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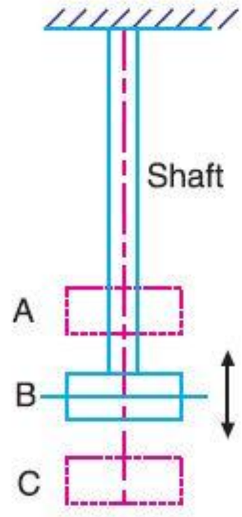
EXAMPLES ON LONGITUDINAL AND TRANSVERSE VIBRATIONS

امثلة تطبيقية على الاهتزازات الحرة الطولية والعرضية

Example .1.

A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m2. Determine the frequency of longitudinal and transverse vibrations of the shaft.

Solution. Given : d = 50 mm = 0.05 m ; l = 300 mm = 0.03 m ; m = 100 kg ; $E = 200\ \text{GN/m}^2 = 200\times10^9\ \text{N/m}^2$



We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4}\times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}\,\text{m}^2$$

and moment of inertia of the shaft,

$$I = \frac{\pi}{64}\times d^4 = \frac{\pi}{64}(0.05)^4 = 0.3\times10^{-6}\,\text{m}^4$$

Frequency of longitudinal vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l}{A.E} = \frac{100\times9.81\times0.3}{1.96\times10^{-3}\times200\times10^9} = 0.751\times10^{-6}\ \text{m}$$

$$\ldots(\because\ W = m.g)$$

∴ Frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575 \text{ Hz} \quad \textbf{Ans.}$$

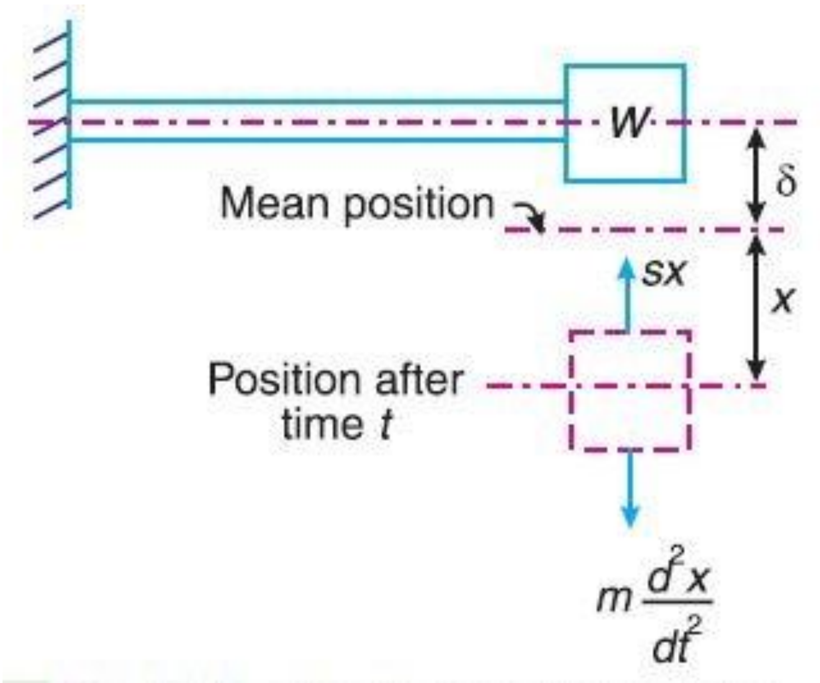
Frequency of transverse vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l^3}{3E.I} = \frac{100 \times 9.81 \times (0.3)^3}{3 \times 200 \times 10^9 \times 0.3 \times 10^{-6}} = 0.147 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.147 \times 10^{-3}}} = 41 \text{ Hz} \quad \textbf{Ans.}$$



Example .2.

A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibration. Assume E = 200 GN/m2 and shaft diameter = 50 mm.

