The voltage and current divider circuits

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Lecture three

Series and Parallel Circuits

3-1 Series Resistors

A series circuit is one in which several resistances are connected one after the other. There is only one path for the flow of current. Consider the resistances shown in the **Fig.** 3.1. The resistance $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, said to be in series.

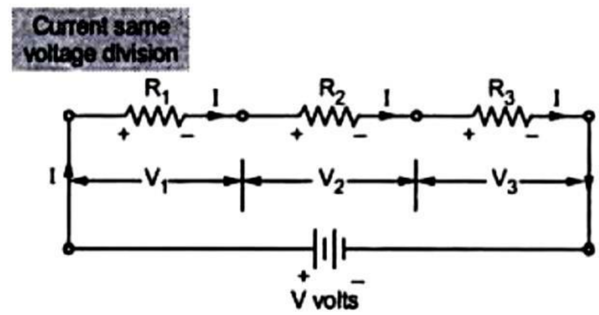


Fig. 3.1 series circuit

i.e. total or equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

For **N** resistances in series, $\mathbf{R = R_1 + R_2 + R_3 + \ldots + R_N}$ (3.1)

If $\mathbf{R_1 = R_2 = \cdot\cdot\cdot = R_N = R}$, then

$$\mathbf{R_{eq} = N \times R} \quad (3.2)$$

3.1.1 Characteristics of Series Circuits

1) The same current flows through each resistance.

2) The supply voltage **V** is the sum of the individual voltage drops across the resistances.

$$\mathbf{V = V_1 + V_2 + V_3 + \ldots + V_N} \quad (3.3)$$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances. i.e. $\mathbf{R > R_1, R > R_2, \ldots R > R_N}$

3.2 PARALLEL RESISTORS

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the **Fig. 2.6**.

R_{eq}= Total or equivalent resistance of the circuit,

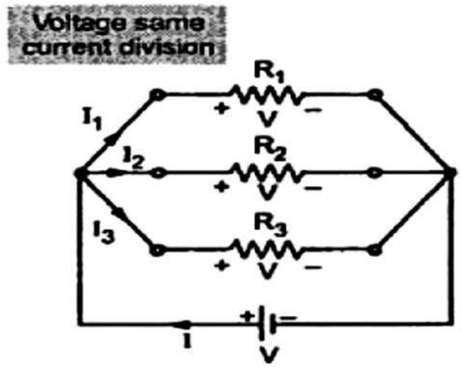


Fig. 2.6 A parallel circuit.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general if '**N**' resistances are in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \quad (3.4)$$

Note that $\mathbf{R_{eq}}$ is always smaller than the resistance of the smallest resistor in the parallel combination. If $\mathbf{R_1 = R_2 = \cdot\cdot\cdot = R_N = R}$, then $\mathbf{R_{eq} = R/N}$

Conductance (G):

It b known that, **1/R = G** (conductance) hence,

$$\mathbf{G = G_1 + G_2 + G_3 + \ldots + G_N} \quad (3.5)$$

Important result:

Now If **N** = 2, two resistance are in parallel then,.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ or } R = \frac{R_1 R_2}{R_1 + R_2} \quad (3.6)$$

3.2.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of the individual currents.

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
4) The equivalent resistance is the smallest of all the resistances $\mathbf{R < R_1, R<R_2, R<R_N}$.
5) The equivalent conductance is the arithmetic addition of the individual conductance's.

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single equivalent resistance **Req**.

Example 3.1: Find $\mathbf{R_{eq}}$ for the circuit shown in **Fig. 1**.

