Lecture 7

Second Stage

Medical Physical Department

Digital Electronics

Lecture7: Numbering System

By

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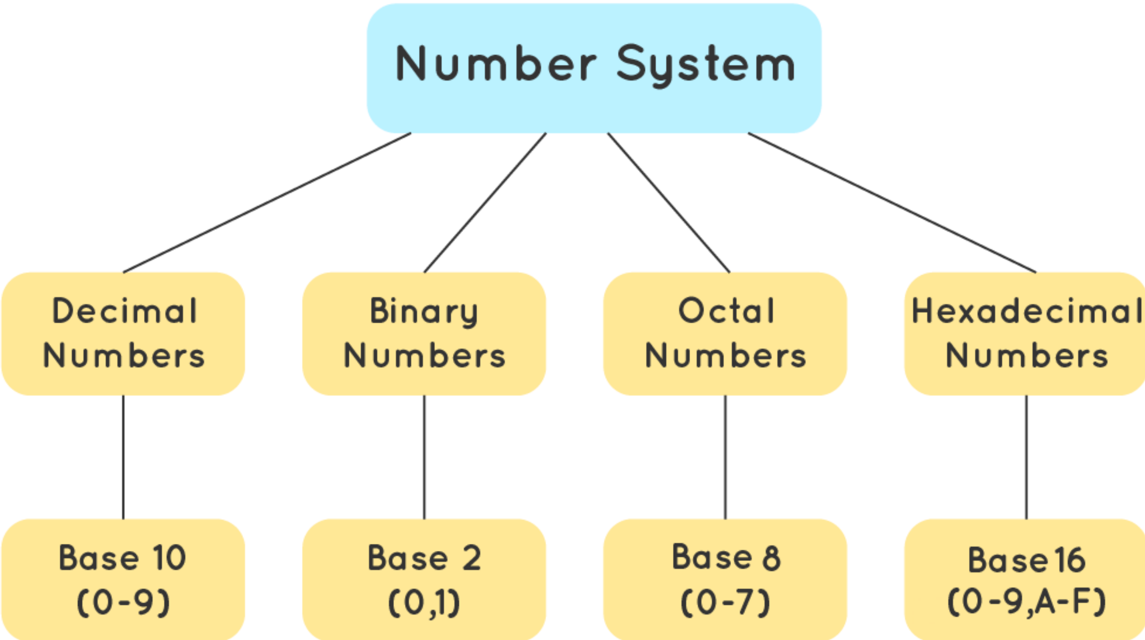
Numbering System

Introduction

A Number System (or system of numeration) is a writing system for expressing numbers, that is a mathematical notation for representing numbers of a given set, using digits or other symbols in a consistent manner. Ideally, a numeral system will represent a useful set of numbers (e.g. all integers, or rational numbers) Give every number represented a unique representation (or at least a standard representation) . Reflect the algebraic and arithmetic structure of the numbers.

1. Types of Number Systems



- ❖ **Decimal system** uses symbols (digits) for the ten values **0, 1, 2, 3, 4, 5, 6, 7, 8, 9**. Humans use decimal number system in daily life for counting and other mathematical calculations. Decimal number system is not suitable for computers and other microprocessor related applications.
- ❖ **Binary System** uses digits for the **two values 0, and 1**. Computers use binary number system. Decimal numbers are converted into binary numbers for digital processing applications.
- ❖ **Octal System** uses digits for ***the eight values*** **0, 1, 2, 3, 4, 5, 6, 7**
- ❖ **Hexadecimal System** uses digits for the **sixteen values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F**. Octal and Hexadecimal Systems are also used for digital processing considering the complexity of processing needs.

1.1 Decimal Number System

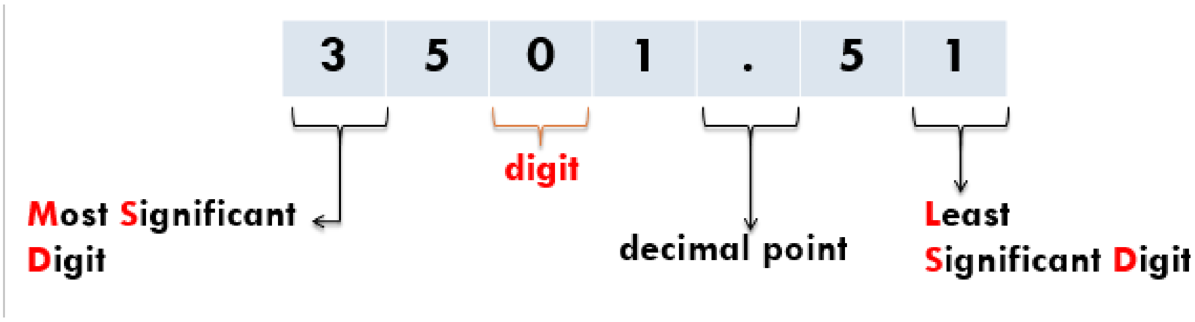
In the **decimal** number system, each of the ten digits **(0 through 9)** represents a certain quantity.

