second & high order derivatives

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$\acute{y} = \frac{dy}{dx}$ is the first Order derivatives of y to x

if $\acute{y}$ is defferentiable with respect to x, so

$\acute{\acute{y}} = \frac{d\acute{y}}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ which is call

second order derivative ($\frac{d^2y}{dx^2}$ or $\acute{\acute{y}}$)

again if $\acute{\acute{y}}$ is a differentiable in x, so

$y''' = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$ is the third order derivative

In general

$y^{(n)} = \frac{d}{dx} y^{(n-1)}$

How to read derivative symbols

y' "y prime"

y'' "y double prime"

y''' "y triple prime"

$y^{(n)}$ "y super n"

Ex: find y', y'', y''' and $y^{(4)}$ for $y = x^3 - 3x^2 + 2$

Ans:

$y' = 3x^2 - 6x$, $y'' = 6x - 6$

$y''' = 6$ $y^{(4)} = 0$

High order derivatives

let $y = f(x)$

1^{st} order derivative $\quad y' = f'(x)$

2^{nd} " " $\quad y'' = f''(x)$

3^{rd} " " $\quad y''' = f'''(x)$

Ex: $y = x^4 + 3x^3 + 2x + 5$

Find y' , y'' and y'''

Ans:

$$\frac{dy}{dx} = y' = 4x^3 + 9x^2 + 2$$

$$\frac{d^2y}{dx^2} = y'' = 12x^2 + 18x$$

$$\frac{d^3y}{dx^3} = y''' = 24x + 18$$

Ex: if $y = x(x+3)^2$, Find y' , y'' , y'''

Ans

$$y' = x * 2(x+3) + (x+3)$$

$$= 2x^2 + 3x + x + 3$$

$$= 2x^2 + 4x + 3$$

$$y'' = 4x^2 + 4$$

$$y''' = 12x$$

High order derivatives of rational function

Ex: Find $\frac{d^2y}{dx^2}$ if $2x^3 - 3y^2 = 7$

Ans:

$$2x^3 - 3y^2 = 7$$

$$6x^2 - 6y\frac{dy}{dx} = 0$$ or can be written

$$6x^2 - yy' = 0 \rightarrow \left(y' = \frac{6x^2}{y}\right)$$

derive again

$$12x - (y \cdot y'' + y' \cdot y') = 0$$

$$12x - yy'' + y'^2 = 0$$

$$-yy'' = -y'^2 - 12x$$

$$yy'' = y'^2 + 12x$$

$$y'' = \frac{(y')^2 + 12x}{y} = \frac{\left(\frac{6x^2}{y}\right)^2 + 12x}{y}$$

$$= \frac{\frac{36x^4}{y^2} + 12x}{y} = \frac{36x^4 \; 12xy^2}{y^3}$$

Exercises 2.6

Derivatives of Rational Powers

Find dy/dx in Exercises 1–10.

1. $y = x^{9/4}$

2. $y = x^{-3/5}$

3. $y = \sqrt[3]{2x}$

4. $y = \sqrt[4]{5x}$

5. $y = 7\sqrt{x+6}$

6. $y = -2\sqrt{x-1}$

7. $y = (2x+5)^{-1/2}$

8. $y = (1-6x)^{2/3}$

9. $y = x(x^2+1)^{1/2}$

10. $y = x(x^2+1)^{-1/2}$

Find the first derivatives of the functions in Exercises 11–18.

11. $s = \sqrt[7]{t^2}$

12. $r = \sqrt[4]{\theta^{-3}}$

13. $y = \sin\left[(2t+5)^{-2/3}\right]$

14. $z = \cos\left[(1-6t)^{2/3}\right]$

15. $f(x) = \sqrt{1-\sqrt{x}}$

16. $g(x) = 2(2x^{-1/2}+1)^{-1/3}$

17. $h(\theta) = \sqrt[3]{1+\cos(2\theta)}$

18. $k(\theta) = (\sin(\theta+5))^{5/4}$

Differentiating Implicitly

Use implicit differentiation to find dy/dx in Exercises 19–32.

