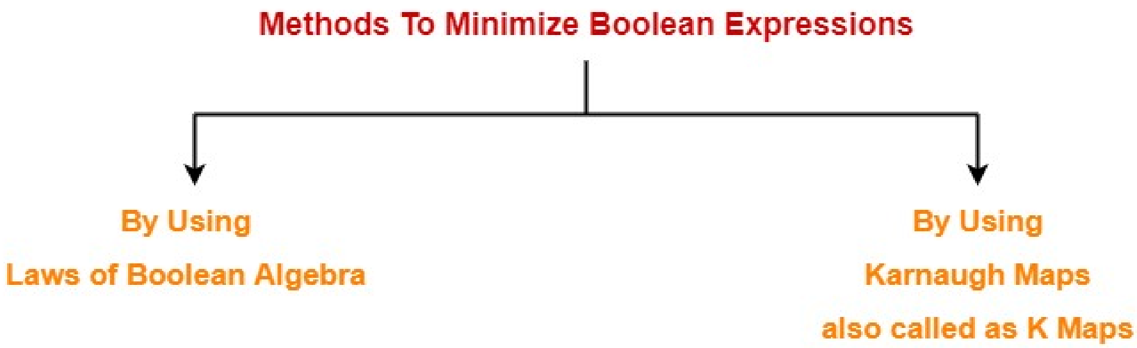
Lec._8

Karnaugh maps



1. By using laws of Boolean Algebra
2. By using Karnaugh Maps also called as K Maps

Karnaugh Map-

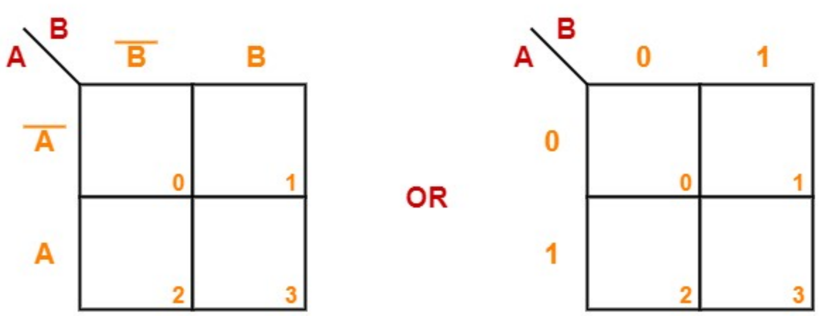
The Karnaugh Map also called as K Map is a graphical representation that provides a systematic method for simplifying the boolean expressions.

For a boolean expression consisting of n-variables, number of cells required in K Map = 2^n cells.

Two Variable K Map-

- Two variable K Map is drawn for a boolean expression consisting of two variables.
- The number of cells present in two variable K Map = 2^2 = 4 cells.
- So, for a boolean function consisting of two variables, we draw a 2 x 2 K Map.

Two variable K Map may be represented as-



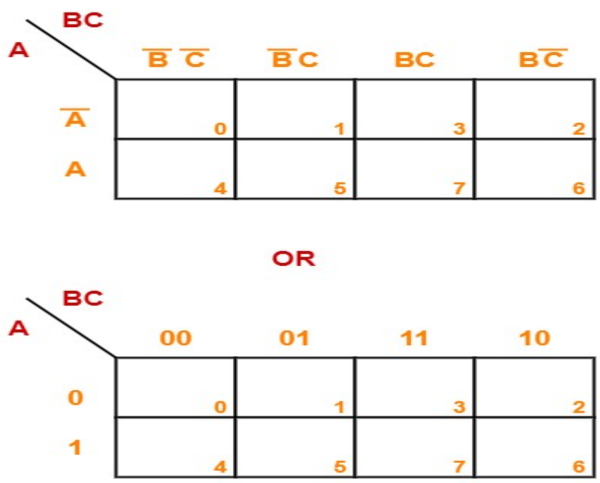
Two Variable K Map

Here, A and B are the two variables of the given boolean function.

Three Variable K Map-

- Three variable K Map is drawn for a boolean expression consisting of three variables.
- The number of cells present in three variable K Map = 2^3 = 8 cells.
- So, for a boolean function consisting of three variables, we draw a 2 x 4 K Map.

