Lecture (1)

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PREREQUISITES FOR CALCULUS (المتطلبات الاساسية للتفاضل و التكامل)

Sets and Intervals (المجموعات و الفترات)

DEFINITIONS:

Set: is a collection of things under certain conditions.

Example 1:

A={1,3,5,8,10};

A is a set, 1,3,5,8,10 are elements.

Real Numbers (R): is a set of all rational and irrational numbers. R= {-∞, +∞},

-∞ ←———————— 0 ————————→ +∞

Integer Numbers (I): a set of all irrational numbers.

I = {-∞,----,-3,-2,-1,0,1,2,3,----,+∞} negative and positive numbers only.

Natural Numbers (N): consist of zero and positive integer numbers only.

N ={0,1,2,3,----,+∞}

Intervals: is a set of all real numbers between two points on the real number line. (it is a subset of real numbers)

1. **Open interval:** is a set of all real numbers between A&B excluded (A&B are not elements in the set). $\{x: A < x < B\}$ or (A, B).

   -∞ ←———— A(———— X ————)B ————→ +∞

2. **Closed interval:** is a set of all real numbers between A&B included (A&B are elements in the set). {x: A ≤ x ≤ B} or [A, B].

   -∞ ←———— A[—— X ——]B ————→ +∞

3. **Half-Open interval (Half-Close):** is a set of all real numbers between A & B with one of the end-points as an element in the set.

   a) (A, B]= {x: A < x ≤ B} -∞ ←———— A(—— X ——]B ————→ +∞

   b) [A, B)= {x: A ≤ x < B} -∞ ←———— A[—— X ——)B ————→ +∞

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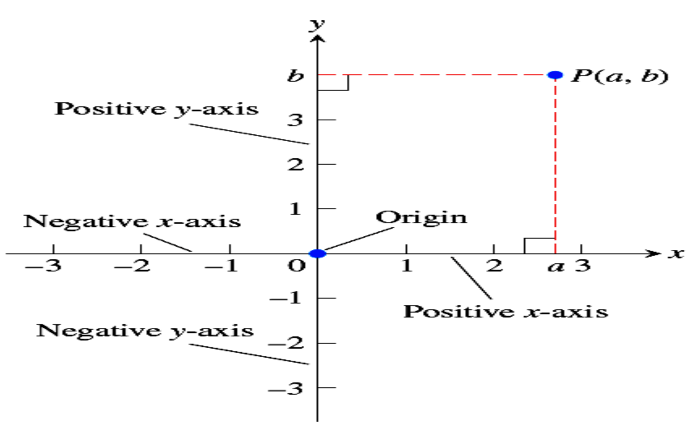
TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	a b
	$[a, b]$	$\{x \mid a \le x \le b\}$	Closed	a b
	$[a, b)$	$\{x \mid a \le x < b\}$	Half-open	a b
	$(a, b]$	$\{x \mid a < x \le b\}$	Half-open	a b
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	a
	$[a, \infty)$	$\{x \mid x \ge a\}$	Closed	a
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	b
	$(-\infty, b]$	$\{x \mid x \le b\}$	Closed	b
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

Coordinate in the Plane (الاحداثيات فى الفراغ او المستوى)

Each point in the plane can be represented with a pair of real numbers (a,b), the number a is the horizontal distance from the origin to point P,
while b is the vertical distance from the origin to point P. The origin divides the x-axis into positive x axis to the right and the negative x-axis to the left, also, the origin divides the y-axis into positive y-axis upward and the negative x-axis downward. The axes divide the plane into four regions called quadrants.



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Distance between Points and (Mid-Point Formula):

Distance between points in the plane is calculated with a formula that comes from Pythagorean Theorem:

- **Distance Formula for Points in the Plane**

The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$\mathbf{d} = \sqrt{(\Delta \mathbf{x})^2 + (\Delta \mathbf{y})^2} = \sqrt{(\mathbf{x_2} - \mathbf{x_1})^2 + (\mathbf{y_2} - \mathbf{y_1})^2}$$

and the mid-point formula:

$$\mathbf{x_0} = \frac{\mathbf{x_1} + \mathbf{x_2}}{\mathbf{2}} \quad , \mathbf{y_0} = \frac{\mathbf{y_1} + \mathbf{y_2}}{\mathbf{2}}$$