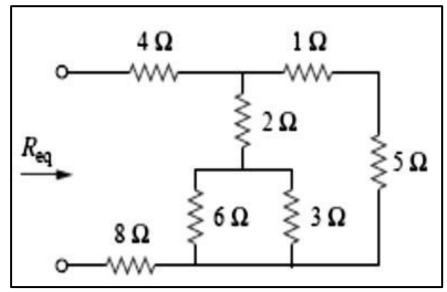


Figure 1

Solution:

To get $\mathbf{R_{eq}}$, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$\mathbf{6\ \Omega \parallel 3\ \Omega = 6 \times 3/(6 + 3) = 2\ \Omega}$$

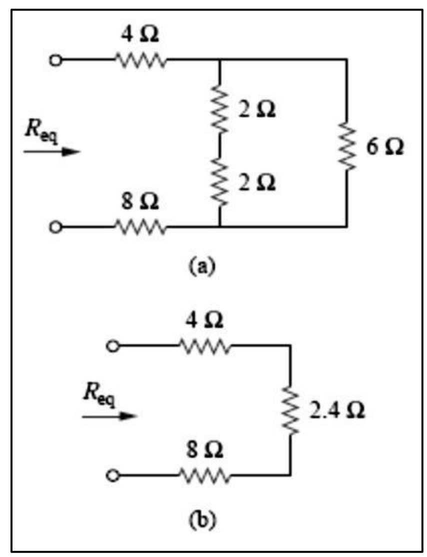
(The symbol || is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$\mathbf{1\ \Omega + 5\ \Omega = 6\ \Omega}$$

Thus the circuit in **Fig. 1** is reduced to that in **Fig. 2(a)**. **In Fig. 2(a)**, we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$\mathbf{2\ \Omega + 2\ \Omega = 4\ \Omega}$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in **Fig. 2 (a)**; their equivalent resistance is



$$\mathbf{4\ \Omega \parallel 6\ \Omega = 4 \times 6/(4 + 6) = 2.4\ \Omega}$$

Figure 1

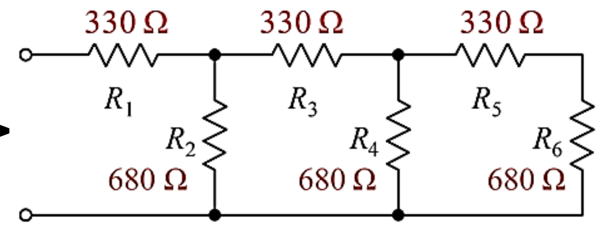
The circuit in **Fig. 2 (a)** is now replaced with that in **Fig. 2 (b)**.

In **Fig. 2 (b)**, the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$\mathbf{Req = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega}$$

Example 3.2

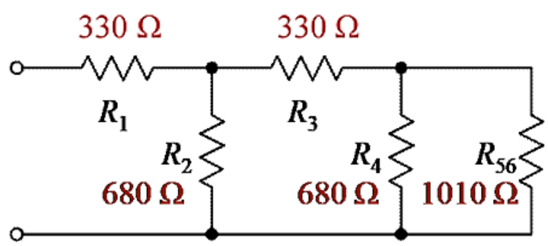
Find the equivalent resistance looking into the indicated port of the "ladder network" shown



Solution:

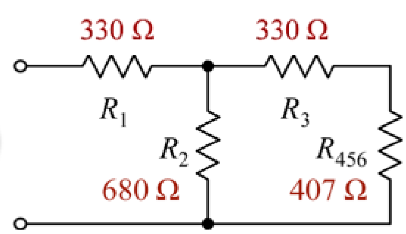
1. Starting at the "far end", we see that R_5 and R_6 are in series.

$$R_{56} = R_5 + R_6 = 1010\ \Omega.$$



2. R_4 is in parallel with R_{56}.

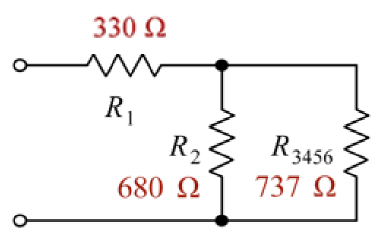
$$R_{456} = (1/R_4 + 1/R_{56})^{-1} = 407\ \Omega.$$



3. R_3 and R_{456} are in series.

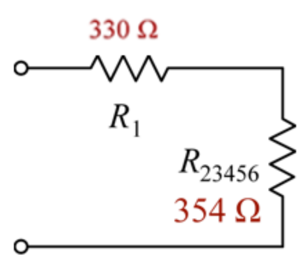
$$R_{3456} = R_3 + R_{456} = 1010\ \Omega.$$

