Basic Laws

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Lecture Two

Basic Laws

2-1 Introduction

Lecture 1 introduced basic concept of circuits. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits.

These laws, known as **Ohm's law** and **Kirchhoff's laws**, form the foundation upon which electric circuit analysis is built. In addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis.

2-2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1. **Network:** Any arrangement of the various, electrical energy source along with the different circuit elements is called an electrical network. Such a network is shown in the **Fig. 2.1**.
2. **Network Element:** Any individual circuit element with two terminals which can be connected to other circuit element is called a network element. Network elements can be either active elements or passive elements.
3. **Branch:** A part of the network which connects the various points of the network with one another is called a branch. In the **Fig. 2.1**, **AB, BC, CD, DA, DE, CF** and **EF** are the various branches. The branch may consist of more than one element.
4. **Junction Point:** A point where three or more branches meet is called a junction point. Points D and C are the junction points in the network shown in the **Fig. 2.1**.
5. **Node:** A point at which two or more elements are joined together is called node. The junction points are also the nodes of the network. In the network shown in the **Fig. 2.1**, **A, B, C, D, E** and **F** are the nodes of the network.

6. **Mesh (or Loop):** Mesh (or Loops) is a set of branches forming a closed path in a network in such way that if one branch is removed then remaining branches do not form a closed path. In the **Fig. 2.1** paths **A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D** etc are the loops of the network.

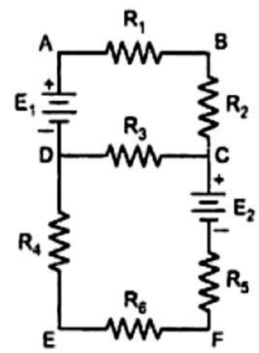


Figure 2.1 An electrical network.

2.3 OHM'S LAW

As shown in lecture 1, the materials in general have a characteristic behavior of resisting the flow of electric charge. The resistance R of any material with a uniform cross-sectional area A depends on A and its length **l**.

The circuit element used to model the current-resisting behavior of a material is the resistor. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit symbol for the resistor is shown in **Fig. 2.**2, where R stands for the resistance of the resistor. The resistor is the simplest passive element. Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

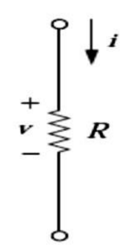


Figure 2.2 Circuit symbol for resistance.

Key Point: Ohm's law states that the voltage v across a resistor is directly proportional to the current I flowing through the resistor.

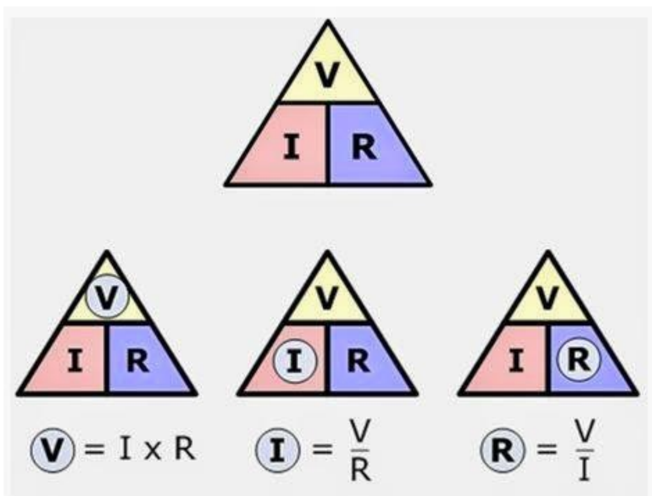
Ohm defined the constant of proportionality for a resistor to be the resistance; R. (The resistance is material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus,

$$\mathbf{V = IR} \tag{2.1}$$

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

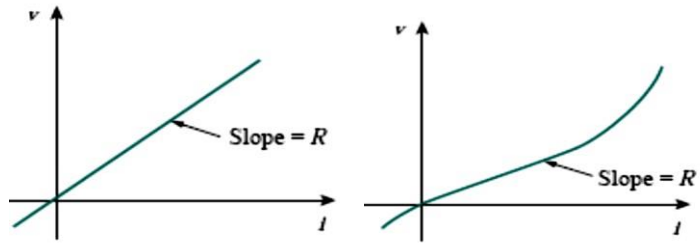
Then $\mathbf{R = V/I}$ (2.2)

so that $\mathbf{1\ \Omega = 1\ V/A}$



It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a linear resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in **Fig. 2.**3**(a)**. A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in **Fig. 2.**3 **(b)**. Examples of devices with nonlinear resistance are the light bulb and the diode. A useful quantity in circuit analysis is the reciprocal of resistance R, known as conductance and denoted by G:

$$\mathbf{G = 1/R = I/V} \tag{2.3}$$



(a) **(b)**

Figure 2.3 The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor**.**

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the mho (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mhos, in this lectures we prefer to use the Siemens (S), the SI unit of conductance:

1 S = 1 ℧ = 1 A/V

Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or Siemens (S).

