عنوان المحاضرة: **Complex Numbers الاعداد المركبة**

Power & Roots of Comp. Numbs Form قوى وجذور الاعداد المركبة

Complex Functions الدوال المركبة

De Moivre's Theorem :-

If $Z = r(\cos\theta + i\sin\theta)$ and (n) is a positive integer number then:

$$Z^n = [r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

$$Z^{1/n} = [r(\cos\theta + i\sin\theta)]^{1/n}$$

$$= r^{1/n}\left[\cos\left(\frac{\theta + 2\pi K}{n}\right) + i\sin\left(\frac{\theta + 2\pi K}{n}\right)\right]$$

where $K = 1, 2, 3, \ldots, n-1$

EX:- Find $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$

Solution:-

$$r = \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2} = \boxed{\frac{1}{\sqrt{2}}}$$

$$\theta = \tan^{-1}\frac{1/2}{1/2} \Rightarrow \theta = 45^\circ = \boxed{\frac{\pi}{4}}$$

$$Z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$\therefore \left(\frac{1}{2} + i\frac{1}{2}\right)^{10} = \left(\frac{1}{\sqrt{2}}\right)^{10}\left(\cos\frac{10\pi}{4} + i\sin\frac{10\pi}{4}\right)$$

$$= \frac{1}{(\sqrt{2})^{10}}\left(\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right)$$

$$= \frac{1}{32}(0 + i)$$

$$= \boxed{\frac{1}{32}i}$$

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Ex:- Find the value of $(-1+i)^{1/3}$.

Solution:-

$r=\sqrt{(-1)^2+(1)^2} = \boxed{\sqrt{2}}$

$\theta = \tan^{-1}\frac{1}{-1} = \boxed{-45^\circ}$

$\theta = 180 - 45 = \boxed{135^\circ}$

$$z^{1/n} = r^{1/n}\left[\cos\left(\frac{\theta+2\pi K}{n}\right) + i\sin\left(\frac{\theta+2\pi K}{n}\right)\right]$$

$$(-1+i)^{1/3} = (\sqrt{2})^{1/3}\left[\cos\left(\frac{135+2\pi K}{3}\right) + i\sin\left(\frac{135+2\pi K}{3}\right)\right]$$

$K = 0, 1, 2 \qquad (n=3 \Rightarrow n-1=2)$

at $K=0 \Rightarrow (-1+i)^{1/3} = (\sqrt{2})^{1/3}\left[\cos\left(\frac{135}{3}\right) + i\sin\left(\frac{135}{3}\right)\right]$

$= (\sqrt{2})^{1/3}(\cos 45 + i\sin 45)$

at $K=1 \Rightarrow (-1+i)^{1/3} = (\sqrt{2})^{1/3}\left[\cos\left(\frac{135+2\pi}{3}\right) + i\sin\left(\frac{135+2\pi}{3}\right)\right]$

$= (\sqrt{2})^{1/3}\left[\cos 165 + i\sin 165\right]$

at $K=2 \Rightarrow (-1+i)^{1/3} = (\sqrt{2})^{1/3}\left[\cos\left(\frac{135+4\pi}{3}\right) + i\sin\left(\frac{135+4\pi}{3}\right)\right]$

$= (\sqrt{2})^{1/3}\left[\cos 285 + i\sin 285\right]$

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Roots of Equations :

If $Z = re^{i\theta}$ is a Complex number different from Zero and (n), then there are precisely (n) different complex numbers $w_0, w_1, \ldots, w_{n-1}$, that are ($n$th) roots of Z.

Let $\omega = \rho e^{i\alpha}$ is (nth) root of $Z = re^{i\theta}$

So $\omega^n = Z \Rightarrow \rho^n e^{in\alpha} = re^{i\theta}$

$\therefore \rho = \sqrt[n]{r}$ is the real positive (nth) root of $(r$

$n\alpha \neq \theta$

we can say they may differ only by an integer multiple of (2π) that is

$$n\alpha = \theta + 2\pi K$$

Where $K = 0, \mp 1, \mp 2, \ldots$

$$\alpha = \frac{\theta}{n} + \frac{2\pi K}{n}$$

The (nth) roots of $Z = re^{i\theta}$ are given by

$$\sqrt[n]{re^{i\theta}} = \sqrt[n]{r} \cdot e^{i\left(\frac{\theta}{n} + \frac{2\pi K}{n}\right)}$$

where $K = 0, \mp 1, \mp 2, \ldots, n-1$

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EX:— Find the sixth roots of $Z=-8$

