Strategy for Solving Related Rate Problems

1. *Draw a picture and name the variables and constants.* Use t for time. Assume all variables are differentiable functions of t.
2. *Write down the numerical information* (in terms of the symbols you have chosen).
3. *Write down what you are asked to find* (usually a rate, expressed as a derivative).
4. *Write an equation that relates the variables.* You may have to combine two or more equations to get a single equation that relates the variable whose rate you want to the variable whose rate you know.
5. *Differentiate with respect to t.* Then express the rate you want in terms of the rate and variables whose values you know.
6. *Evaluate.* Use known values to find the unknown rate.

Applications

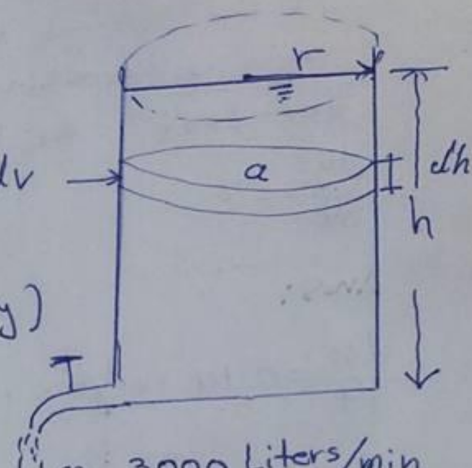
~~dv=a~~

1) **pumping out tank**

a) Determine the rate of hieght (h) drop.

b) the height (h) after $t = 100$ min

c) determine t when $h = 0$ (tank empty)



$Q = 3000$ Liters/min
$= 3\ m^3/min$

$r = 5\ m$

$h = 10\ m$

Ans:

$dv = a\,dh$ (a: tank area) $a = \pi r^2$

$dV = \pi r^2 dh$

V = volume of the tank

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ $\left(\text{but } \frac{dV}{dt} = Q = 3000 \frac{\text{Liters}}{\text{min}} = 3\ m^3/min\right)$

$3 \frac{m^3}{min} = \pi r^2 \frac{dh}{dt}$

a) $\frac{dh}{dt} = \frac{-3}{\pi r^2}$ (m/min) {height drop rate}

b) $dh = -\frac{3}{\pi r^2}\,dt$

$h = \frac{-3}{\pi r^2} t + c$ [at $h = 10$, $t = 0$]

$10 = c$

$$\boxed{h = -\frac{3}{\pi r^2} t + 10}$$

after $t = 100$ min

$h = -\frac{3}{\pi (5)^2} * 100 + 10 =$

$= -3.82 + 10 = 6.12\ m$

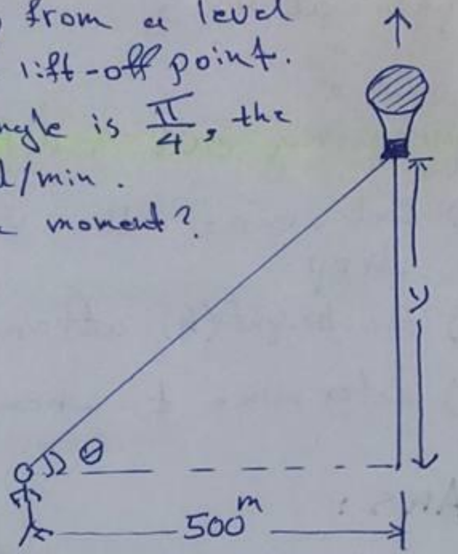
c) $0 = -\frac{3}{\pi 5^2} t + 10$

$\frac{3}{\pi 25} t = 10$

$t = \frac{250\pi}{3} = 261.8$ min

A rising ballon

EX: A hot-air ballon rising straight up from a level feild is tracked by a ranger 500 m from lift-off point. At the moment the ranger elevation angle is $\frac{\pi}{4}$, the angle is increasing at rate of 0.14 rad/min. How fast is the balloon rising at the moment?



$\frac{dy}{dt} = ?$

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Ans:

$$\frac{d\theta}{dt} = 0.14 \text{ rad/min}, \quad \theta = \frac{\pi}{4}$$

$$\tan\theta = \frac{y}{500}$$

$$y = 500 \tan\theta$$

$$\frac{dy}{d\theta} = 500 \sec^2\theta \quad \Rightarrow \quad dy = 500 \sec^2\theta \, d\theta$$

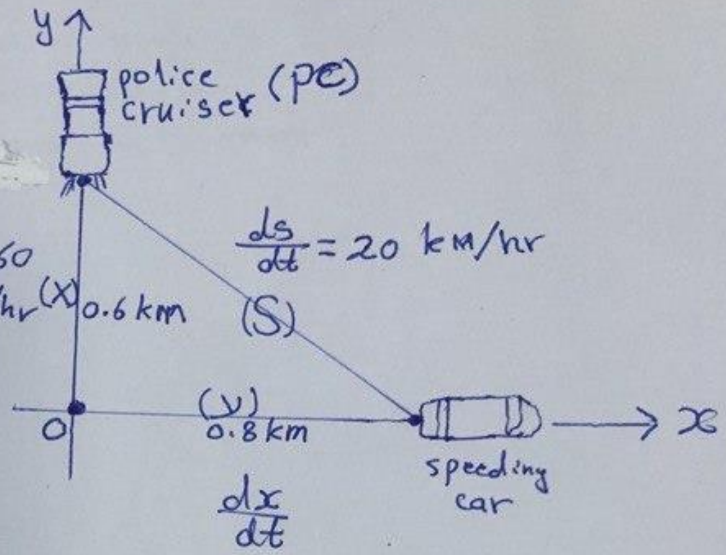
$$\frac{dy}{dt} = 500 \sec^2\theta \, \frac{d\theta}{dt} \qquad \left[\frac{d\theta}{dt} = 0.14 \frac{\text{rad}}{\text{min}}, \text{ when } \theta = \frac{\pi}{4}\right]$$