From **Eq. (2.3)**, we may write

$$\mathbf{I = GV} \tag{2.4}$$

The power dissipated by a resistor can be expressed in terms of **R**. Using **Eq**. **(2.1)**,

$$\mathbf{P = VI = I^2R = V^2/R} \tag{2.5}$$

The power dissipated by a resistor may also be expressed in terms of **G** as

$$\mathbf{P = VI = V^2G = I^2/G} \tag{2.6}$$

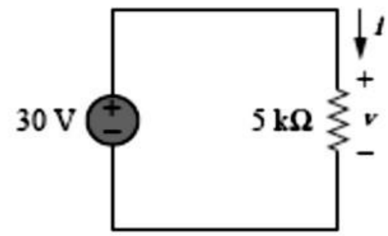
We should note two things from **Eqs. (2.5)** and **(2.6)**:

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.

2. Since R and G are positive quantities, the power dissipated in a resistor is always positive.

Thus, a resistor always absorbs power from the circuit.

Example 2.1: In the circuit shown below, calculate the current i, the conductance G, and the power **P**. **Solution:**



The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$\mathbf{I = V / R = 30 / 5 \times 10^3 = 6\ mA}$

The conductance is $\mathbf{G = 1/ R = 1/ (5 \times 10^3) = 0.2\ mS}$

We can calculate the power in various ways using either **Eqs. (2.5)**, or **(2.6)**.

$$\mathbf{P = VI = 30 \times (6 \times 10^{-3}) = 180\ mW}$$

2.4 Kirchhoff's Laws

Ohm's law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff's two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

2.4.1 Kirchhoff's current law

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero or the sum of the currents entering a node is equal to the sum of the currents leaving the node. Mathematically, **KCL** implies that

$$\sum_{n=1}^{N} i_n = 0 \qquad (2.7)$$

where **N** is the number of branches connected to the node and in is the nth current entering (or leaving) the node.

Consider the node in **Fig. 2.7**. Applying KCL gives

$$\mathbf{i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0} \qquad (2.8)$$

since currents $\mathbf{i_1}$, $\mathbf{i_3}$, and $\mathbf{i_4}$ are entering the node, while currents $\mathbf{i_2}$ and $\mathbf{i_5}$ are leaving it. By rearranging the terms, we get

$$\mathbf{i_1 + i_3 + i_4 = i_2 + i_5} \tag{2.9}$$

Class: Second

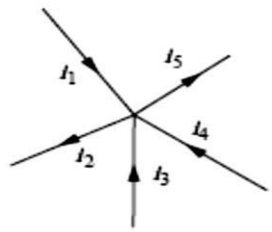


Figure 2.4 Currents at a node illustrating KCL.

A simple application of KCL is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in **Fig. 2.5(a)** can be combined as in **Fig. 2.5(b)**. The combined or equivalent current source can be found by applying KCL to node **a**.

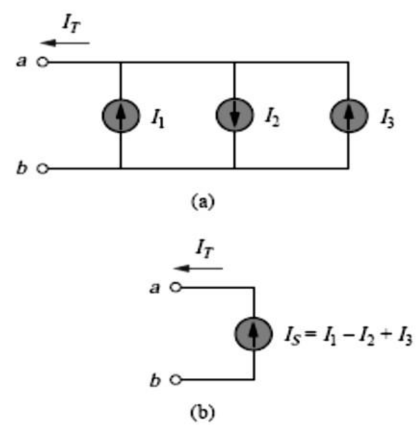


Figure 2.5 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

$\mathbf{I_T + I_2 = I_1 + I_3}$

or

$\mathbf{I_T = I_1 - I_2 + I_3}$

A circuit cannot contain two different currents, $\mathbf{I_1}$ and $\mathbf{I_2}$, in series, unless $\mathbf{I_1} = \mathbf{I_2}$; otherwise **KCL** will be violated.

2.4.2 Kirchhoff's voltage law

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0 \tag{2.10}$$

Where **M** is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.

To illustrate KVL, consider the circuit in **Fig. 2.6**. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and

$+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, KVL yields

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \tag{2.11}$$

Rearranging terms gives

$$v_2 + v_3 + v_5 = v_1 + v_4 \tag{2.12}$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises (2.13)

This is an alternative form of KVL. Notice that if we had traveled counterclockwise, the result would have been $+v_1$, $-v_5$, $+v_4$, $-v_3$, and $-v_2$, which is the same as before, except that the signs are reversed. Hence, **Eqs. (2.11) and (2.12)** remain the same.

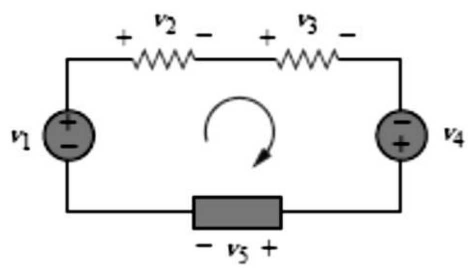


Figure 2.6 A single-loop circuit illustrating KVL.

When voltage sources are connected in series, KVL can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources.