4. R2 is in parallel with R_{3456}



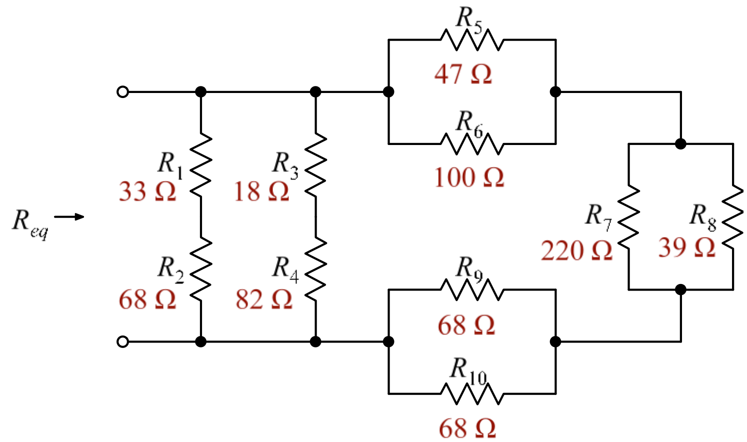
$$R_{23456} = \frac{1}{\frac{1}{R2} + \frac{1}{R3456}} = 354\Omega$$

5. Finally, R_{eq} is the series combination of R_1 and R_{23456}.



$$R_{eq} = 330\ \Omega + 354\ \Omega = 684\ \Omega.$$

Example 3.3 Find the equivalent resistance looking into the indicated port of the circuit shown below



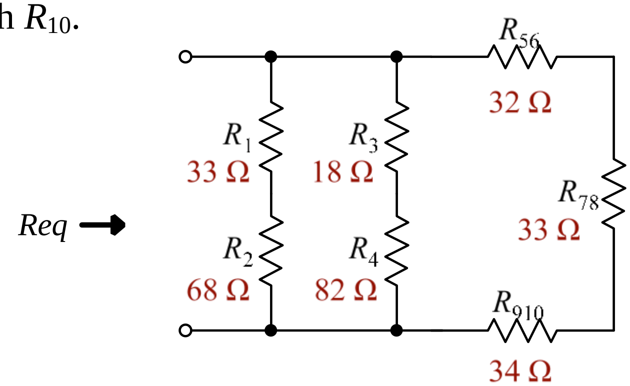
Solution:

At first glance, this looks very difficult, but it's not so bad. We can pick it apart piece by piece. Start by noting that R_7 is in parallel with R_8.

$$R78 = \frac{1}{\frac{1}{R7}+\frac{1}{R8}} = 33.1\Omega$$

Similarly, R_5 is in parallel with R_6 and R_9 is in parallel with R_{10}.

$$R_{56} = \frac{1}{\frac{1}{R5}+\frac{1}{R6}} = 32.0\Omega \qquad R_{910} = \frac{1}{\frac{1}{R9}+\frac{1}{R10}} = 34\Omega$$



Next, we note that there are several series combinations

R_1 in series with R_2: $R_a = R_1 + R_2 = 101\Omega$ R_3 in series with R_4:

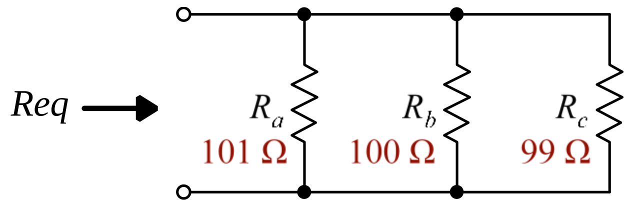
$R_b = R_3 + R_4 = 100\Omega$

R_{56}, R_{78}, and R_{910} all in series:

$R_c = R_{56} + R_{78} + R_{910} = 99\Omega.$

Finally, we see that the equivalent resistance is just the parallel combination of R_a, R_b, and R_c.

Req = $1/R_a + 1/R_b + 1/R_c = 33.3\Omega$



Thank You