CH-8- : Gyroscop

جهاز الحركة الهوائية (الدوامية)
او الحركة الفلكية

* Introdution

مقدمة

1) **Active force** :

القوة النشطة: هي القوة الخارجية (المركزية) التي تؤثر على الجسم الذي يتحرك على مسار منحني بسرعة خطية منتظمة والتي تؤثر باتجاه التعجيل المركزي :

It's an external force (Centripetal force) applied on a body, which moves along a curved path with a uniform linear velocity in direction of centripetal accleration.

2) **Reactive force** :

قوة رد الفعل: هي قوة الطرد المركزي التي تحاول سحب الجسم قطرياً الى الخارج

It's a Centrifugal force act on the body, which moves along a circular path with uniform linear velocity, and try to move it radialy outward direction.

* Precessional angular motion

الحركة الفلكية الزاوية (السبقية)

لتوضيح اخذ حركة مغزلية

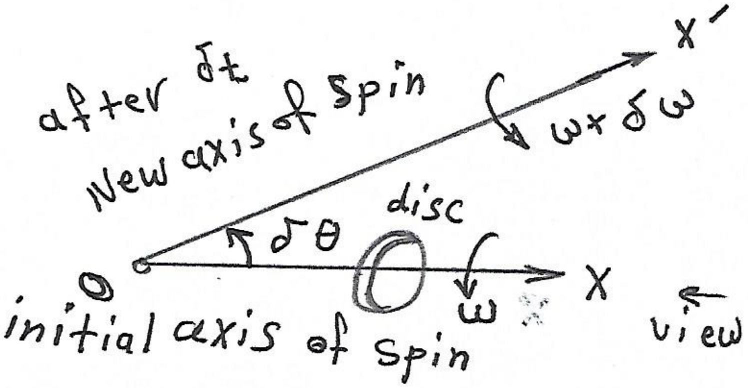
For discussing this motion, consider a disc is spinning about the axis of spin (OX) with angular velocity (ω) in a plane right angles to the paper. ($\perp$) عمودي على الورقة

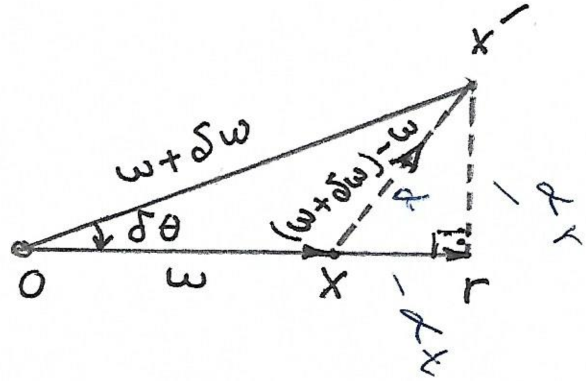
فترة

after short interval of time (δt), the disc be spining about new axis ($O\bar{X}$), ~~and~~ with angle $\delta\theta$ from OX and with angular velocity ($\omega + \delta\omega$),

باستخدام ابهام اليد اليمنى

* By using right hand screw rule, we can rep-resent the ① initial angular velocity of the disc (ω) with vector (OX), ② the final angular velocity ($\omega + \delta\omega$) with vector ($O\bar{X}$), and ③ the chang of angular velocity in time δt, which equal to angular acceleration with ($X\bar{X}$), as shown in fig(a).





fig(a) Precessional angular motion.

$\therefore$ change in angular velocity w.r.t δt = angular accele-
-ration.

$\therefore \; XX' = \alpha \quad \Rightarrow \frac{\delta\omega}{\delta t}$ التعجيل = مقدار التغير بالسرعه على الزمن

But the angular acceleration (α) can be resolved into Component, parallel and perpendicular to oX.

$\therefore \; \alpha = \alpha_t + \alpha_r$, and from fig(a)

$$\alpha_t = \frac{xr}{\delta t} = \frac{or - ox}{\delta t} = \frac{ox' \cos\delta\theta - ox}{\delta t} = \frac{(\omega+\delta\omega)\cos\delta\theta - \omega}{\delta t} =$$

But from $\triangle\, orx' \Rightarrow \cos\delta\theta = \frac{or}{ox'} \Rightarrow \therefore or = ox' \cdot \cos\delta\theta$

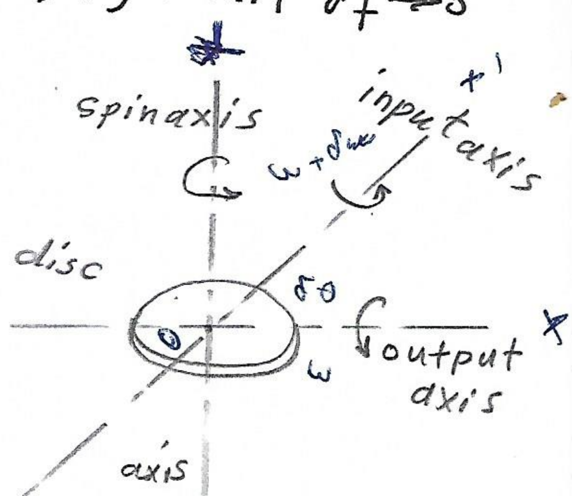
$$= \frac{\omega\cos\delta\theta + \delta\omega\cos\delta\theta - \omega}{\delta t}$$

Because of $\delta\theta \rightarrow$ very small $= 0$, $\therefore \cos\delta\theta \simeq 1$, and

$$\alpha_t = \frac{\omega + \delta\omega - \omega}{\delta t} = \frac{\delta\omega}{\delta t}$$, by limit when $\delta t \rightarrow 0$

$$\therefore \alpha_t = \lim_{\delta t \to 0}\left(\frac{\delta\omega}{\delta t}\right) = \frac{d\omega}{dt}$$

$$\therefore \boxed{\alpha_t = \frac{d\omega}{dt}} \quad --- \; (1)$$



and the perpendicular Component of angular acceleration α to oX be.