19. $x^2y + xy^2 = 6$

20. $x^3 + y^3 = 18xy$

21. $2xy + y^2 = x + y$

22. $x^3 - xy + y^3 = 1$

23. $x^2(x-y)^2 = x^2 - y^2$

24. $(3xy+7)^2 = 6y$

25. $y^2 = \dfrac{x-1}{x+1}$

26. $x^2 = \dfrac{x-y}{x+y}$

27. $x = \tan y$

28. $x = \sin y$

29. $x + \tan(xy) = 0$

30. $x + \sin y = xy$

31. $y\sin\left(\dfrac{1}{y}\right) = 1 - xy$

32. $y^2\cos\left(\dfrac{1}{y}\right) = 2x + 2y$

Find $dr/d\theta$ in Exercises 33–36.

33. $\theta^{1/2} + r^{1/2} = 1$

34. $r - 2\sqrt{\theta} = \dfrac{3}{2}\theta^{2/3} + \dfrac{4}{3}\theta^{3/4}$

35. $\sin(r\theta) = \dfrac{1}{2}$

36. $\cos r + \cos\theta = r\theta$

Higher Derivatives

In Exercises 37–42, use implicit differentiation to find dy/dx and then d^2y/dx^2.

37. $x^2 + y^2 = 1$

38. $x^{2/3} + y^{2/3} = 1$

39. $y^2 = x^2 + 2x$

40. $y^2 - 2x = 1 - 2y$

41. $2\sqrt{y} = x - y$

42. $xy + y^2 = 1$

43. If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point $(2, 2)$.

44. If $xy + y^2 = 1$, find the value of d^2y/dx^2 at the point $(0, -1)$.

Solve all

Slopes, Tangents, and Normals

In Exercises 45 and 46, find the slope of the curve at the given points.

45. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$

46. $(x^2+y^2)^2 = (x-y)^2$ at $(1, 0)$ and $(1, -1)$

In Exercises 47–56, verify that the given point is on the curve and find the lines that are (a) tangent and (b) normal to the curve at the given point.

47. $x^2 + xy - y^2 = 1$, $(2, 3)$

48. $x^2 + y^2 = 25$, $(3, -4)$

49. $x^2y^2 = 9$, $(-1, 3)$

50. $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$

51. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$, $(-1, 0)$

52. $x^2 - \sqrt{3}xy + 2y^2 = 5$, $(\sqrt{3}, 2)$

53. $2xy + \pi\sin y = 2\pi$, $(1, \pi/2)$

54. $x\sin 2y = y\cos 2x$, $(\pi/4, \pi/2)$

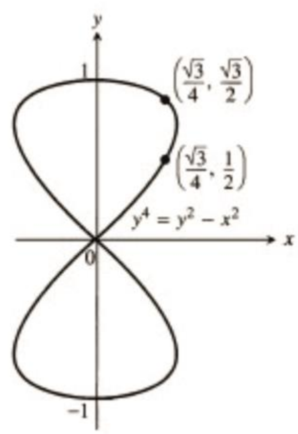
55. $y = 2\sin(\pi x - y)$, $(1, 0)$

56. $x^2\cos^2 y - \sin y = 0$, $(0, \pi)$

57. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

58. Find points on the curve $x^2 + xy + y^2 = 7$ (a) where the tangent is parallel to the x-axis and (b) where the tangent is parallel to the y-axis. In the latter case, dy/dx is not defined, but dx/dy is. What value does dx/dy have at these points?

59. *The eight curve.* Find the slopes of the curve $y^4 = y^2 - x^2$ at the two points shown here.