$$= 500^m (\sqrt{2})^2 * 0.14 \text{ rad/min} \qquad \left[\sec\frac{\pi}{4} = \sqrt{2}\right]$$

$$= 140 \text{ m/min}$$

EX: Highway chase مطاردة المرور السريع

A polic cruiser is chasing a speeding car which is now moving straight est. When PC at 0.6 kM north the (O), the police radar determine the distance between them and the car is increasing 20 kM/hr. If the cruiser is moving at 60 kM/hr, What the speed of the Care.



Ans:

By Pythagoras theorem

$S^2 = x^2 + y^2$, derive with respect to (t), t: time

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \quad \text{— (1)}$$

at the moment of chasing

$\frac{ds}{dt} = 20$ km/hr (distance increasing)

$\frac{dy}{dt} = -60$ kM/hr (cruiser speed in negative direction)

$x = 0.6$ kM (cruiser axis above or north the intersection •)

$y = 0.6$ km (car axis east •)

$S = \sqrt{0.8^2 + 0.6^2} = 1$ km

sub in (1)

$$2 * 1 * 20\frac{kM}{hr} = 2 * 0.8 * \frac{dx}{dt} + 2 * 0.6 * \left(-60\frac{kM}{hr}\right)$$

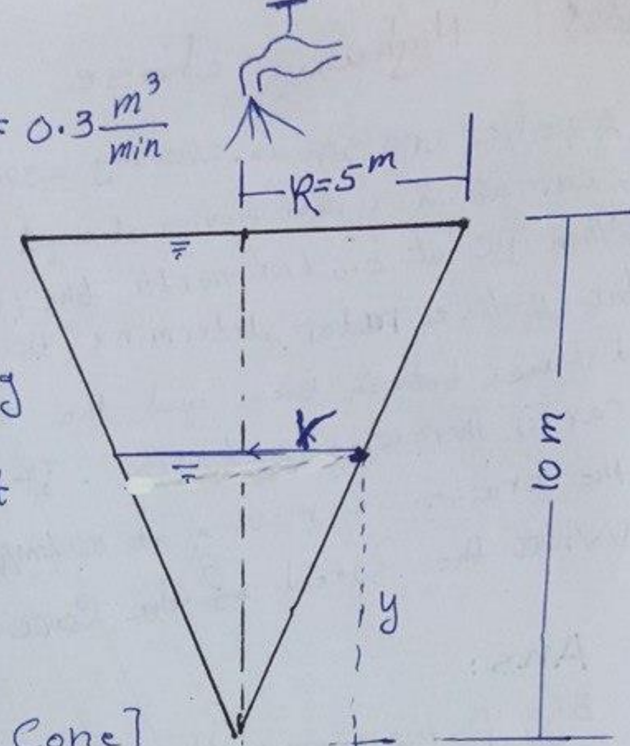
$$40 = 1.6\frac{dx}{dt} - 72$$

$$\therefore \frac{dx}{dt} = \frac{112}{1.6} = 70 \text{ kM}$$

EX : Water tank

$$\frac{dv}{dt} = Q = 0.3 \frac{m^3}{min}$$

water run into a conial tank at a rate $0.3\ m^3/hr$. The tank height is 10 m. The radius of the tank base is 5m. 1- How fast is the water level rising up
2- what is the rising speed at height $y = 6\ m$



Ans:

1) $V = \frac{\pi}{3} r^2 y$ — ① [volume of Cone]

but $\frac{r}{R} = \frac{y}{10} \Rightarrow \frac{r}{5} = \frac{y}{10}$.

$\therefore r = \frac{y}{2}$ sub in ①

$$V = \frac{\pi}{3} \left(\frac{y}{2}\right)^2 y$$

$$V = \frac{\pi}{12} y^3$$

derive with respect to time

$$\frac{dV}{dt} = \frac{\pi}{\cancel{12}_4} * \cancel{3} y^2 \frac{dy}{dt}$$

but $\frac{dV}{dt} = Q = 0.3 \frac{m^3}{min}$

$$\therefore 0.3 = \frac{\pi}{4} y^2 \frac{dy}{dt}$$

$$\therefore \frac{dy}{dt} = \frac{1.2}{\pi y^2}$$

2) $$\frac{dy}{dt} = \frac{1.2}{\pi (6)^2} = 0.001137\ m/min = 1.137\ mm/min = 68.2\ mm/hr$$
$$= 6.82\ cm/hr$$

Home work

Solve problems 13-39, page 177-178