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Vertical asymptotes:

Vertical asymptotes of a function are “invisible” vertical lines that the graph of the function is always approaching but never touching. They serve as boundaries of the function’s graph. Vertical asymptotes are found much in the same manner as the domain, except they take on the values of the independent variable excluded from the function’s domain. To locate the vertical asymptote(s):

الخطوط المحاذية العمودية للدالة هي خطوط عمودية "غير مرئية" يقترب منها الرسم البياني للدالة دائمًا ولكن لا يلمسها أبدًا. إنها بمثابة حدود للرسم البياني للدالة. يتم العثور على الخطوط المحاذية العمودية بنفس طريقة المجال، باستثناء أنها تأخذ قيم المتغير المستقل المستبعد من مجال الدالة. لتحديد الخط المقارب العمودي

Finding vertical Asymptotes:

1. Set the denominator equal to zero.

2. Solve for x. This may involve a variety of solving methods.

3. These values are the equations of the vertical lines that are the vertical asymptotes.

Example: $f(x) = \frac{1}{x-4}$

Sol.:

Set denominator equal to zero $x - 4 = 0$

The solution is $x = 4$

The vertical line $x = 4$ is the vertical asymptote for the graph of the function.

Example: $f(x) = \frac{x}{x^2-1}$

Sol.:

Set denominator equal to zero $x^2 - 1 = 0$

The solution is $x = \mp 1$

The vertical line $x = 1$, $x = -1$ are the vertical asymptote for the graph of the function.

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Horizontal asymptotes:

Horizontal asymptotes of a function are "invisible" horizontal lines that the graph of the function is always approaching but never touching at the extreme ends. A graph may cross a horizontal asymptote in the "middle" of the graph. Like vertical asymptotes, they also serve as the boundaries of the function's graph. To determine the horizontal asymptote for a function:

الخطوط المحاذية الأفقية للدالة هي خطوط أفقية "غير مرئية" يقترب منها الرسم البياني للدالة دائمًا ولكنها لا تلمسها أبدًا عند الأطراف القصوى. قد يتقاطع الرسم البياني مع الخط المحاذي الأفقي في "منتصف" الرسم البياني. مثل الخطوط المحاذية العمودية، فهي أيضًا بمثابة حدود الرسم البياني للدالة. لتحديد الخط المحاذي الأفقي للدالة:

Finding Horizontal Asymptotes:

1- Look at the **degree** of polynomials in both the **numerator** and the **denominator**. The degree is the highest exponent on the independent variable.

2- There are three possibilities:

A) If the numerator has **lower degree** than the denominator, then the horizontal asymptote is the line **y=0** which is the x-axis

Example: $f(x) = \frac{x-1}{x^2-x-6}$

The degree in the numerator is **1** which is less than the denominator which is **2**, so y=0 is the horizontal asymptote.

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B) If the **numerator** and **denominator** have the same degree, then the horizontal asymptote is the line $y = \frac{a}{b}$ where **a** is the leading coefficient in the numerator and **b** is the leading coefficient in the denominator.

Example: $\boldsymbol{f(x) = \frac{4x^2-1}{x^2-3x-5}}$

The degree in the numerator is the same as the denominator, so $y = \frac{4}{1}$ which simplifies y=4 to is the horizontal asymptote.

C) If the degree of the **numerator** is higher than the degree of the **denominator**, then the graph of the function has **NO horizontal asymptote**

Example: $\boldsymbol{f(x) = \frac{4x^3-5}{3x^2-2x-5}}$

The degree of the numerator is greater than the degree of the denominator, so there is NO horizontal asymptote

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Continuity

The function **y= f (x)** is continuous at **x=c** if and only if the following statements are true:

1- **f (c) exists**
2- **$\lim_{x\to c} f(x)$ exists**
3- **$f(c) = \lim_{x\to c} f(x)$**

Example: did the function $f(x) = 8 - x^3 - 2x^2$ is continuous at the x=2 ?

Sol:

$$f(2) = 8 - 2^3 - 2*(2)^2 = 8$$

$$\lim_{x\to 2}[\,8 - x^3 - 2x^2\,] = 8 - 2^3 - 2*(2)^2 = 8$$

$$f(2) = \lim_{x\to 2} f(x).$$

So the function is continuous at x=2.

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Example: did the function $f(x) = \frac{(x^2-4)}{x-2}$ is continuous at the x=2 ?

Sol:

$f(2) = \frac{(2^2-4)}{2-2} = \frac{0}{0}$ not exists

So the function is not continuous at x=2.

H.W:

1- did the function $f(x) = \frac{(x^2-9)}{x-3}$ is continuous at the x=3 ?

2- Find the limit of the function $f(x) = \frac{(x^2-1)}{x-\sqrt{1}}$ is continuous at the x=$\sqrt{1}$?