$$\alpha_c = \frac{rx'}{\delta t}$$, But fro right Δ $orx' \Rightarrow rx' = ox' \sin\delta\theta$

$$\therefore \alpha_c = \frac{ox' \sin\delta\theta}{\delta t} = \frac{(\omega+\delta\omega)\sin\delta\theta}{\delta t} = \frac{\omega \sin\delta\theta + \delta\omega \sin\delta\theta}{\delta t}$$

since $\delta\theta \to$ very Small $\Rightarrow \sin\delta\theta = \delta\theta$, and

$$\alpha_c = \frac{\omega\,\delta\theta + \delta\omega \cdot \delta\theta}{\delta t} = \frac{\omega\delta\theta}{\delta t}$$, but $\delta\omega \cdot \delta\theta = 0 \to$ very Small.

$\therefore$ By Limit, when $\delta t \to 0$

Assum ω_p

$$\therefore \alpha_c = \lim_{\delta t \to 0}\left(\frac{\omega \cdot \delta\theta}{\delta t}\right) = \omega * \frac{d\theta}{dt} = \omega * \omega_p$$

whene $\frac{d\theta}{dt} = \omega_p$ - angular velocity of pression motion.

$$\therefore \boxed{\alpha_c = \omega * \omega_p} \quad \text{--- (2)}$$

$\therefore$ Total acceleration of p-n m-n of disc.

$$\boxed{\alpha = \frac{d\omega}{dt} + \omega * \omega_p} \quad \text{--- (3)}$$

whene | ω - angular velocity of spin motion
$\omega_p = \frac{d\theta}{dt}$ - angular velocity of precession motion سرعة الحركة البيضية (التطليبة)
α - gyroscopic acceleration of pre-n m-n.

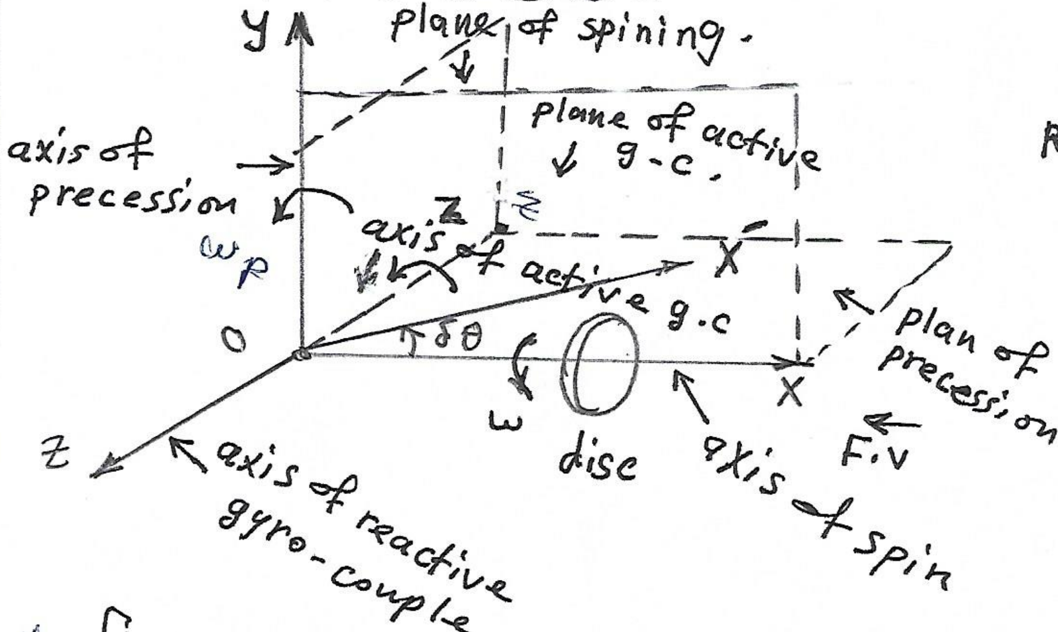
Not(1) : the axis of precession m-n is $\perp$ spin m-n

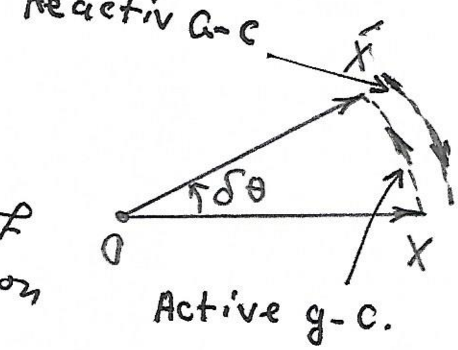
(2) when ω = const in all position, the $\omega_p = \frac{d\theta}{dt} = 0$ and $\alpha = 0$

3) when $\omega \to$ chang in direction, but ω = const

the $\alpha = \underbrace{\frac{d\omega}{dt}}_{=0} + \omega \frac{d\theta}{dt} = \omega \cdot \omega_p$

* Gyroscopic couple
العزم الفلكي (الكايروسكوبي)





* fig(a) gyroscopic couple
العزم الكايروسكوبي

For determining the gyroscopic couple, let we have a disc spining with angular velocity (ω) rad/s about the axis of spin (OX) in anticlockwise direction (front-view) fig(a). The plane of spin XOZ is ⊥ to the axis OX, OZ, and rotate about OY with angular velocity (ω_p) (rad/s), therefore the horizontal plane (XOZ) (plane of spin) is called plane of precession and OY is the axis of presession.

∴ angular momentum of the disc.