Example 2: find the distance between **P(-1,2)** and **Q(3,4)** and find the mid-point:

Sol.:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-1))^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$x_0 = \frac{x_1+x_2}{2} \quad , x_0 = \frac{-1+3}{2} = 1 \text{ and } \quad y_0 = \frac{y_1+y_2}{2} \quad , y_0 = \frac{2+4}{2} = 3.$$

Example 3: find the distance between **R(2,-3)** and **S(6,1)** and find the mid-point:

Sol.:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6 - 2)^2 + (1 - (-3))^2} = \sqrt{16 + 16} = \sqrt{32} = 2\sqrt{8}$$

$$x_0 = \frac{x_1+x_2}{2} \quad , x_0 = \frac{2+6}{2} = 4 \text{ and } \quad y_0 = \frac{y_1+y_2}{2} \quad , y_0 = \frac{-3+1}{2} = -1.$$

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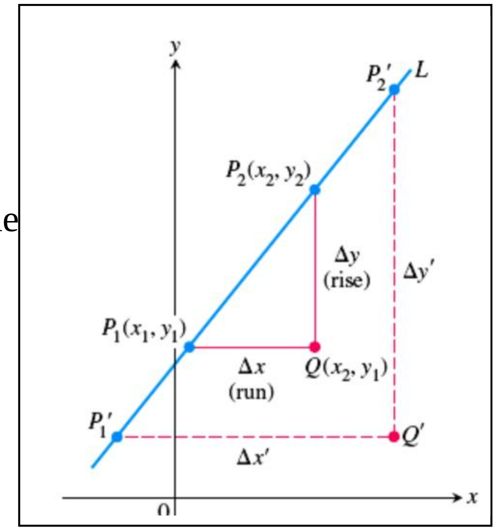
Slope and Equation of Line

- **Slope (الميل):** The constant

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the slope of non-vertical line $P_1 P_2$.



Note1: Horizontal line have (**m=0**) (**Δy** =0), and the vertical line has no slope or the slope of vertical line is undefined (**Δx** =0).

Note2: Parallel lines have the same slope
In the the lines are parallel then **(m1= m2).**

Note3: If two non-vertical lines L1and L2 are perpendicular, their slopes **m1** and **m2** satisfy

m1*m2 = -1,

so each slope is the negative reciprocal of the other.

$$m_1 = \frac{-1}{m_2} \; and \; m_2 = \frac{-1}{m_1}$$

Example 4: Find the slope of the straight line through the two points **P(3,2)** and **Q(4,4)** :

Sol.:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{4-3} = 2.$$

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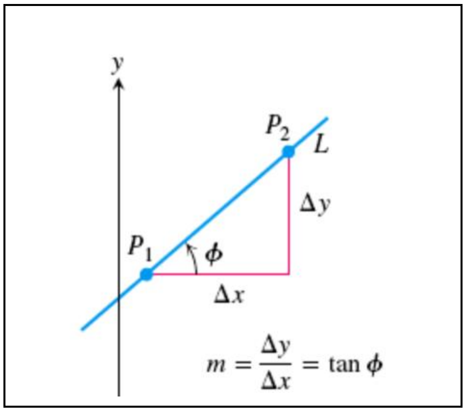
❖ Point-Slope Equation:



We can write an equation for a non-vertical straight line L if we know its slope m and the coordinate of one point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is any other point on L, then we can use two points P_1 and P to compute the slope,

$$m = \frac{y - y_1}{x - x_1}$$

so that $y - y_1 = m(x - x_1)$

or $y = y_1 + m(x - x_1)$

The equation $\mathbf{y = y_1 + m(x - x_1)}$

is the point-slope equation of the line that passes through the point $\mathbf{P_1(x_1, y_1)}$ and has slope m.