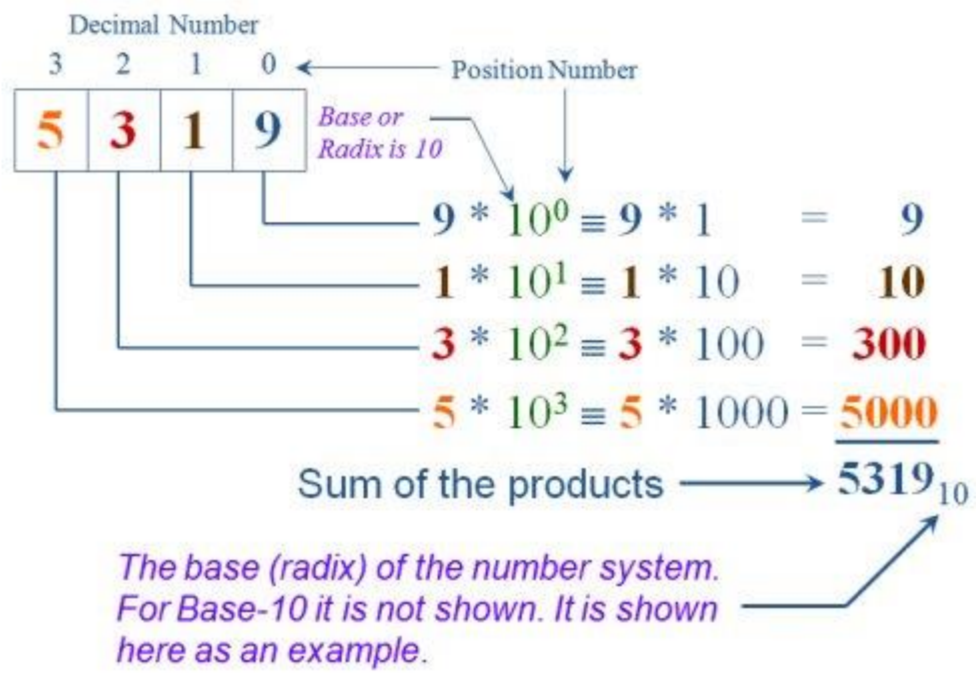
The position of each digit in a decimal number indicates the magnitude of the quantity represented and can be assigned a weight. The weights for whole numbers are positive powers of ten that increase from right to left, beginning with $\mathbf{10^0 = 1}$.

….	10^4	10^3	10^2	10^1	10^0	.	10^{-1}	10^{-2}	10^{-3}	…..
….	10000	1000	100	10	1	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	….

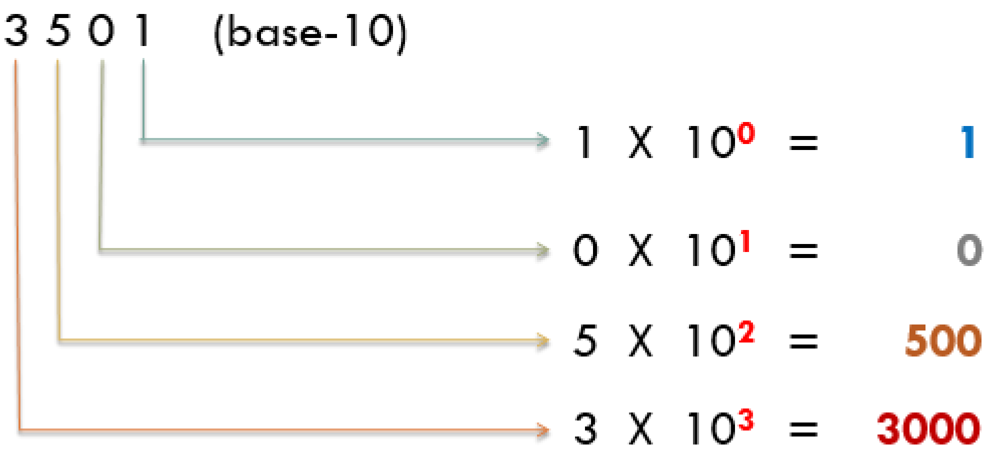
For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1}.