$M = I \cdot \omega$

where $\begin{cases} I - \text{mass moment of inertia about OX, and} \\ \omega - \text{angular velocity of the disc.} \end{cases}$

Since the angular momentum is a vector quantity, therefore may be represent by vector $\overrightarrow{OX}$, and when the spin axis ox turn in the plane xoz with angle $\delta\theta$ rad/s to the new position ox', through time δt second, and assuming $\omega = const$ then the angular momentum be represented by vector ox', and the chang in angular momentum be

$$= ox' - ox = xx' = ox \cdot \delta\theta \cdot$$

$$= I \cdot \omega \cdot \delta\theta$$

and the rate of chang of angular momentum w.r.t δt be.

$$= I \cdot \omega \cdot \frac{\delta\theta}{\delta t}$$

then the couple causing the precession be -

$$C = \lim_{\delta t \to 0} I \cdot \omega \cdot \frac{\delta\theta}{\delta t} = I \cdot \omega \underbrace{\frac{d\theta}{dt}}_{\omega_p} = I \cdot \omega \cdot \omega_p$$

العزم الفلكي النشط

$\therefore$ $\boxed{C = I \cdot \omega \cdot \omega_p}$ — — ① activ gyroscopic Couple.

$I = m \cdot k^2$, $k = \frac{r^2}{2}$ for disc radius of gyration. العزم الذاتي للعجلة

where $\boxed{I = \frac{m r^2}{2}}$. $kg \cdot m^2$ mass moment of inertia.

المزدوج هو مقدار التغير بالعزم

Note (1): C - represent change in angular Momentum and act in direction of vector xx', which lie in plane xoz with ang $\delta\theta$,

But, when $\delta\theta \to$ very Small, vector xx' be $\perp$ plane xoy, which is called P. of A.g.C about axis oz.

عندما يدور محور الدوران بنفس حركة السبق يتولد عزم مضاد

Note (2): when axis of spin moves with ω_p on the disc is subjected reactive Couple = C in magnitude but in opposite direction with axis oz

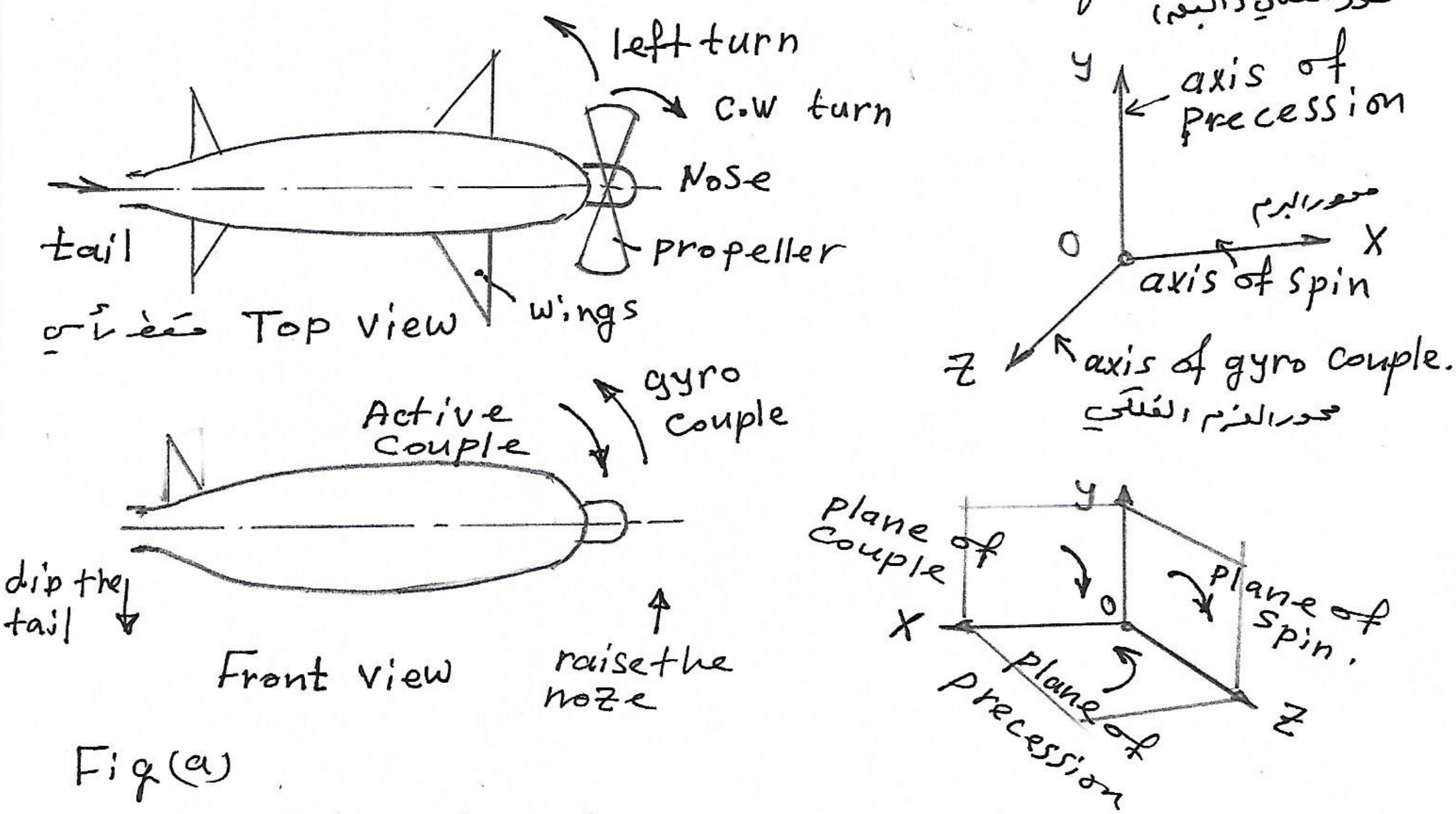
عزم السبق يؤثر على الكراسي بعزم مضاد ومساوي له

Note (3): C also applied in bearings support, which resist equal and apposite couple.