Three variable K Map may be represented as-



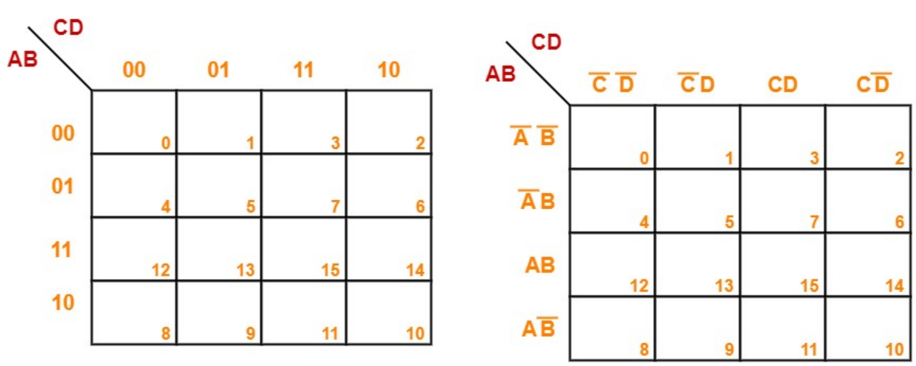
Three Variable K Map

Here, A, B and C are the three variables of the given boolean function.

Four Variable K Map-

- Four variable K Map is drawn for a boolean expression consisting of four variables.
- The number of cells present in four variable K Map = 2^4 = 16 cells.
- So, for a boolean function consisting of four variables, we draw a 4 x 4 K Map.

Four variable K Map may be represented as-



Four Variable K Map

OR

Here, A, B, C and D are the four variables of the given boolean function.

Karnaugh Map Simplification Rules-

To minimize the given boolean function,

- We draw a K Map according to the number of variables it contains.
- We fill the K Map with 0's and 1's according to its function.
- Then, we minimize the function in accordance with the following rules.

Rule-01:

- We can either group 0's with 0's or 1's with 1's but we can not group 0's and 1's together.
- X representing don't care can be grouped with 0's as well as 1's.

NOTE

There is no need of separately grouping X's i.e. they can be ignored if all 0's and 1's are already grouped.

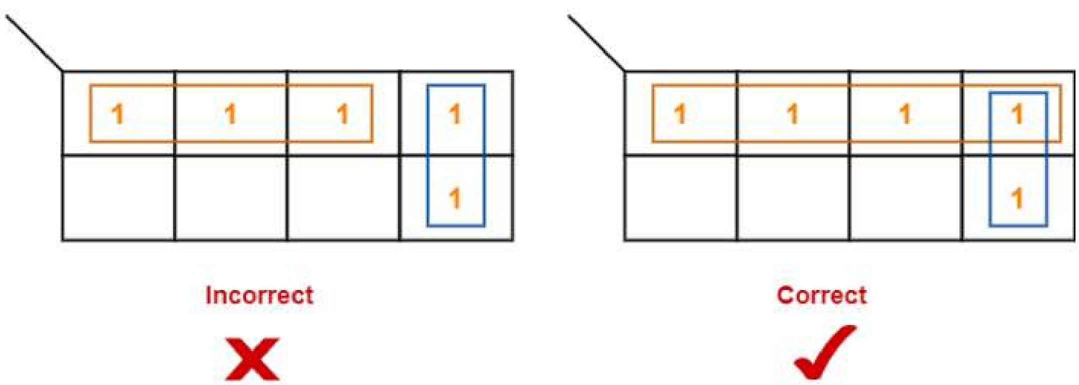
Rule-02:

- Groups may overlap each other.

Rule-03:

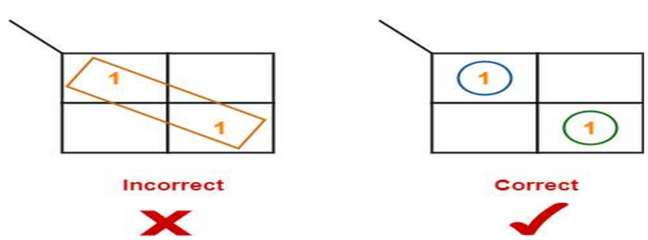
- We can only create a group whose number of cells can be represented in the power of 2.
- In other words, a group can only contain 2^n i.e. 1, 2, 4, 8, 16 and so on number of cells.