How to read the symbols for derivatives

y'	"y prime"	y''	"y double prime"
$\frac{d^2y}{dx^2}$	"d squared y dx squared"		
y'''	"y triple prime"		
$y^{(n)}$	"y super n"		
$\frac{d^ny}{dx^n}$	"d to the n of y by dx to the n"		

EXAMPLE 13 The first four derivatives of $y = x^3 - 3x^2 + 2$ are

First derivative: $y' = 3x^2 - 6x$

Second derivative: $y'' = 6x - 6$

Third derivative: $y''' = 6$

Fourth derivative: $y^{(4)} = 0.$

The function has derivatives of all orders, the fifth and later derivatives all being zero. ❑

Solve as much as you can

Exercises 2.2

Derivative Calculations

In Exercises 1–12, find the first and second derivatives.

1. $y = -x^2 + 3$
2. $y = x^2 + x + 8$
3. $s = 5t^3 - 3t^5$
4. $w = 3z^7 - 7z^3 + 21z^2$
5. $y = \frac{4x^3}{3} - x$
6. $y = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4}$
7. $w = 3z^{-2} - \frac{1}{z}$
8. $s = -2t^{-1} + \frac{4}{t^2}$
9. $y = 6x^2 - 10x - 5x^{-2}$
10. $y = 4 - 2x - x^{-3}$
11. $r = \frac{1}{3s^2} - \frac{5}{2s}$
12. $r = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4}$

In Exercises 13–16, find y' (a) by applying the Product Rule and (b) by multiplying the factors to produce a sum of simpler terms to differentiate.

13. $y = (3 - x^2)(x^3 - x + 1)$
14. $y = (x - 1)(x^2 + x + 1)$
15. $y = (x^2 + 1)\left(x + 5 + \frac{1}{x}\right)$
16. $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

Find the derivatives of the functions in Exercises 17–28.

17. $y = \frac{2x + 5}{3x - 2}$
18. $z = \frac{2x + 1}{x^2 - 1}$
19. $g(x) = \frac{x^2 - 4}{x + 0.5}$
20. $f(t) = \frac{t^2 - 1}{t^2 + t - 2}$
21. $v = (1 - t)(1 + t^2)^{-1}$
22. $w = (2x - 7)^{-1}(x + 5)$
23. $f(s) = \frac{\sqrt{s} - 1}{\sqrt{s} + 1}$
24. $u = \frac{5x + 1}{2\sqrt{x}}$
25. $v = \frac{1 + x - 4\sqrt{x}}{x}$
26. $r = 2\left(\frac{1}{\sqrt{\theta}} + \sqrt{\theta}\right)$
27. $y = \frac{1}{(x^2 - 1)(x^2 + x + 1)}$
28. $y = \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$

Find the derivatives of all orders of the functions in Exercises 29 and 30.

29. $y = \frac{x^4}{2} - \frac{3}{2}x^2 - x$
30. $y = \frac{x^5}{120}$

Find the first and second derivatives of the functions in Exercises 31–38.

31. $y = \frac{x^3 + 7}{x}$
32. $s = \frac{t^2 + 5t - 1}{t^2}$
33. $r = \frac{(\theta - 1)(\theta^2 + \theta + 1)}{\theta^3}$
34. $u = \frac{(x^2 + x)(x^2 - x + 1)}{x^4}$
35. $w = \left(\frac{1 + 3z}{3z}\right)(3 - z)$
36. $w = (z + 1)(z - 1)(z^2 + 1)$
37. $p = \left(\frac{q^2 + 3}{12q}\right)\left(\frac{q^4 - 1}{q^3}\right)$
38. $p = \frac{q^2 + 3}{(q - 1)^3 + (q + 1)^3}$

Using Numerical Values

39. Suppose u and v are functions of x that are differentiable at $x = 0$ and that

$$u(0) = 5, \quad u'(0) = -3, \quad v(0) = -1, \quad v'(0) = 2.$$

Find the values of the following derivatives at $x = 0$.

a) $\frac{d}{dx}(uv)$ **b)** $\frac{d}{dx}\left(\frac{u}{v}\right)$ **c)** $\frac{d}{dx}\left(\frac{v}{u}\right)$ **d)** $\frac{d}{dx}(7v - 2u)$