Note(4) : Gyroscope instrument is used in cars, monorail, aeroplane and ships to minimize the rolling and pitching effect of waves. لمقاومة تأثير التدحرج والانحدار

* **Effect of the gyroscopic Couple on an Aeroplane** تأثير العزم الفلكي على الطائرة :

For determining the effect of gyroscopic cople on an aeroplane, we take top and front view, while the engine or (propeller shaft) محور الدوران rotates clockwise seen from rear خلف, and the aeroplane turn to the left, as shown in fig(a).



Fig(a)

Gyroscop coule acting on the aeroplane be;

$C = I \cdot \omega \cdot \omega_p$, N.m --- (1)

where; | ω - rad/s - angular velocity of engine,
| m - kg - mass of engine and propeller shaft.,
| k - m - radius of gyration,
| I - kg.m^2 - Mass moment of inertia العزم الفلكي of engine and,
| wher: $I = mk^2$ propeller shaft.
| V - m/s - Linear velocity of aeroplane,

1) Befor aeroplane turn to left → the angular momentum vector is represented by vector OX.

2) Afer it turn to left, the active gyroscopic couple change the direction of anglure momentum vector to OX́ with angle $\delta\theta$, XX́ - represent the chang in angular momentum (or the active gyro couple) by limit XX́, when $\delta\theta \to 0$, it will be ⊥ OX, and by applying right hand screw rule to vector XX́, we find its direction a clockwise, as in fig (b) in the axis OZ.

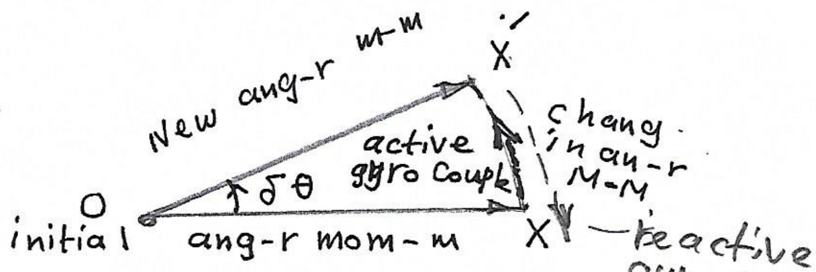


fig (b) tern to left.

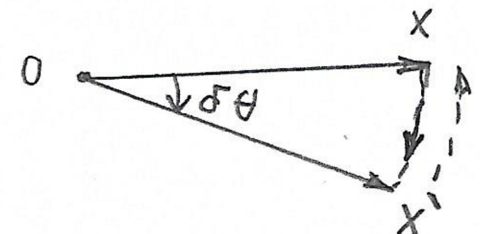


fig (c) turn to right

There for the reactive gyro-p couple which is induc-ed is equal and opposite in direction to the active gyro-couple, and will be anticlock wise direction. there for the effect of this couple raise the nose ↑ and tip the tail ↓

Note (1): when the aeroplane turn to right by simillar way, then the effect of reative gyro couple dip the noze ↓ and raise the tail ↓ fig (c). engine turn C.W.

Note (2): when engine rotate anticlockwise → seen from tail, and the aeroplane turn to left. the effect of R.g.c dip the noze ↓ and raise the tail ↑.

Note (3): when engine turn anti C.W. and the aeroplane turn to right → raise noze ↑ and dip tail ↓.

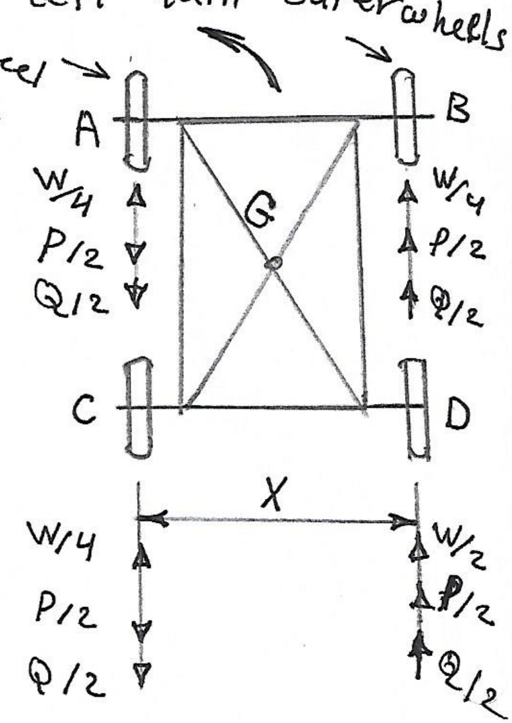
Note (4): when engine rotate clockwise, with view from front, and the aeroplane turn to left → then the tail raise ↑ and dip the noze ↓.

Note (5); when engine rotate clockwise, and aeroplane turn to right → front view → then the noze raise ↑ and the tail dip ↓ down.

استقرارية مركبة ذات اربع عجلات على طريق منحني .

* Stability of a four wheel Drive moving in a curved path

For determining the stability, we take four wheel automobile Locomotive, which turn towards to left, and the automobile with a following date (fig(a)



fig(a)

- Inner wheels A, C
- outer wheels B, D
- Center of gravity (C.G), with height (h - metrs from the road surface.
- $W = mg$, N weight of vehicle.
- r_w - m radius of wheel
- R - m radius of curviture and ($R > r_w$)
- X - mrs - width of vehicle.
- I_w - mass moment of one wheele in $Kg \cdot m^2$, where $I_w = m \frac{r_w^2}{2}$
- W_w - Angular velocity of wheels or velocity of spin in rad/s
- I_E = mass moment of rotating parts or engine, $Kg \cdot m^2$.