Example-

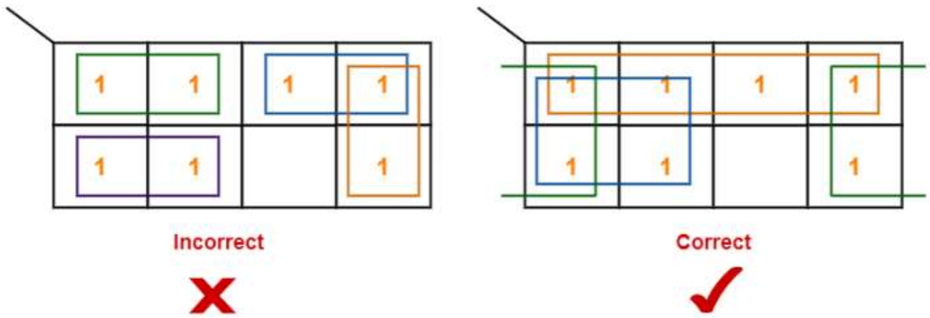


Rule-04:

- Groups can be only either horizontal or vertical.
- We can not create groups of diagonal or any other shape.



Example-



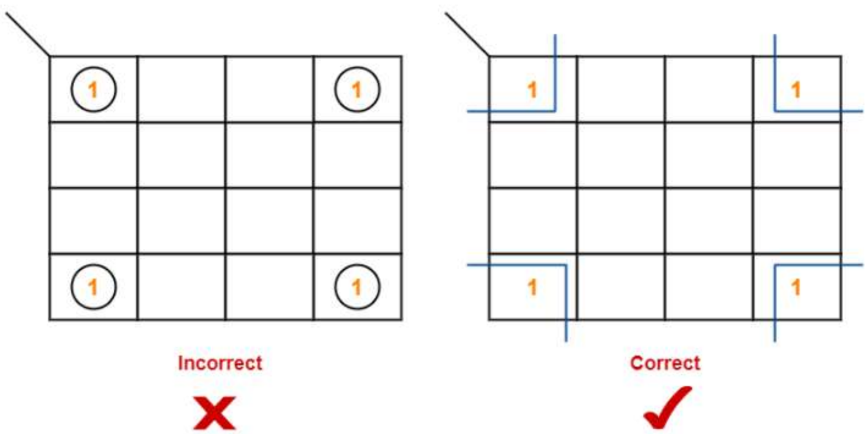
Rule-05:

- Each group should be as large as possible.

Rule-06:

- Opposite grouping and corner grouping are allowed.
- The example of opposite grouping is shown illustrated in Rule-05.
- The example of corner grouping is shown below.

Example-



Rule-07:

- There should be as few groups as possible.

PROBLEMS BASED ON KARNAUGH MAP-

Problem-01:

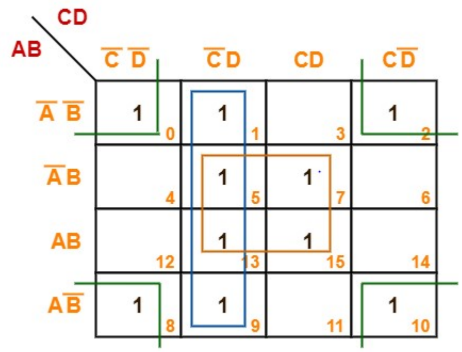
Minimize the following boolean function-

$$F(A, B, C, D) = \Sigma m(0, 1, 2, 5, 7, 8, 9, 10, 13, 15)$$

Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C, D)

= (A’B + AB)(C’D + CD) + (A’B’ + A’B + AB + AB’)C’D + (A’B’ + AB’)(C’D’ + CD’)

= BD + C’D + B’D’

Thus, minimized boolean expression is-

F(A, B, C, D) = BD + C’D + B’D’

Problem-02:

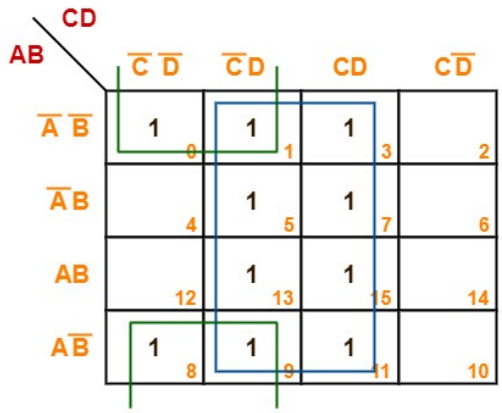
Minimize the following boolean function-

F(A, B, C, D) = Σm(0, 1, 3, 5, 7, 8, 9, 11, 13, 15)

Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C, D)

= (A’B’ + A’B + AB + AB’)(C’D + CD) + (A’B’ + AB’)(C’D’ + C’D)

= D + B’C’

Thus, minimized boolean expression is-

F(A, B, C, D) = B’C’ + D

Problem-03:

