

Sums, Difference, Product and Quotients of Functions:

جمع، طرح، ضرب وقسمة الدوال

Definition: If F and G are functions, then we define the functions

- ✓ Sum $\rightarrow (F+G)(x)= F(x)+G(x)$
- ✓ Difference $\rightarrow (F - G)(x)= F(x) - G(x)$
- ✓ Product $\rightarrow (F * G)(x)= F(x) *G(x)$
- ✓ Quotient $\rightarrow (F / G)(x)= F(x) /G(x)$, where $g(x) \neq 0$

Example 1: Combining Functions Algebraically

The function defined by the formulas

$f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$

Function	Formula
$f + g$	$(f + g)(x) = \sqrt{x} + \sqrt{1-x}$
$f - g$	$(f - g)(x) = \sqrt{x} - \sqrt{1-x}$
$g - f$	$(g - f)(x) = \sqrt{1-x} - \sqrt{x}$
$f \circ g$	$(f \circ g)(x) = f(x)g(x) = \sqrt{x(1-x)} = \sqrt{x-x^2}$
f/g	$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \sqrt{\frac{x}{1-x}}$
g/f	$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \sqrt{\frac{1-x}{x}}$

H.W: Combining Functions Algebraically The function defined by the formulas **$f(x) = 3x$ and $g(x) = 1 - x^2$** .

Composition of Functions: (تركيب الدوال)

DEFINITION: If f and g are functions, the composite $(f \circ g)$ ((f composed with g)) or $g \circ f$ ((g composed with f)) are defined by:
 $(f \circ g)(x) = f(g(x))$ and $(g \circ f)(x) = g(f(x))$ respectively

Examples 1: Find the formula for $(f \circ g)(x)$ and $(g \circ f)(x)$ if $g(x) = x^2$ and $f(x) = x - 7$, then find the value of $f(g(2))$ and $g(f(2))$.

SOL:

A: $(f \circ g)(x) = f(g(x)) = f(x^2) = x^2 - 7$.
 $f(g(2)) = 2^2 - 7 = -3$.

B: $(g \circ f)(x) = g(f(x)) = g(x - 7) = (x - 7)^2$.
 $g(f(2)) = (2 - 7)^2 = (-5)^2$.

Examples 2: Find the formula for $(f \circ g)(x)$ and $(g \circ f)(x)$ if $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$, then find the value of $f(g(3))$ and $g(f(3))$.

SOL:

A: $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$.
 $f(g(3)) = 3 + 1 = 4$.

B: $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1}$.
 $g(f(3)) = \sqrt{3^2 + 1} = \sqrt{10}$.

H.W: Finding formulas for composites If $f(x) = x$ and $g(x) = x + 1$, Find:

- (a)** $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **(c)** $(f \circ f)(x)$ **(d)** $(g \circ g)(x)$

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Even Function, Odd Function: (الدالة الفردية والدالة الزوجية)

A function $y = f(x)$ is an

even function of x if $f(-x) = f(x)$

odd function of x if $f(-x) = -f(x)$

for every x in the function's domain.

Examples: Recognizing Even and Odd functions

1) $f(x) = x^2$

$$f(-x) = (-x)^2 = x^2$$

$$-f(x) = -(x)^2$$

Even function

2) $f(x) = x^2 + 1$

$$f(-x) = (-x)^2 + 1 = x^2 + 1$$

$$-f(x) = -(x)^2 - 1$$

Even function

3) $f(x) = x$

$$f(-x) = -x$$

$$-f(x) = -x$$

odd function

4) $f(x) = x + 1$

$$f(-x) = -x + 1$$

$$-f(x) = -x - 1$$

Not Even function , Not odd function

5) $f(x) = x - 1$
 $f(-x) = -x - 1$
 $-f(x) = -x + 1$

Not Even function , Not odd function

6) $f(x) = x^3$
 $f(-x) = (-x)^3 = -(x)^3$
 $-f(x) = -(x)^3$

Odd function

H.W:

- 1) $f(x) = x^3 - 3$.
- 2) $f(x) = x^3 + x^2 - 3$
- 3) $f(x) = x^2 - x$
- 4) $f(x) = \frac{1}{x}$