ω_E - Angular velocity of rotating parts (engine) in rad/s.

G - gear ratio $= \omega_E / \omega_w$

v - linear velocity of the vehicle in m/s $= \omega_w \cdot r_w$.

(1) * At first we know that the wieight of the vehicle is equaily distributed of four wheels and act down wards, the reation of road surface equal to the weight and act upwards.

$\therefore R = \frac{W}{4} = \frac{mg}{4}$, N reaction of road surface.

2) for determing the stability we must discused the effect of gyroscopic and centrifugal couple on the vehicle as follow:.

I) Effect of gyroscopic Couple ; تأثير العزم الفلكي على العجلة

The net gyroscopic couple act on the vehicle be: $C = C_W \pm C_E = 4 I_W . \omega_W . \omega_P \pm J_E . \omega_E . \omega_P$

$C = C_W \pm C_E = 4 J_W \, \omega_W \cdot \omega_P \pm J_E \, G \, \omega_W \cdot \omega_P$

whene; $\therefore \boxed{C = \omega_W \cdot \omega_P (4 J_W \pm G J_E)}$ --- (1) العزم الفلكي للعجلات

C_W - Gyroscopic Couple due to 4 wheels, and be.

$C_W = 4 J_W \, \omega_W \cdot \omega_P$ --- (2) السرعة الدورانية للعجلة

whene $\omega_W = \frac{V}{r_w}$, rad/s, angular-V- of wheels or of (spin)

$\omega_P = \frac{V}{R}$, rad/s, angular-V- of precession السرعة الدورانية الفلكية (السبقية)

and:

العزم الفلكي للمحرك

$C_E = J_E \cdot \omega_E \cdot \omega_P = J_E \cdot G \cdot \omega_W \cdot \omega_P$ --- (3)

نسبة السرعة للتروس

whene: $G = \frac{\omega_E}{\omega_W} \Rightarrow \omega_E = G . \omega_W$ --- (4) gear ratio

Note: (1) ; A) when $\overleftarrow{\omega_W}$ and $\overleftarrow{\omega_E}$ rotate in same direction the $C = C_W + C_E$

B) and when $\overleftarrow{\omega_W}$, $\overrightarrow{\omega_E}$ rotate in opposite direction the $C = C_W - C_E$

Note(2):

Vertical reation on the raad surface duo to gyroscopic couple for inner or outer wheel. be:

$$P * X = C \Rightarrow P = \frac{C}{X}, \text{ Newton.}$$

and for each two wheel. inner or outer be

$$\frac{P}{2} = \frac{C}{2X} \quad \to N \quad (5)$$

رد الفعل للعجل الواحد

Note(3):

when: $C = C_W + C_E$

then the reaction on the outer wheels act upwards ↑, and the reaction on the inner wheels act downwards ↓

Note(4):

when $C = C_W - C_E$, and $C_E > C_W$

$\therefore \quad C = -C$

and the reaction on the outer wheels act vertically downwards ↓, and on the inner wheels act vertically upwards ↑

II) Effect of the Centrifugal Couple

أثر مزدوج الطرد المركزي

Since the vehicle move along curved path therefore on which act centrifugal force outwards at the center of gravity of the vehicle and be:

$$F_c = m r \omega^2 = m * \frac{V^2}{R} \quad -- (1)$$

and the couple due to the centrifugal force tend to overturn the vehicle, and name overturning couple, and be:

$$C_o = F_c * h = \frac{mV^2}{R} * h \quad -- (2)$$

overturning couple is balanced by vertical reation duo to centrifugal force, which act vertically upwards on the outer wheels ↑, and vertically downwards on the inner wheels ↓ and be:

$$Q * X = C_o \rightarrow Q = \frac{C_o}{X} = \frac{mv^2 \cdot h}{R \cdot X}$$

and for two inner or outer wheels. be:

$$\frac{Q}{2} = \frac{m \cdot v^2 \cdot h}{2R \cdot X} \quad \text{--- (3)}$$

∴ The total vertical reaction on outer wheels be:

$$P_o = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2} \quad \text{--- (4)}$$

and for inner wheels

$$P_i = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2} \quad \text{--- (5)}$$

Note(S):

When the vehicle run at high speed $\omega_w \uparrow$ max

$P_i \rightarrow$ zero, and the inner wheels → tend to overturn the vechiche, and tend to leave the gnoud.

therefore $\frac{P}{2} + \frac{Q}{2} < \frac{W}{4}$ must be less

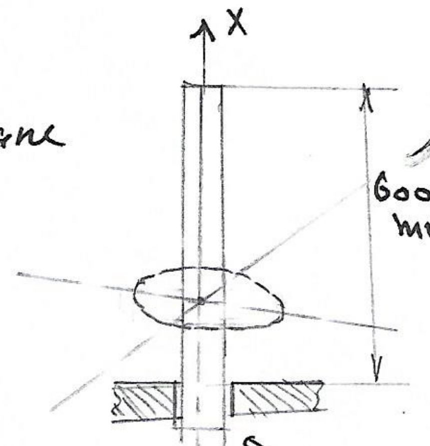
to have contact between ground and inner wheels, and the vehicle not be overturn.

Examples on gyroscop — امثلة تطبيقية على الجايروسكوب

Example (1) :

A uniform disc of diameter (300)mm and mass of (5) kg is mounted on one end of an arm of length (600) mm. The other end of the arm is free in a universal bear-ing. If the disc rotates about the arm with a speed of (300) r.p.m clockwise. looking from the front, with what speed will it presess about the vertical axis.