2.4.3 Steps to Apply Kirchhoff. Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1: Draw the circuit diagram from the given information and insert all the value of sources with appropriate polarities and all the resistances.

Step 2: Mark all the branch currents with assumed directions using KCL at various nodes and junction points. Kept the number of unknown currents as minimum as far as possible to limit the mathematical calculations required to solve them later on. Assumed directions may be wrong; in such case answer of such current will be mathematically negative which indicates the correct direction of the current.

Step 3: Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistance of the network. This is necessary for application of KVL to various closed loops.

Step 4: Apply KVL to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any preview equation.

2.5 Solving Simultaneous Equations and Cramer's Rule

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error.

Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equations happens to be very large, use of a digital computer can make the task easy. Let us assume that set of simultaneous equations obtained is, as follows,

$$a_{11}\, x_1 + a_{12}\, x_2 + \ldots\ldots + a_{1n}\, x_n = C_1$$

$$a_{21}\, x_1 + a_{22}\, x_2 + \ldots\ldots\ldots\ldots + a_{2n}\, x_n = C_2$$

$$\vdots$$

$$a_{n1}\, x_1 + a_{n2}\, x_2 + \ldots\ldots\ldots\ldots + a_{nn}\, x_n = C_n$$

where $\mathbf{C_1, C_2, \ldots\ldots, C_n}$ constants. Then Cramer's rule says that form a system determinant $\mathbf{\Delta}$ or **D** as,

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = D$$

Then obtain the sub determinant **Dj** by replacing **j**th column of **Δ** by the column of constants existing on right hand side of equations i.e. $C_1, C_2, \ldots C_n$;

$$D_1 = \begin{bmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_n & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \qquad D_2 = \begin{bmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{bmatrix}$$

and

$$D_n = \begin{bmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & C_n \end{bmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D}, \quad X_2 = \frac{D_2}{D}, \cdots, X_n = \frac{D_n}{D}$$

Where D_1, D_2, …, D_n and **D** are values of the respective determents

Example 2.2 : Apply Kirchhoff's laws to the circuit shown in figure 1 below Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative?

If yes, explain the significance of the negative sign.

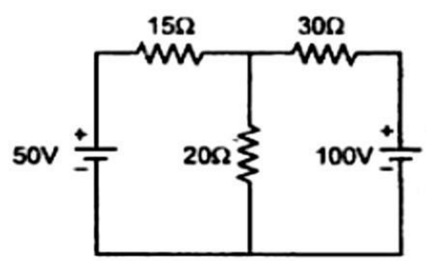


Figure 1

Solution: Application Kirchhoff's laws:

Step 1and 2: Draw the circuit with all the values which are same as the given network

Mark all the branch currents starting from **+ve** of any of the source, say +ve of 50 V source

Step 3: Mark all the polarities for different voltages across the resistance. This is combined with step 2 shown in the network below in Fig. 1 (a).

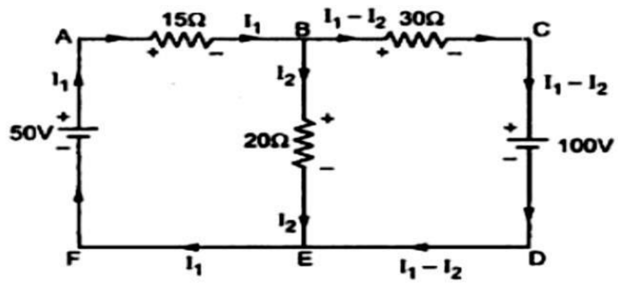


Figure 1

(a) Step 4: Apply **KVL** to different loops.

Loop 1: **A-B-E-F-A**, $\mathbf{-15\, I_1 - 20\, I_2 + 50 = 0}$ Loop 2: **B-C-D-E-D**, $\mathbf{-30\,(I_1 - I_2) - 100 + 20\, I_2 = 0}$

Rewriting all the equations, taking constants on one side,

$$\mathbf{15\, I_1 + 20\, I_2 = 50, \qquad -30\, I_1 + 50\, I_2 = 100}$$

Apply Cramer's rule, $\boldsymbol{D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350}$

Calculating $\mathbf{D_1}$, $\boldsymbol{D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500}$

$$\boldsymbol{I_1 = \frac{D_1}{D} = \frac{500}{1350} = 0.37\ A}$$

Calculating $\mathbf{D_2}$, $\boldsymbol{D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000}$

$$\boldsymbol{I_2 = \frac{D_2}{D} = \frac{3000}{1350} = 2.22\ A}$$

For $\mathbf{I_1}$ and $\mathbf{I_2}$ as answer is positive, assumed direction is correct.

For $\mathbf{I_1}$ answer is **0.37** A. For $\mathbf{I_2}$ answer is **2.22** A

$$\mathbf{I_1 - I_2 = 0.37 - 2.22 = -1.85\ A}$$

Negative sign indicates assumed direction is wrong.

i.e. $\mathbf{I_1 - I_2}$ = 1.85 A flowing in opposite direction to that of the assumed direction.

Thank You