



عنوان المحاضرة:

Triple Integrals التكاملات الثلاثية

Triple Integrals :-

The integral $\iiint_D f(x,y,z) dV$ is called the volume integral and it is evaluated in the manner that

$$\iiint_D f(x,y,z) \cdot dV = \iint_R \left(\int_{z_1}^{z_2} f(x,y,z) dz \right) \cdot dA$$

The volume of a closed bounded region D in Space is

$$V_D = \iiint_D dV$$

Ex:- Evaluate $\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz \cdot dx \cdot dy$

Solution:-

$$\int_0^{\sqrt{2}} \int_0^{3y} \left[z \right]_{x^2+3y^2}^{8-x^2-y^2} \cdot dx \cdot dy = \int_0^{\sqrt{2}} \int_0^{3y} [(8-x^2-y^2) - (x^2+3y^2)] \cdot dx \cdot dy$$

$$= \int_0^{\sqrt{2}} \int_0^{3y} (8-2x^2-4y^2) \cdot dx \cdot dy = \int_0^{\sqrt{2}} [8x - \frac{2x^3}{3} - 4y^2x]_0^{3y} \cdot dy$$

$$= \int_0^{\sqrt{2}} [(8 \times 3y - \frac{2}{3}(3y)^3 - 4y^2 \times 3y)] dy = \int_0^{\sqrt{2}} (24y - 18y^3 - 12y^3) \cdot dy$$

$$= \int_0^{\sqrt{2}} (24y - 30y^3) \cdot dy = \left[\frac{24y^2}{2} - \frac{30y^4}{4} \right]_0^{\sqrt{2}}$$

$$= 12(\sqrt{2})^2 - \frac{15}{2}(\sqrt{2})^4 = 24 - 30 = \boxed{-6}$$



H-W

① Evaluate the integral :

$$a - \int_0^2 \int_{-1}^1 (x-y) dy dx$$

$$b - \int_0^1 \int_1^2 xy e^x dy dx$$

$$c - \int_0^{\ln 2} \int_1^{\ln 5} e^{2x+y} dy dx$$

② Evaluate the double integral over the region R

$$a / \iint_R \frac{\sqrt{x}}{y^2} dA \quad R: 0 \leq x \leq 4 \quad \& \quad 1 \leq y \leq 2$$

$$b / \iint_R xy \cos y \cdot dA \quad R: -1 \leq x \leq 1 \quad \& \quad 0 \leq y \leq \pi$$

$$c / \iint_R \frac{xy^3}{x^2+1} dA \quad R: 0 \leq x < 1 \quad \& \quad 0 \leq y \leq 2$$

③ Sketch the region of integration and write an equivalent double integral with the order of integration reversed

$$a / \int_0^1 \int_2^{4-2x} dy dx$$

$$b / \int_0^1 \int_{1-x}^{1-x^2} dy dx$$

$$c / \int_0^1 \int_y^{\sqrt{y}} dx dy$$

④ Sketch the region bounded by the parabolas $x=y^2$ and $x=2y-y^2$. Then express the region's area as an iterated double integral and evaluate the integral.

$$⑤ Evaluate \int_0^{\ln 2} \int_0^e dx dy$$



6) A thin plate cover the triangle region bounded by the x-axis and the line $x=1$ and $y=2x$ in the first quarter. The plate density is $\delta(x,y) = 6x+6y+6$. find

- The mass of the body.
- The center of mass.
- The radii of gyration.

7) Evaluate

a/ $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz \cdot dy \cdot dx$

b/ $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz \cdot dy \cdot dx$

c/ $\int_0^{\pi/6} \int_0^1 \int_{-2}^3 y \sin z \cdot dx \cdot dy \cdot dz$



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----- نهاية محاضرة " التكاملات الثلاثية Triple Integrals " -----