Solution.



mass moment of inertia is ⊥ disc plane

$$I = \frac{mr^2}{2} = 5 * \frac{(0,15)^2}{2} = 0,056 \ kg \cdot m^2$$

and couple due to mass of disc.

$$C = W \cdot l = m \cdot g \cdot l = 5 * 9.81 * 0,056 =$$
$$= 29.43 \ N \cdot m.$$

precessional speed be.

$$C = I \cdot \omega \cdot \omega_p \quad , \omega = \frac{2\pi n}{60}$$

$$29.43 = 0,056 * \frac{2*3.14*300}{60} * \omega_p$$

$$\therefore \omega_p = \frac{29.34}{1.76} = 16.7 \ rad/s$$

Example (2) :

A uniform disc of (150) mm diameter has a mass of (5) kg. It is mounted centrally in bearings which maintain its axile in a horizontal plane. The disc spins about it axile with a constant speed of (1000) r.p.m while the axile presses uniformly about the vertical at (60) r.p.m. The direction of rotat-ion are as shown in fig (a). If the distance between the bearings is (100) mm. find the resultant reaction at each bearing due to the mass and gyroscopic effects.

solution: axis of precession

x - axis of spin

y - axis of precession

z - axis of active couple

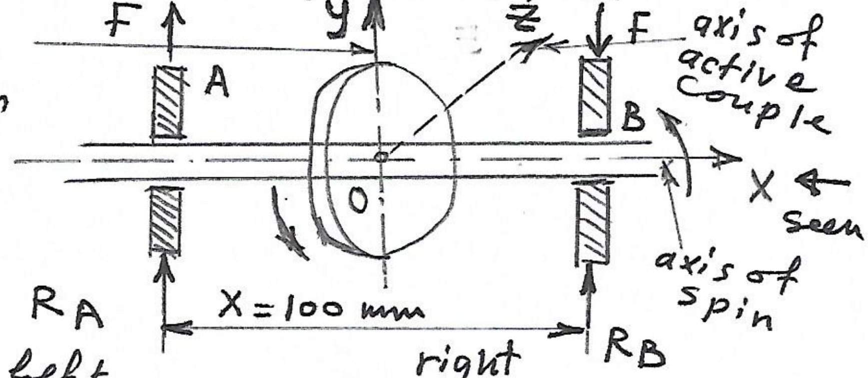
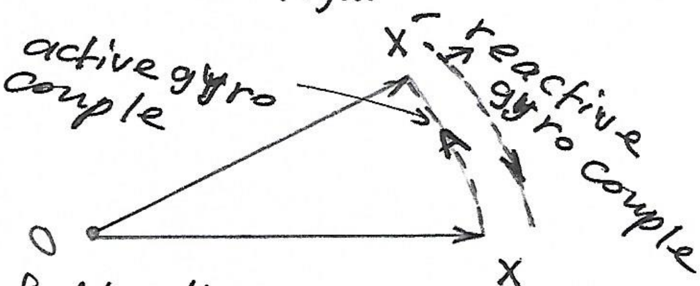


Fig (a)



1) mass moment of inertia of the disc about the ox ⊥ to the center of disc be:

$I = m\frac{r^2}{2} = \frac{5 \times (0.075)^2}{2} = 0.014\ m^2$, and

2) Gyroscopic couple act on the disc be.

$C = I \cdot \omega \cdot \omega_p$ where

$\omega = \frac{2\pi n}{60} = \frac{2 \times 3.14 \times 1000}{60} = 104.7$ rad/s

$\omega_p = \frac{2\pi n_p}{60} = \frac{2 \times 3.14 \times 60}{60} = 6.284$ rad/s

$\therefore C = 0.014 \times 104.7 \times 6.284 = 9.2$ N.m

3) Reactive gro-couple act in opposite direction and be on each bearing.

$C = f \times X$, and

$F = \frac{C}{\ } = \frac{9.2}{\ } = 92\ N$

4) when the weight of the disc is mounted Centrally in bearing, there fore the reaction at bearings A, and B are equally.

$\therefore R_A = R_B = \frac{m\_disc}{2} = \frac{5}{2} = 2,5$ kg $* 9,81 = 24.5$ N

5) let: R_{Ai} - Resultant reaction on bearing A
R_{Bi} - " " " " B

But because of reactive gyro-couple acts clockwise → then it's increase reaction on bearing (A) at left → and decrease on bearing (B) at right, as follow → by seen from front.

$\therefore R_{Ai} = R_A + F = 92 + 24.5 = 116.5$ N ↑
$R_{Bi} = R_B - f = 92 - 24.5 = 67.5$ N ↓

example (3):

An aeroplane makes a complete half circle of (50) metres radius, towards left. when flying at 200 km/h. The rotary engine and the propeller of the plane has a mass of (400) kg, and a radius of gyration of (0,3) m, The engine rotates at (2400) r.p.m, clock-wise, when viewed from the rear. find the gyroscopic Couple on the aircraft and state its effect on it

Solution:

1) The gyroscopic Couple act on the airplane be:
$C = I \cdot \omega \cdot \omega_p$

$$\omega = \frac{2\pi n}{60} = \frac{2 * 3.14 * 2400}{60} = 251 \text{ rad/s}$$

$$\omega_p = \frac{V}{R} = \frac{\frac{200 * 1000}{1 * 60 * 60}}{50} = 1,11 \text{ m/s}$$

$$I = m k^2 = 400 * (0,3)^2 = 36 \text{ kg.m}^2$$ → for rotating parts

$$\therefore C = 36 * 251.4 * 1.11 = 100.46 \text{ N.m}$$

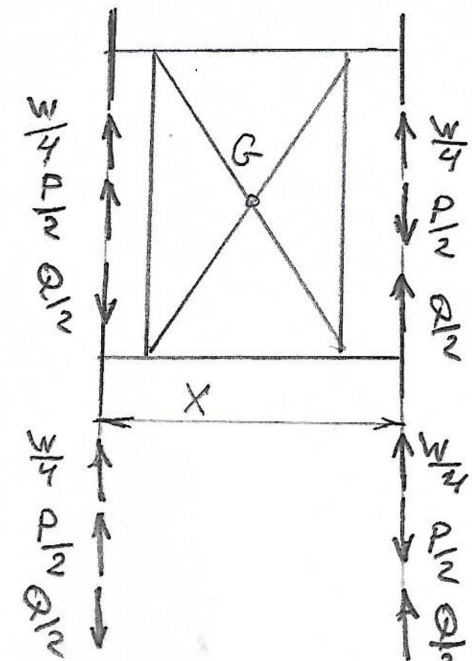
From abouve the effect of gyro - couple when, It turns toward left and seen from the rear → is lift the nose ↑ upwards and tip the tail down wards ↓