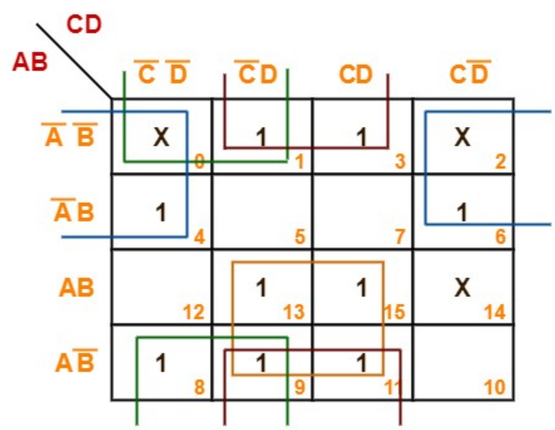
Minimize the following boolean function-

F(A, B, C, D) = Σm(1, 3, 4, 6, 8, 9, 11, 13, 15) + Σd(0, 2, 14)

Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C, D)

= (AB + AB’)(C’D + CD) + (A’B’ + AB’)(C’D + CD) + (A’B’ + AB’)(C’D’ + C’D) + (A’B’ + A’B)(C’D’ + CD’)

= AD + B’D + B’C’ + A’D’

Thus, minimized boolean expression is-

F(A, B, C, D) = AD + B’D + B’C’ + A’D’

Problem-04:

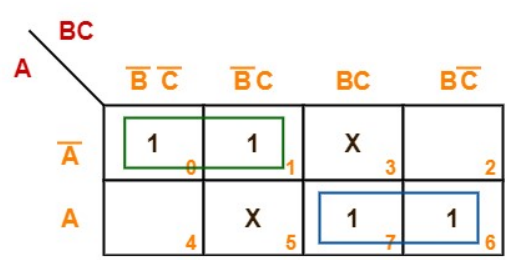
Minimize the following boolean function-

F(A, B, C) = Σm(0, 1, 6, 7) + Σd(3, 5)

Solution-

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C)

= A'(B'C' + B'C) + A(BC + BC')

= A'B' + AB

Thus, minimized boolean expression is-

F(A, B, C) = AB + A'B'

NOTE-

- It may be noted that there is no need of considering the quad group.
- This is because even if we consider that group, we will have to consider the other two duets.
- So, there is no use of considering that quad group.

Problem-05:

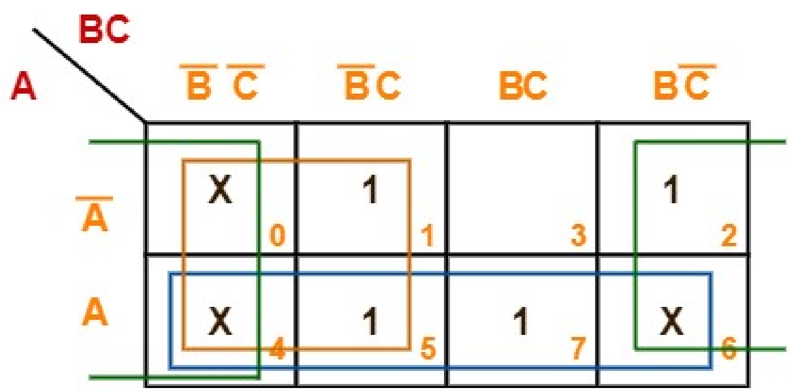
Minimize the following boolean function-

F(A, B, C) = Σm(1, 2, 5, 7) + Σd(0, 4, 6)

Solution-

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C)

= (A + A')(B'C' + B'C) + A(B'C' + B'C + BC + BC') + (A + A')(B'C' + BC')

= B' + A + C'

Thus, minimized boolean expression is-

F(A, B, C) = A + B' + C'

Problem-06:

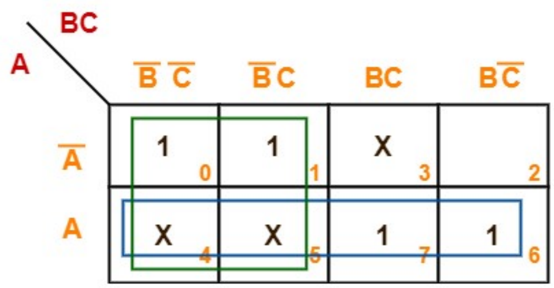
Minimize the following boolean function-

F(A, B, C) = Σm(0, 1, 6, 7) + Σd(3, 4, 5)