Example : 3501.51





Example: $(3501)_{10}$



$$3000 + 500 + 0 + 1 = 3501$$

Example: consider the four digit decimal number, **4689.567**.

The positional weight of decimal number is used in the radix representation.

Decimal number	4	6	8	9	5	6	7
Positional weight	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

$$4689.576 = 4 \times 10^3 + 6 \times 10^2 + 8 \times 10^1 + 9 \times 10^0 + 5 \times 10^{-1} + 6 \times 10^{-2} + 7 \times 10^{-3}$$

Example: $(3752.46)_{10} = 3 \times 1000 + 7 \times 100 + 5 \times 10 + 2 \times 1 + 4 \times \frac{1}{10} + 6 \times \frac{1}{100}$

EXAMPLE 1: Express the decimal number 47 as a sum of the values of each digit.

Solution:

Decimal Number:	4	7
Decimal Weight:	10^1	10^0 **Base**

$\mathbf{47} = (4 \times 10^1) + (7 \times 10^0)$

$= (4 \times 10) + (7 \times 1) = \mathbf{40} + \mathbf{7}$

Note: The digit 4 has a weight of 10, which is 10^1, as indicated by its position. The digit 7 has a weight of 1, which is 10^0, as indicated by its position.

EXAMPLE 2: Express the decimal number 568.23 as a sum of the values of each digit.

Solution:

Decimal Number:	5	6	8	.	2	3
Decimal Weight:	10^2	10^1	10^0	.	10^{-1}	10^{-2}

568.23 $= (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3\times10^{-2})$

$= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01)$

$= \mathbf{500 + 60 + 8 + 0.2 + 0.03}$

Note: The whole number digit 5 has a weight of 100, which is 10^2, the digit 6 has a weight of 10, which is 10^1, the digit 8 has a weight of 1, which is 10^0, the fractional digit 2 has a weight of 0.1, which is 10^{-1}, and the fractional digit 3 has a weight of 0.01, which is 10^{-2}.

Example: Decimal Number Quantity (fractional number) $(.581)_{10}$

. 5 8 1 (base-10)

$5 \times 10^{-1} = 5\times0.1 = 0.5$

$8 \times 10^{-2} = 8\times0.01 = 0.08$

$1 \times 10^{-3} = 1\times0.001 = 0.001$

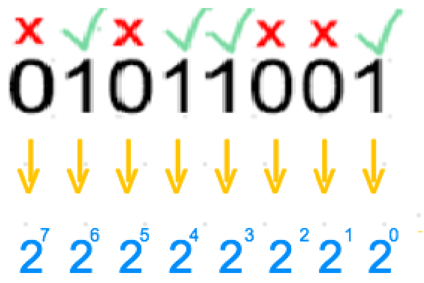
$0.5 + 0.08 + 0.001 = 0.581$

1.2 Binary Number

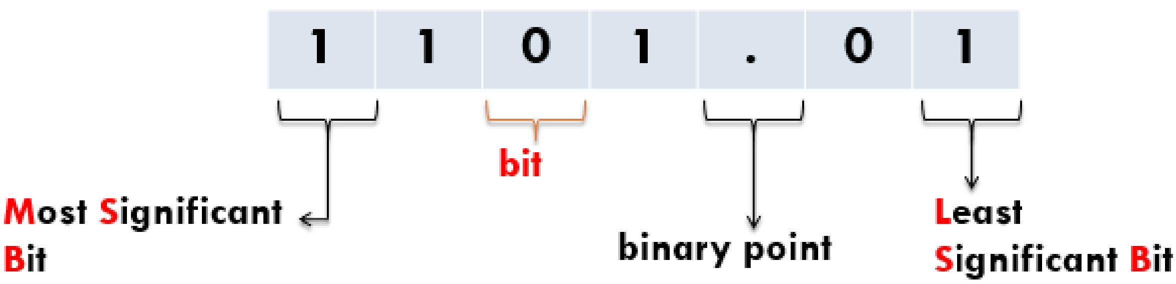
- The binary number system is another way to represent quantities. It is less complicated than the decimal system because the binary system has only two digits. The two binary digits (bits) are 1 and 0. The Weighting Structure of Binary Numbers:
- A binary number is a weighted number. The right-most bit is the LSB (least significant bit) in a binary whole number. The weights increase from right to left by a power of two for each bit. The left-most bit is the MSB (most significant bit); its weight depends on the size of the binary number.