* example (4)

A four wheel trolley Car of mass (2500 kg) runs on rails, which are (1.5) m aparat and travels around a Curve of (30) m radius at 24 km/h. The rails are at the same level. Each wheel of the trolley is (0.75) m in diameter, and each of the two axles is driven by a motor running in a direction opposite to that of the wheels at a speed of five times the speed of rotation of the wheels. The moment of inertia of each axle with gear and wheels is (18) $kg \cdot m^2$. Each motor with shaft and gear pinion has a moment of inertia of (12) $kg \cdot m^2$. The Centre of gravity of the Car is (0,9) m above the rail level. Determine the vertical force exerted by each wheel on the rails taking into consideration the centrifugal and gyroscopic effect. State the centrifugal and gyroscopic effects on the trolley, take gear ratio ($G = \frac{\omega_E}{\omega_W} = 5$).

Solution:

The weight of the trolley is equally distributed on four wheel:



∴ 1) Road reaction on one wheel be:

$$P = \frac{W}{4} = \frac{mg}{4} = \frac{2500 * 9.81}{4} = \underline{\underline{6131.25\,N}}$$

2) Gyro-Couple due to inner or outer wheel with axle

$$C_W = 2\, J_W \cdot \omega_W \cdot \omega_P$$

where: $\omega_W = \frac{V}{r_W} = \frac{\frac{24 \times 1000}{1 \times 60 \times 60}}{0,375} = \frac{6.67}{0,375} = 17.8\, rad/s$

and $\omega_P = \frac{V}{R} = \frac{\frac{24 * 1000}{1 * 60 \times 60}}{30} = 0,22\, rad/s$

$J_W = 18\, kg\text{-}m^2$

and $C_W = 2 * 18 * 17.8 * 0,22 = \underline{\underline{141\, N \cdot m}}$

3) Gyro-couple due to the engine and rotating parts and gears. for pair

$$C_E = 2\, J_E \cdot \omega_E \cdot \omega_P = 2\, J_E \cdot G\, \omega_W \cdot \omega_P$$

where: $G = \frac{\omega_E}{\omega_W} \Rightarrow \omega_E = G \cdot \omega_W \rightarrow$ gear ratio.

and $J = 12\, kg \cdot m^2$

$$\therefore C_E = 2 * 12 * 5 \times 17.8 \times 0,22 = 470\, N \cdot m$$

4) Net gyro-couple be:

$$C = C_W - C_E = 141 - 470 = -\underline{\underline{329}}\ N.m$$

where -ve singn duo to opposite direction of motor turn)

Since $C_E > C_W$

5) → reaction on the outer wheels vertically down wards $\downarrow \frac{P}{2}$

→ and reaction on the inner wheels vertically upwards $\uparrow \frac{P}{2}$

and there magnitude be: $C = \frac{P}{2} * 2X$

$$\therefore \frac{P}{2} = \frac{C}{2X} = \frac{329}{2*1.5} = 109.7 N.m$$

6) Centrifugal force be:

$$F_C = \frac{m . v^2}{R} = \frac{2500 * (6.67)^2}{30} = 3707\ N$$

7) over turning couple be:

$$C_o = F_c * h = 3707 * 0,9 = 3336.3\ N-m$$

The overturn couple be balanced by vertical reaction, which act verticall upward on outer wheels $\uparrow$ and down wards vertically on inner wheels $\downarrow$.

8) magnitud of reation duo to overturn couple be: $C_o = \frac{Q}{2} * 2X \rightarrow \frac{Q}{2} = \frac{C_o}{2X}$

$$\therefore \frac{Q}{2} = \frac{C_o}{2X} = \frac{3336.3}{2*1.5} = 1112.1\ N$$

9) ∴ Vertical total force (reaction) on outer wheels

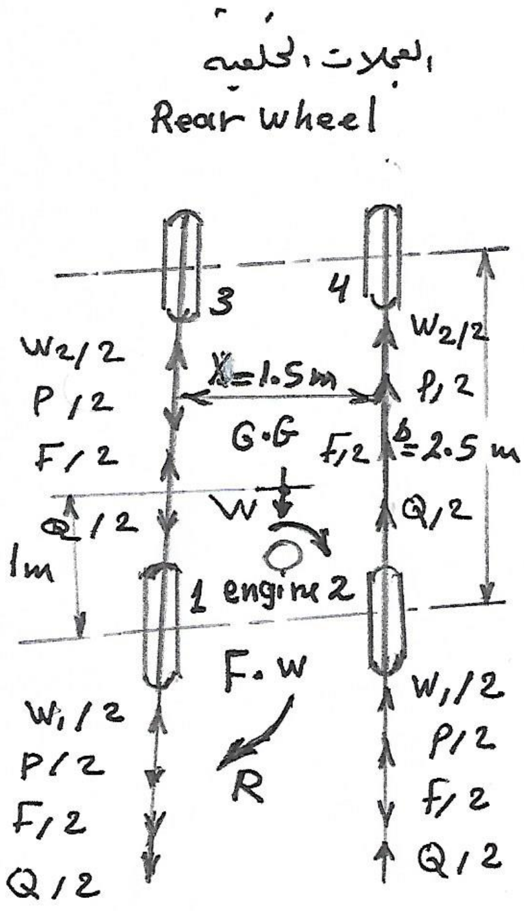
$$P_o = \frac{W}{2} - \frac{P}{2} + \frac{Q}{2} = 6131.25 - 109.7 + 1112.1 = \underline{\underline{7142.65}}\ N \downarrow$$

