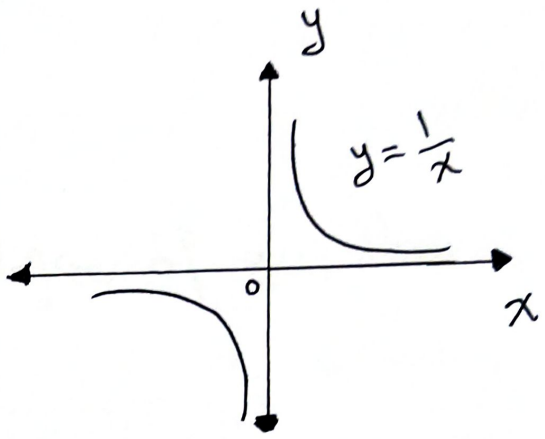
Lecture Two :

Limits Involving Infinity :-

For example , If we have the function $y = \frac{1}{x}$

$\lim_{x \to \infty} \frac{1}{x} = 0$ and $\lim_{x \to -\infty} \frac{1}{x} = 0$



Another example is a constant function $y = K$: $\lim_{x \to \infty} K = K$

$\lim_{x \to -\infty} K = K$

Note : the limit of $\frac{\sin \theta}{\theta}$ as θ approaches $\mp \infty$ is 0

Ex :- prove that $\lim_{\theta \to \infty} \frac{\sin \theta}{\theta} = 0$

Sol :-

$$-1 \leq \sin \theta \leq 1$$

$$[\div \theta] \quad \frac{-1}{\theta} \leq \frac{\sin \theta}{\theta} \leq \frac{1}{\theta}$$

Now : $\lim_{x \to \infty} \frac{-1}{\theta} = 0$ and $\lim_{\theta \to \infty} \frac{1}{\theta} = 0$

From Sandwich theorem $\Rightarrow \lim_{\theta \to \infty} \frac{\sin \theta}{\theta} = 0$

Limits of rational functions as x approaches $\mp\infty$.

Rules :- For the rational function $\frac{f(x)}{g(x)}$

1. If degree of $f(x)$ Less than degree of $g(x)$ then $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = 0$.

2. If degree of $f(x)$ equals degree of $g(x)$ then $\lim_{x\to\pm\infty} \frac{f(x)}{g(x)}$ is finite.

3. If degree of $f(x)$ greater than degree of $g(x)$ then $\lim_{x\to\mp\infty}$ is infinite.

- To solve the limit problem of rational functions as x approaches $\mp\infty$, divide both the numerator and the denominator by the highest power of x in denominator.

Ex :-

1. $\lim_{x\to\infty} \frac{3x+1}{x^2-5} = 0$ [deg. of numerator less than deg. of denominator]

2. $\lim_{x\to\infty} \frac{-x}{7x+3} = \lim_{x\to\infty} \frac{-x/x}{(7x/x + 3/x)}$

$= \lim_{x\to\infty} \frac{-1}{7+3/x} = -1/7$

3. $\lim_{x\to\infty} \frac{-4x^3+7x}{2x^2-3x-10}$

$\lim_{x\to\infty} \frac{(-4x^3/x^2)+(7x/x^2)}{(2x^2/x^2)-(3x/x^2)-(10/x^2)}$

$\lim_{x\to\infty} \frac{-4x+7/x}{2-3/x-10/x^2}$

$= \frac{-\infty+0}{2-0-0} = \frac{-\infty}{2} = -\infty$

4. $\lim_{x\to\infty} \frac{\cos 1/x}{1+1/x} = \frac{\cos 0}{1+0} = 1$

5. $\lim_{x\to\infty} x \sin 1/x$

$\lim_{x\to\infty} x \sin\frac{1}{x}$

let $\theta = \frac{1}{x}$

as $x \to \infty$ then $\theta \to 0$

$= \lim_{\theta\to 0} \frac{1}{\theta} \sin\theta = 1$