = k$

2- If $\lim_{x\to a} f_1(x) = L_1$ $\lim_{x\to a} f_2(x) = L_2$

a) Sum rule: $\lim_{x\to a}[f_1(x) + f_2(x)] = L_1 + L_2$

b) Difference rule: $\lim_{x\to a}[f_1(x) - f_2(x)] = L_1 - L_2$

c) Product rule : $\lim_{x\to a}[f_1(x) * f_2(x)] = L_1 * L_2$

d) Quotient rule: $\lim_{x\to a}[\frac{f_1(x)}{f_2(x)}] = \frac{L_1}{L_2}$

3- Polynomial $\lim_{x\to a}(c_0 + c_1x + c_2x^2 + \cdots + c_nx^n) = c_0 + c_1a + c_2a^2 + \cdots + c_na^n$

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Example: Find the limits of the following:

1- $\lim_{x\to 2}(x^2 - 4x) = 2^2 - 4 * 2 = -4$

2- $\lim_{x\to 1}(x^3 - 2x^2) = 1^3 - 2 * 1^2 = -1$

3- $\lim_{x\to 1}[\frac{(3x-1)^2}{(x+1)^3}] = \frac{(3*1-1)^2}{(1+1)^3} = \frac{(2)^2}{(2)^3} = \frac{4}{8}$

4- $\lim_{x\to 2}[\frac{(x^2-4)}{x-2}] = \frac{0}{0}$ (Indeterminate quantities كمية غير محددة)

So $\lim_{x\to 2}[\frac{(x-2)(x+2)}{x-2}] = \lim_{x\to 2}(x+2) = 2 + 2 = 4$

5- $\lim_{x\to 2}[\frac{(x^2-4)}{x^2-5x+6}] = \frac{0}{0}$ (Indeterminate quantities كمية غير محددة)

So $\lim_{x\to 2}[\frac{(x-2)(x+2)}{(x-2)(x-3)}] = \lim_{x\to 2}\left[\frac{(x+2)}{(x-3)}\right] = \frac{4}{-1} = -4.$

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