Example 5: write an equation for the line pass through the point **(2,3)** with slope **(-3/2).**

Sol.: we substitute $x_1 = 2$, $y_1 = 3$, and m = -3/2 into the point-slope equation and obtain

$$\mathbf{y = y_1 + m(x - x_1)}$$

$$\mathbf{y = 3 + \frac{-3}{2}(x - 2)}$$

$$\mathbf{y = -\frac{3}{2}x + 6.}$$

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Example 6: A line pass through two points: write an equation for the line through
(-2,-1) and **(3,4)**

Sol.: The line's slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{-2-3} = \frac{-5}{-5} = 1.$$

We can use this slope with either of the two given points in the point-slope equation;

With (x1,y1) = (–2, –1)

y = –1 + 1 . (x–(–2))

y = –1 + x + 2

y = x + 1

With (x2,y2) = (3, 4)

y = 4 + 1 . (x–3)

y = 4 + x –3

y = x + 1

Note: The equation:

y = mx + b

is called the **slope-intercept equation** of the line with slope m and y-intercept b

Note: **The general form of straight line equation** is

Ax + By + C= 0

Example 7: finding the slope and y-Intercept for the line 8x + 5y = 20.

Sol.: solve the equation for y to put it in slope-intercept form :

$$8x + 5y = 20$$
$$5y = -8x + 20$$
$$y = -8/5\, x + 4.$$

The slope **m = –8/5**. the y-intercept is **b = 4**.

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Example 8: Find the line L1 passes through the point P(1,2) and parallel the line L2: x + 2y = 3.

SOL: L1: P(1,2) M=???

L2: x + 2y = 3.

L1 parallel the line L2 so that m1=m2.

x + 2y = 3

y = -1/2 X + 3/2

then m2= -1/2 so that m1=-1/2

$$\mathbf{y = y_1 + m(x - x_1)}$$

$$\mathbf{y = 2 + (-\frac{1}{2})(x - 1)}$$

$$\mathbf{y = 2 + (-\frac{1}{2}x + \frac{1}{2})}$$

$$\mathbf{y = -\frac{1}{2}x + \frac{5}{2}}$$

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The Distance from a Point to a Line:

The distance (d) between the line L is Ax + By + C= 0 and the point $P(x_1, y_1)$:

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Example 9: Find the distance from the point P(2,1) to the line y = x + 2

SOL: 1- put the line in the general form Ax + By + C= 0

y = x + 2

-x+y-2=0

so that A=-1 ,B=1 , C=-2 , $x_1 = 2$, $y_1 = 1$

$$d = \frac{|\mathbf{A}x_1 + \mathbf{B}y_1 + \mathbf{C}|}{\sqrt{A^2 + B^2}} = \frac{|-1 * (2) + 1 * (1) + (-2)|}{\sqrt{(-1)^2 + (1)^2}}$$

$$= \frac{|-3|}{\sqrt{2}} = \frac{3}{\sqrt{2}}.$$

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H.W:

1. finding the slope and y-Intercept for the line 4x + 2y = 4.
2. write an equation for the line pass through (-1,-1) and (1,5).
3. write an equation for the line pass through the point (1,-1) with slope (5).
4. Find the slope of the straight line through the two points P(5,-2) and Q(3,6).
5. write an equation for the horizontal line pass through the point (1,-1) .
6. Find the line L1 passes through the point (-2,2) and perpendicular to the line L2 : 2x + y = 4.
7. Find the distance from the point P(3,2) to the line y = 3x – 4.
8. Find the distance from the point P(-4,1) to the line y = -2x +1.
9. Find the following:
   * The slope of the line 2x+3y-5=0?
   *The distance from the above line to the point P(-1,0).

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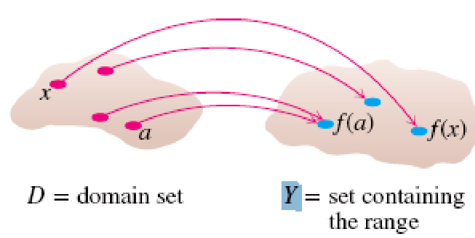
Functions الدوال

DEFINITION: Function

A **Functions** is a set D (domain) to a set R (range) is a rule that assigns to unique (single) element f(x)□R to each element x□D.

$$F: X \rightarrow F(X)$$ it means that f sends x to f(x)=y



- **The set of x is called the "Domain" of the function (Df).**
- **The set of y is called the "Range" of the function (Rf).**

Domain (Df): is the set of all possible inputs (x-values).
Range (Rf): is the set of all possible outputs (y-values).

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Note: To find Domain (Df) and the Range (Rf) the following points must be noticed:

1- The denominator in a function must not equal zero (never divide by zero).
2- The values under even roots must be positive.

