



Class: Third Class  
Subject: Theory of Machines  
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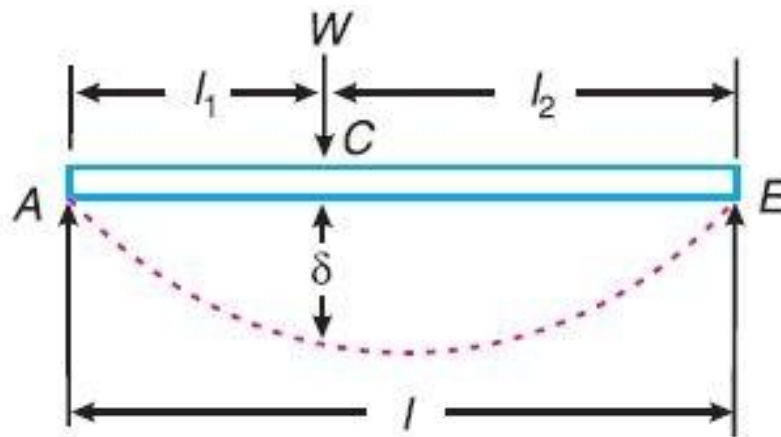
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## **NATURAL FREQUENCY OF FREE TRANSVERSE VIBRATIONS DUE TO A POINT LOAD ACTING OVER A SIMPLY SUPPORTED SHAFT**

التردد الطبيعي للاهتزازات الطولية تحت تأثير الحمل لنقطه واحده على العمود  
البسيط المثبت بطرق مختلفة

Consider a shaft AB of length  $l$ , carrying a point load  $W$  at C which is at a distance of  $l_1$  from A and  $l_2$  from B, as shown in Fig.1. A little consideration will show that when the shaft is deflected and suddenly released, it will make transverse vibrations. The deflection of the shaft is proportional to the load  $W$  and if the beam is deflected beyond the static equilibrium position then the load will vibrate with simple harmonic motion (as by a helical spring). If  $\delta$  is the static deflection due to load  $W$ , then the natural frequency of the free transverse vibration is



**Fig. 1** Simply supported beam with a point load.

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz} \quad \dots \text{ (Substituting, } g = 9.81 \text{ m/s}^2\text{)}$$

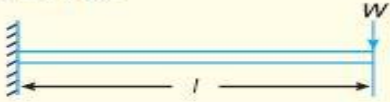
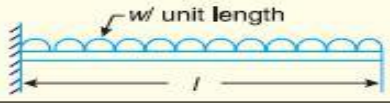

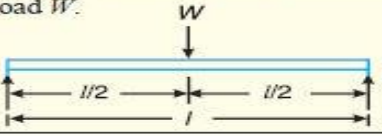
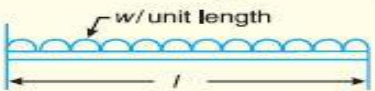
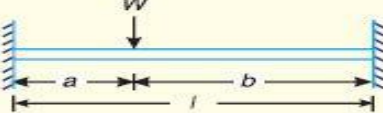
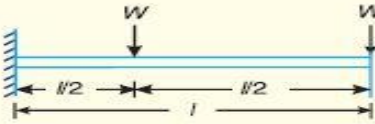
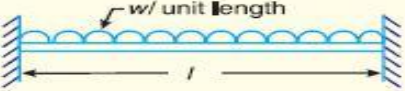
$$t_p = \frac{1}{f_n}$$

$$\omega_n = 2\sqrt{\frac{g}{\delta}}$$

Some of the values of the static deflection for the various types of beams and under various load conditions are given in the following table .

**Table.1.** Values of static deflection ( $\delta$ ) for the various types of beams and under various load conditions.

جدول رقم (1) حساب التشويه الاستاتيكي للأعمدة حسب نوع الحمل والتثبيت العمود .

S.No.	Type of beam	Deflection ( $\delta$ )
1.	Cantilever beam with a point load $W$ at the free end. 	$\delta = \frac{Wl^3}{3EI}$ (at the free end)
2.	Cantilever beam with a uniformly distributed load of $w$ per unit length. 	$\delta = \frac{wl^4}{8EI}$ (at the free end)
3.	Simply supported beam with an eccentric point load $W$ . 	$\delta = \frac{Wa^2b^2}{3EIl}$ (at the point load)
4.	Simply supported beam with a central point load $W$ . 	$\delta = \frac{Wl^3}{48EI}$ (at the centre)
5.	Simply supported beam with a uniformly distributed load of $w$ per unit length. 	$\delta = \frac{5}{384} \times \frac{wl^4}{EI}$ (at the centre)
6.	Fixed beam with an eccentric point load $W$ . 	$\delta = \frac{Wa^3b^3}{3EIl}$ (at the point load)
7.	Fixed beam with a central point load $W$ . 	$\delta = \frac{Wl^3}{192EI}$ (at the centre)
8.	Fixed beam with a uniformly distributed load of $w$ per unit length. 	$\delta = \frac{wl^4}{384EI}$ (at the centre)