Solution. Given : $l = 0.75$ m ; $m = 90$ kg ; $a = AC = 0.25$ m ; $E = 200$ GN/m^2 = 200×10^9 N/m^2; $d = 50$ mm = 0.05 m

The shaft is shown in Fig. 2

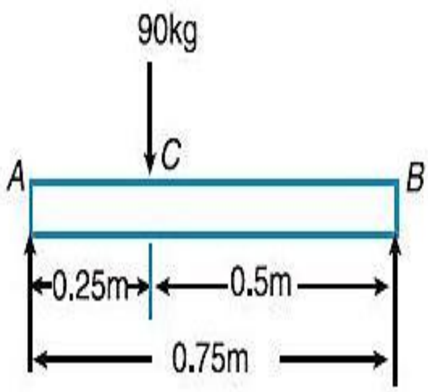


Fig. 2

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 \text{ m}^4$$

$$= 0.307 \times 10^{-6} \text{ m}^4$$

and static deflection at the load point (*i.e.* at point *C*),

$$\delta = \frac{Wa^2b^2}{3EIl} = \frac{90 \times 9.81(0.25)^2(0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 0.75} = 0.1 \times 10^{-3} \text{ m}$$

$$\ldots (\because b = BC = 0.5 \text{ m})$$

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Example.3.

A flywheel is mounted on a vertical shaft as shown in Fig.3. The both ends of the shaft are fixed and its diameter is 50 mm. The flywheel has a mass of 500 kg. Find the natural frequencies of longitudinal and transverse vibrations. Take E = 200 GN/m2.

Solution. Given : $d = 50$ mm $= 0.05$ m ; $m = 500$ kg ; $E = 200$ GN/m^2 $= 200 \times 10^9$ N/m^2

We know that cross-sectional area of shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3} \text{ m}^2$$

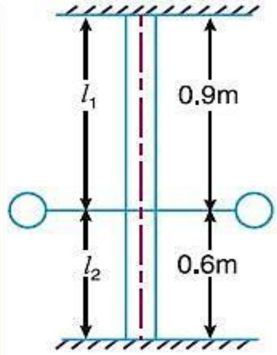

Fig. 3

and moment of inertia of shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6} \text{m}^4$$

Natural frequency of longitudinal vibration

Let m_1 = Mass of flywheel carried by the length l_1.

$\therefore$ $m - m_1$ = Mass of flywheel carried by length l_2.

We know that extension of length l_1

$$= \frac{W_1 . l_1}{A.E} = \frac{m_1 . g . l_1}{A.E} \quad \ldots (i)$$

Similarly, compression of length l_2

$$= \frac{(W - W_1)\, l_2}{A.E} = \frac{(m - m_1)\, g . l_2}{A.E} \quad \ldots (ii)$$

Since extension of length l_1 must be equal to compression of length l_2, therefore equating equations (*i*) and (*ii*),

$$m_1 . l_1 = (m - m_1) l_2$$

$$m_1 \times 0.9 = (500 - m_1) 0.6 = 300 - 0.6 m_1 \text{ or } m_1 = 200 \text{ kg}$$

$\therefore$ Extension of length l_1,

$$\delta = \frac{m_1 . g . l_1}{A.E} = \frac{200 \times 9.81 \times 0.9}{1.96 \times 10^{-3} \times 200 \times 10^9} = 4.5 \times 10^{-6} \text{ m}$$

We know that natural frequency of longitudinal vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{4.5 \times 10^{-6}}} = 235 \text{ Hz} \quad \textbf{Ans.}$$

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Natural frequency of transverse vibration

We know that the static deflection for a shaft fixed at both ends and carrying a point load is given by

$$\delta = \frac{Wa^3b^3}{3EIl^3} = \frac{500 \times 9.81 (0.9)^3 (0.6)^3}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} (1.5)^3} = 1.24 \times 10^{-3} \text{ m}$$

. . . (Substituting $W = m.g$; $a = l_1$, and $b = l_2$)

We know that natural frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{1.24 \times 10^{-3}}} = 14.24 \text{ Hz}$$ **Ans.**