....	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}	2^{-3}	.....
....	16	8	4	2	1	.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	....

Example: 01011001

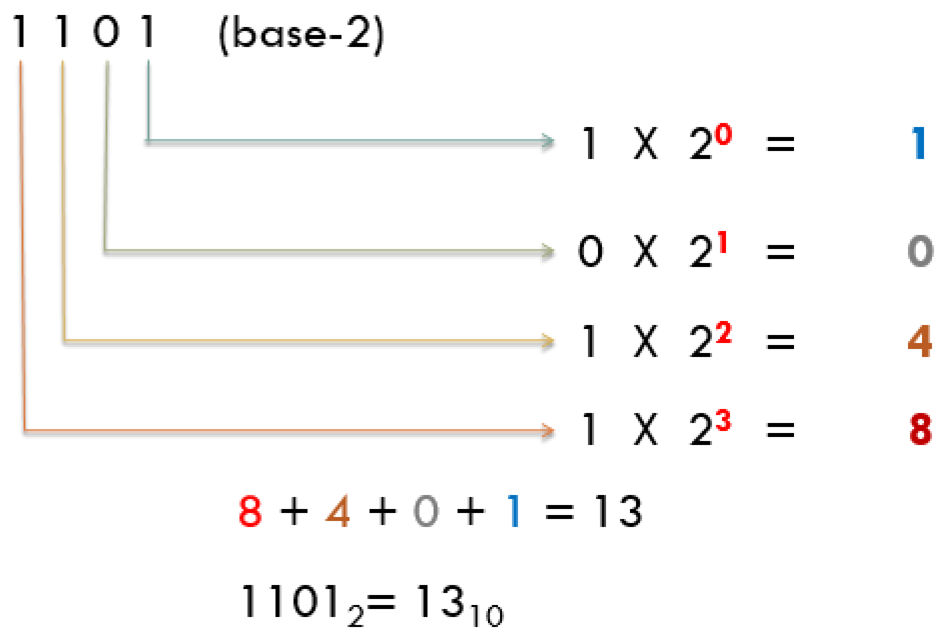


Example : 1101.01



Example: $(1101.11)_2 = 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 + 1 \times \frac{1}{2} + 1 \times \frac{1}{4}$

Example: $(1101)_2$



2. Number Base Conversion

2.1 Decimal-to-Binary Conversion

A. Sum-of-Weights Method:

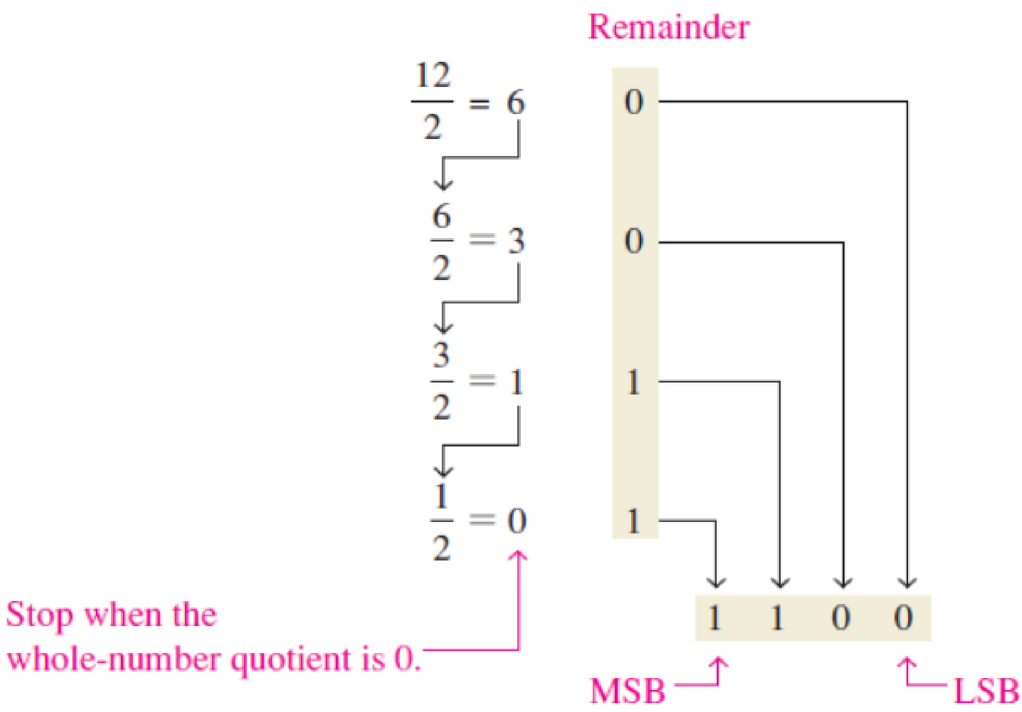
❖ One way to find the binary number that is equivalent to a given decimal number is to determine the set of binary weights whose sum is equal to the decimal number