10) Vertical total force (reaction) on inner wheels be

$$P_i = \frac{W}{2} + \frac{P}{2} - \frac{Q}{2} = 6131.25 + 109.7 - 1112.1 = 5128.85\ N \uparrow$$

Example (5):

A four wheeled car of mass (2000) kg has a (2,5) m wheel base, track width (1,5) m and height of centre of gravity (500) mm above the ground level and lies at (1) metre from the front axle. Each wheel has an effective diameter of (0,8) m and moment of inertia of (0,8) $kg \cdot m^2$. The drive shaft, engine, flywheel and transmission are rotating at (4) times the speed of road wheel, in a clockwise direction when viewed from the front, and is equivalent to a mass of (75) kg having a radius of gyration of (100) mm. If the car is taking a right turn of (60) m radius at (60) km/h, find the load on each wheel:



Solution: رد فعل الوزن على العجلة الواحدة الامامية او الخلفية ↑

I) Road reaction due to weight on each wheel, act upwards ↑ and equal to

$\frac{W_1}{2}$ — for front wheel ↑

$\frac{W_2}{2}$ → for rear wheel ↑

Because of the center of gravity is not in the center of vechicle, therefore $W_1 \neq W_2$ and may be determine by taking moment about front wheel as follow;

$\Sigma M_1 = 0$

$\therefore W_2 * 2,5 - W * 1 = 0$

$\therefore W_2 * 2,5 = W * 1 = m.g = 2000 * 9.81 = 19620\, N \uparrow$

$\therefore W_2 = \frac{19620}{2,5} = 7848\, N$

But $W = W_1 + W_2$

$\therefore W_1 = W - W_2 = 19620 - 7848 = 11772\, N \uparrow$

$\therefore \begin{cases} \frac{W_1}{2} = \frac{11772}{2} = 5886\, N \\ \frac{W_2}{2} = \frac{7848}{2} = 3924\, N \end{cases}$

II) Gyroscopic couple due to four wheel.

$C_W = 4\, J_w\, \omega_W \cdot \omega_p$

Where: $\omega_W = \frac{V}{r_w} = \frac{16.67}{0,4} = 41.675\ rad/s$

$V = \frac{60 * 1000}{60 \times 60} = 16,67\ m/s$

$\omega_p = \frac{V}{R} = \frac{16.67}{60} = 0,278\ rad/s$

$J_w = 0,8\ kg.m^2$

$\therefore C_W = 4 * 0,8 * 41,675 * 0,278 = 37.1\ N.m$

III) The reaction of the gyro couple of wheel act vertically down wards on inner wheels ↓ and vertically upwards on the outer wheels ↑, or in other words lift the inner and press the outer wheels, and it's magnitude on inner and outer wheels be:

$\frac{P}{2} = \frac{C_W}{2X} = \frac{37.1}{2 * 1.5} = 12.37\ N$

IV/ Gyro Couple duo to engine and rotating parts + drive shaft + flywheel.

$$C_E = 4 J_E \cdot \omega_E \cdot \omega_P = J_E \cdot G\omega_W \cdot \omega_P$$

where : $G = \frac{\omega_E}{\omega_W} \Rightarrow \omega_E = G \cdot \omega_W$

$$J_E = m_E \cdot (k_E)^2 = 75 * (0,1)^2 = 0,75 \ kg.m^2$$

$\therefore C_E = 0.75 * 4 * 41.675 * 0.278 = 34.7 \ N.m$

V/ The reaction of the gyro couple duo to engine and rotating parts act vertically downwards on the front wheels and act vertically upwards on the rear wheels, or in other wards lift the rear wheels and press the front wheels, and there magnitude be for two wheels be;

$$\frac{F}{2} = \frac{C_E}{2b} = \frac{34.7}{2*2,5} = 6.94 \ N$$

$\therefore \frac{f}{2}$ - ↓ for front wheel

$\frac{f_2}{2}$ - ↑ for rear wheel.

VI - Centrifugal Couple (over turn couple) act on the Car.

$$C_o = f_c * h$$

where ; $f_c = m \cdot \frac{V^2}{R} = 2000 * \frac{(16.6)^2}{60} = 9264 \ N$

$\therefore C_o = 9264 * 0,5 = 4631.5 \ N.m$

VII - The reaction of the over turning Couple act vertically downwards ↓ on the inner wheels and vertically upwards on the outer wheels, and there magnitude for inner and outer wheels be:

$$\frac{Q}{2} = \frac{C_o}{2x} = \frac{4631}{2 * 1.5} = 1543 \text{ N}$$

where $\frac{Q}{2} \rightarrow \downarrow$ for inner wheels

$\frac{Q}{2} \rightarrow \uparrow$ for outer wheels.

∴ The load on evry wheel be.

1) Load on the front wheel No 1, 2, 3, 4

$$\left\{\begin{aligned}
P_{t_1} &= \frac{W_1}{2} - \frac{P}{2} - \frac{f}{2} - \frac{Q}{2} \\
P_{t_2} &= \frac{W_1}{2} + \frac{P}{2} - \frac{f}{2} + \frac{Q}{2} \\
P_{t_3} &= \frac{W_2}{2} - \frac{P}{2} + \frac{f}{2} - \frac{Q}{2} \\
P_{t_4} &= \frac{W_2}{2} + \frac{P}{2} + \frac{f}{2} + \frac{Q}{2}
\end{aligned}\right.$$