40. Suppose u and v are differentiable functions of x and that

$$u(1) = 2, \quad u'(1) = 0, \quad v(1) = 5, \quad v'(1) = -1.$$

Find the values of the following derivatives at $x = 1$.

a) $\frac{d}{dx}(uv)$ **b)** $\frac{d}{dx}\left(\frac{u}{v}\right)$ **c)** $\frac{d}{dx}\left(\frac{v}{u}\right)$ **d)** $\frac{d}{dx}(7v - 2u)$

Section 2.2, pp. 129–131

1. $\frac{dy}{dx} = -2x,\ \frac{d^2y}{dx^2} = -2$

3. $\frac{ds}{dt} = 15t^2 - 15t^4,\ \frac{d^2s}{dt^2} = 30t - 60t^3$

5. $\frac{dy}{dx} = 4x^2 - 1,\ \frac{d^2y}{dx^2} = 8x$

7. $\frac{dw}{dz} = -6z^{-3} + \frac{1}{z^2},\ \frac{d^2w}{dz^2} = 18z^{-4} - \frac{2}{z^3}$

9. $\frac{dy}{dx} = 12x - 10 + 10x^{-3},\ \frac{d^2y}{dx^2} = 12 - 30x^{-4}$

11. $\frac{dr}{ds} = \frac{-2}{3s^3} + \frac{5}{2s^2},\ \frac{d^2r}{ds^2} = \frac{2}{s^4} - \frac{5}{s^3}$

13. $y' = -5x^4 + 12x^2 - 2x - 3$ **15.** $y' = 3x^2 + 10x + 2 - \frac{1}{x^2}$

17. $y' = \frac{-19}{(3x-2)^2}$ **19.** $g'(x) = \frac{x^2 + x + 4}{(x+0.5)^2}$

21. $\frac{dv}{dt} = \frac{t^2 - 2t - 1}{(1+t^2)^2}$ **23.** $f'(s) = \frac{1}{\sqrt{s}(\sqrt{s}+1)^2}$

25. $v' = -\frac{1}{x^2} + 2x^{-3/2}$ **27.** $y' = \frac{-4x^3 - 3x^2 + 1}{(x^2-1)^2(x^2+x+1)^2}$

29. $y' = 2x^3 - 3x - 1,\ y'' = 6x^2 - 3,\ y''' = 12x,\ y^{(4)} = 12,$ $y^{(n)} = 0$ for $n \geq 5$

31. $y' = 2x - 7x^{-2},\ y'' = 2 + 14x^{-3}$

33. $\frac{dr}{d\theta} = 3\theta^{-4},\ \frac{d^2r}{d\theta^2} = -12\theta^{-5}$

35. $\frac{dw}{dz} = -z^{-2} - 1,\ \frac{d^2w}{dz^2} = 2z^{-3}$

37. $\frac{dp}{dq} = \frac{1}{6}q + \frac{1}{6}q^{-3} + q^{-5},\ \frac{d^2p}{dq^2} = \frac{1}{6} - \frac{1}{2}q^{-4} - 5q^{-6}$

39. a) 13 b) -7 c) $7/25$ d) 20 **41.** a) $y = -\frac{x}{8} + \frac{5}{4}$ b) $m = -4$ at $(0, 1)$ c) $y = 8x - 15,\ y = 8x + 17$

43. $y = 4x,\ y = 2$ **45.** $a = 1, b = 1, c = 0$ **47.** a) $y = 2x + 2,$ c) $(2, 6)$ **49.** $\frac{dP}{dV} = -\frac{nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$

51. The Product Rule is then the Constant Multiple Rule, so the latter is a special case of the Product Rule.

55. a) $\frac{3}{2}x^{1/2}$, b) $\frac{5}{2}x^{3/2}$, c) $\frac{7}{2}x^{5/2}$, d) $\frac{d}{dx}\left(x^{n/2}\right) = \frac{n}{2}x^{(n/2)-1}$