Binary Weight:	2^3	2^2	2^1	2^0
	8	4	2	1
Binary Number:	1	1	0	0
	↓ MSB			↓ LSB

$\therefore (12)_{10} = (1100)_2$

B. Repeated Division-by-2 Method:

❖ A systematic method of converting whole numbers from decimal to binary is the repeated division-by-2 process until there is a 0 whole-number quotient. The remainders generated by each division form the binary number. The **first remainder** to be produced is the **LSB**, and the **last remainder** to be produced is the **MSB**



2.2 Decimal Fractions-to-Binary Conversion

A. Sum-of-Weights Method:

❖ The sum-of-weights method can be applied to fractional decimal numbers, as shown in the following example:

Binary Weight:	0	.	2^{-1}	2^{-2}	2^{-3}
		.	0.5	0.25	0.125
Binary Number:	0	.	1	0	1

$\therefore (\mathbf{0.625})_{10} = (\mathbf{0.101})_2$

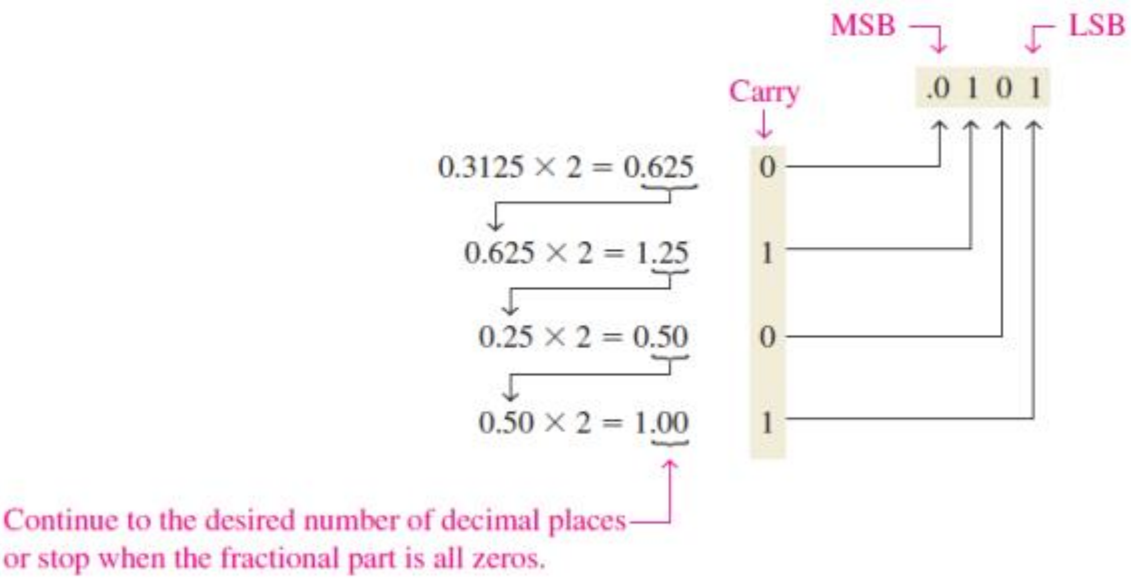
<u>في المثال اعلاه كل قيمة بعد الفارزة تضرب في العدد 2 كالاتي</u>

- ❖ 0.625 x 2= 1.25 , integral part 1
- ❖ 0.25 x 2= 0.5 , integral part 0
- ❖ 0.25 x 2= 1.0 , integral part 1

ثم نكتب الرقم من الاعلى للاسفل وتسمى الطريقة كما في الاسفل

B. Repeated Multiplication by 2 Method:

Decimal fractions can be converted to binary by repeated multiplication by 2. The carry digits, or carries, generated by the multiplications produce the binary number. The **first carry** produced is the **MSB**, and the **last carry** is the **LSB**.