Examples: Find the Domain (Df) and Range (Rf) of the following functions:

1- $y = f(x) = \frac{1}{x}$

Sol: denominator must not equal zero

$$x \neq 0$$

✓ Df =R/{0}

To find Rf : we must convert the function from y=f(x) into x=f(y).

$y = \frac{1}{x} \quad \rightarrow \quad x = \frac{1}{y}$

✓ Rf =R/{0}.

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2- $y = \sqrt{3 - X}$

$$3 - X \geq 0 \rightarrow \quad 3 \geq X$$

✓ $Df = \{x \in R \,/\, x \leq 3\}$

To find Rf : we must convert the function from y=f(x) into x=f(y).

$$y = \sqrt{3 - x}$$
$$y^2 = 3 - x$$
$$x = 3 - y^2$$

✓ $Rf = \{y \in R\}$.

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H.W: Find the Domain (Df) and Range (Rf) of the following functions:

1- $y = \frac{1}{x^2}$
2- $y = 2x^2$
3- $y = \sqrt{5 - 2X}$.

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Absolute Value Function: it is defined as:

$$y = |x| = \sqrt{x^2} = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

Absolute Value Properties

1.	$\lvert -a \rvert = \lvert a \rvert$	A number and its additive inverse or negative have the same absolute value.
2.	$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	The absolute value of a product is the product of the absolute values.
3.	$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}$	The absolute value of a quotient is the quotient of the absolute values.
4.	$\lvert a + b \rvert \le \lvert a \rvert + \lvert b \rvert$	The **triangle inequality**. The absolute value of the sum of two numbers is less than or equal to the

Absolute Values and Intervals

If a is any positive number, then

5.	$\lvert x \rvert = a$	if and only if	$x = \pm a$
6.	$\lvert x \rvert < a$	if and only if	$-a < x < a$
7.	$\lvert x \rvert > a$	if and only if	$x > a$ or $x < -a$
8.	$\lvert x \rvert \le a$	if and only if	$-a \le x \le a$
9.	$\lvert x \rvert \ge a$	if and only if	$x \ge a$ or $x \le -a$

Examples: Solve the following for x?

1. $|x| = 3$ **sol.**: So $x=3$ & $x=-3$ $D_f=\{3,-3\}$

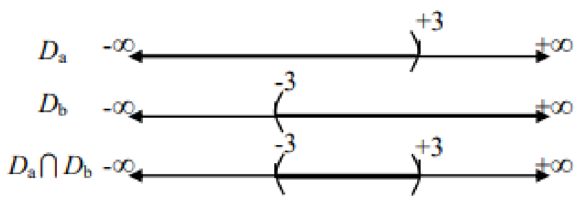
2. $|x| < 3$ **sol.**: So a: $x < 3$

$\Rightarrow$ $D_a=(-\infty,3)$

and b: $-x < 3$ (multiply by -1)

$\Rightarrow x > -3 \Rightarrow D_b = (-3,\infty)$

$\therefore D_f = D_a \cap D_b = (-3,3)$



3. $|x| \ge 3$

sol.: *Note* (solution must consist the remaining part of the real numbers line of the previous example)

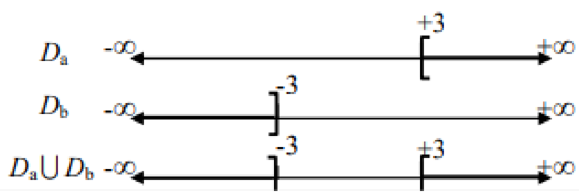
So a: either $x \ge 3$ $\Rightarrow$ $D_a=[3,\infty)$

b: or $-x \ge 3$ (multiply by -1)$\Rightarrow x \le -3$

$\Rightarrow$ $D_b=(-\infty,-3]$

$\therefore D_f = D_a \cup D_b = (-\infty,-3] \cup [3, \infty)$

$= R \backslash (-3,3)$



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4. $|2x-3| \le 7$

sol.: $-7 \le 2x-3 \le 7$

$-4 \le 2x \le 10 \quad \Rightarrow \quad -2 \le x \le 5$

$\therefore D_f = [-2,5]$

5. $|x-9| > 3$

sol.: $x-9 > 3 \quad$ or $\; x-9 < -3$

$\Rightarrow x > 12 \quad$ or $\; x < 3$

$\therefore D_f = (-\infty,3) \cup (12, \infty)$