Solution:—

$Z=-8+0i \Rightarrow r=\sqrt{(-8)^2+(0)^2}=\boxed{8}$

$\theta=\tan^{-1}\frac{0}{8} \Rightarrow \boxed{\theta=\pi}$

$Z=re^{i\theta} \Rightarrow \boxed{Z=8e^{i\pi}}$

$$\omega_k=\sqrt[n]{r}\, e^{i\left(\frac{\theta+2\pi k}{n}\right)}=\sqrt[n]{r}\left[\cos\left(\frac{\theta+2\pi k}{n}\right)+i\sin\left(\frac{\theta+2\pi k}{n}\right)\right]$$

$n=6 \Rightarrow k=0,1,2,3,4,5$

at $\underline{k=0} \Rightarrow \omega_0=\sqrt[6]{8}\left[\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right]=\sqrt{2}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

at $\underline{k=1} \Rightarrow \omega_1=\sqrt[6]{8}\left[\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right]=\sqrt{2}\,i$

at $\underline{k=2} \Rightarrow \omega_2=\sqrt[6]{8}\left[\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right]=\sqrt{2}\left(\frac{-\sqrt{3}}{2}+\frac{1}{2}i\right)$

at $\underline{k=3} \Rightarrow \omega_3=\sqrt[6]{8}\left[\cos\frac{7\pi}{0}+i\sin\frac{7\pi}{6}\right]=\sqrt{2}\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}i\right)$

at $\underline{k=4} \Rightarrow \omega_4=\sqrt[6]{8}\left[\cos\frac{9\pi}{6}+i\sin\frac{9\pi}{6}\right]=-\sqrt{2}\,i$

at $\underline{k=5} \Rightarrow \omega_5=\sqrt[6]{8}\left[\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right]=\sqrt{2}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$.

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Ex:- Find the roots of the equation $Z^3 = -1+i$

Solution:-

$-1+i = r(\cos\theta + i\sin\theta)$

$r = \sqrt{(-1)^2+(1)^2} = \boxed{\sqrt{2}}$

$\theta = \tan^{-1}\frac{1}{-1} \Rightarrow \theta = \boxed{\frac{3\pi}{4}}$

$Z^3 = -1+i \Rightarrow Z = \sqrt[3]{-1+i} = (-1+i)^{1/3}$

$\therefore (-1+i)^{1/3} = \left[\sqrt{2}\left(\cos\frac{3\pi}{4}+2\pi k\right) + i\sin\frac{3\pi}{4}+2\pi k)\right]^{1/3}$

$= (\sqrt{2})^{1/3}\left[\cos\left(\frac{3\pi}{4n}+\frac{2\pi k}{n}\right) + i\sin\left(\frac{3\pi}{4n}+\frac{2\pi k}{n}\right)\right]$

$k = 0, 1, 2$ & $n = 3$

at $\underline{k=0} \Rightarrow Z_0 = (\sqrt{2})^{1/3}\left[\cos\frac{3\pi}{12} + i\sin\frac{3\pi}{12}\right] = (\sqrt{2})^{1/3}(1+i)$

at $\underline{k=1} \Rightarrow Z_1 = (\sqrt{2})^{1/3}\left[\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right]$

at $\underline{k=2} \Rightarrow Z_2 = (\sqrt{2})^{1/3}\left[\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right]$

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H.W

Complex Numbers

(1) Find

a/ i^2

b/ i^3

c/ $(4-2i)+(-6+5i)$

d/ $\frac{-5+5i}{4-3i}$

(2) Prove that $|Z_1-Z_2|^2+|Z_1+Z_2|^2=2|Z_1|^2+2|Z_2|^2$

(3) Write the following in polar forms

a/ $4-3i$

b/ $\sqrt{-i}$

c/ Z^4

d/ $Z/\bar{Z}$

(4) Find the value of

a/ $(1+i)^8$

b/ $(i)^{1/4}$

c/ $\sqrt{Z}$

(5) Find x and y from the

$(x^2y-2)+i(x+2xy-5)=0$

(6) Find the fourth roots of (-1).

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