2.3 Binary-to-Decimal Conversion

❖ The decimal value of any binary number can be found by adding the weights of all bits that are 1 and discarding the weights of all bits that are 0.

Binary Weight:	2^6	2^5	2^4	2^3	2^2	2^1	20
	64	32	16	8	4	2	1
Binary Number:	1	1	0	1	1	0	1
	↓ MSB						↓ LSB

$$\mathbf{1101101} = 2^6 + 2^5 + 2^3 + 2^2 + 2^0$$

$$= 64 + 32 + 8 + 4 + 1 = \mathbf{109}$$

$$\therefore (\mathbf{1101101})_2 = (\mathbf{109})_{10}$$

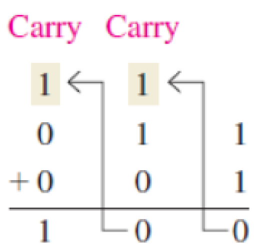
3. Binary Arithmetic

Binary arithmetic is essential in all digital computers and in many other types of digital systems. To understand digital systems, you must know the basics of **binary addition**, **subtraction**, **multiplication**, and **division**.

A. Binary Addition:

The four basic rules for adding binary digits (bits) are as follows:

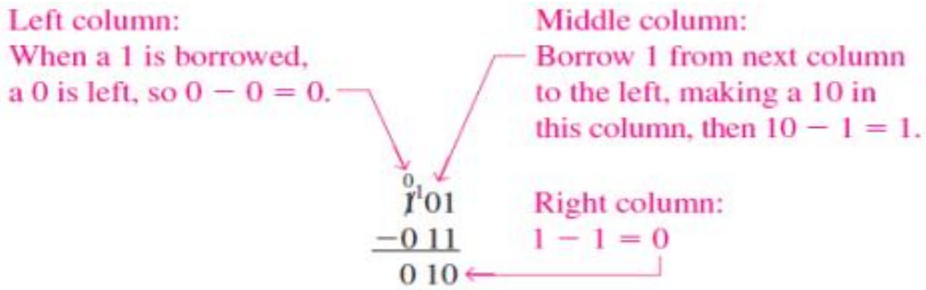
Addition		Result	Carry
0 + 0	=	0	0
0 + 1	=	1	0
1 + 0	=	1	0
1 + 1	=	0	1



B. Binary Subtraction:

The four basic rules for subtracting bits are as follows:

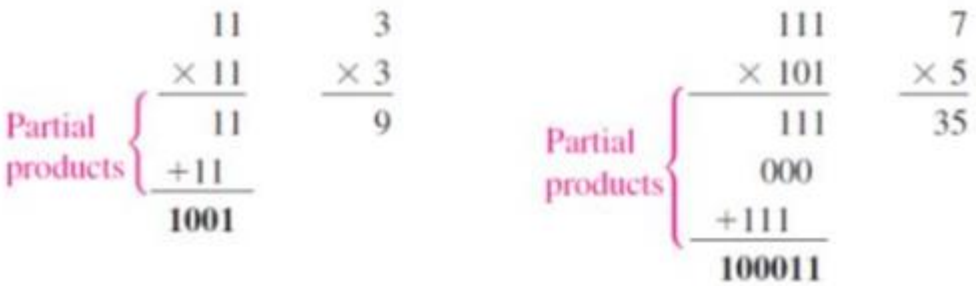
Input A	Input B	Subtract S = A-B	Borrow B
0	0	0	0
0	1	0	1
1	0	1	0
1	1	0	0



C. Binary Multiplication:

The four basic rules for multiplying bits are as follows:

Input A	Input B	Multiply (M) AxB
0	0	0
0	1	0
1	0	0
1	1	1



D. Binary Division:

Division in binary follows the same procedure as division in decimal.

Example: divide the number (11011) on (101)

```
          1 0 1
101  | 1 1 0 0 1
      −1 0 1
        -----
        0 0 1 0 1
      −     1 0 1
            -----
            0 0 0
```

$$\begin{array}{r} \mathbf{10} \\ 11\overline{)110} \\ \underline{11} \\ 000 \end{array} \qquad \begin{array}{r} 2 \\ 3\overline{)6} \\ \underline{6} \\ 0 \end{array} \qquad\qquad \begin{array}{r} \mathbf{11} \\ 10\overline{)110} \\ \underline{10} \\ 10 \\ \underline{10} \\ 00 \end{array} \qquad \begin{array}{r} 3 \\ 2\overline{)6} \\ \underline{6} \\ 0 \end{array}$$