Solution-

- Since the given boolean expression has 3 variables, so we draw a 2 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C)

= (A + A')(B'C' + B'C) + A(B'C' + B'C + BC + BC')

= B' + A

Thus, minimized boolean expression is-

F(A, B, C) = A + B'

Problem-07:

Minimize the following boolean function-

$$F(A, B, C, D) = \Sigma m(0, 2, 8, 10, 14) + \Sigma d(5, 15)$$

Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-

Now,

F(A, B, C, D)

= (AB + AB')CD' + (A'B' + AB')(C'D' + CD')

= ACD' + B'D'

Thus, minimized boolean expression is-

F(A, B, C, D) = ACD' + B'D'

Problem-08:

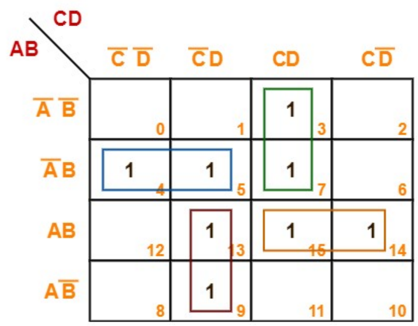
Minimize the following boolean function-

$$F(A, B, C, D) = \Sigma m(3, 4, 5, 7, 9, 13, 14, 15)$$

Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(A, B, C, D)

= A'B(C'D' + C'D) + (A'B' + A'B)(CD) + (AB + AB')(C'D) + AB(CD + CD')

= A'BC' + A'CD + AC'D + ABC

Thus, minimized boolean expression is-

F(A, B, C, D) = A'BC' + A'CD + AC'D + ABC

It is important to note that we are not considering the quad group because we have to consider the duets anyhow.

Problem-09:

Consider the following boolean function-

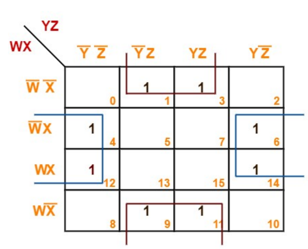
$$F(W, X, Y, Z) = \Sigma m(1, 3, 4, 6, 9, 11, 12, 14)$$

This function is independent ________ number of variables. Fill in the blank.

Solution-

- Since the given boolean expression has 4 variables, so we draw a 4 x 4 K Map.
- We fill the cells of K Map in accordance with the given boolean function.
- Then, we form the groups in accordance with the above rules.

Then, we have-



Now,

F(W, X, Y, Z)

= (W'X + WX)(Y'Z' + YZ') + (W'X' + WX')(Y'Z + YZ)

= XZ' + X'Z

= X ⊕ Z

Thus, minimized boolean expression is-

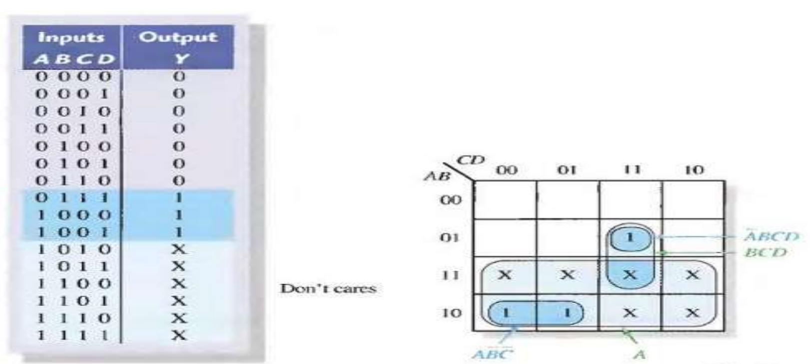
F(W, X, Y, Z) = X ⊕ Z

Clearly, the given boolean function depends on only two variables X and Z.

Hence, it is independent of other two variables W and Y.

"Don't Care" Conditions

Sometimes a situation arises in which some input variable combinations are not allowed. For example, recall that in the BCD code there are six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111. Since these unallowed states will never occur in an application involving the BCD code, they can be treated as "don't care" terms with respect to their effect on the output. That is, for these "don't care" terms either a 1 or a 0 may be assigned to the output: it really does not matter since they will never occur. The "don't care" terms can be used to advantage on the Karnaugh map. Fig.(4-9) shows that for each "don't care" term, an X is placed in the cell. When grouping the 1 s, the Xs can be treated as 1s to make a larger grouping or as 0s if they cannot be used to advantage. The larger a group, the simpler the resulting term will be.



Inputs A B C D	Output Y
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

(a) Truth table

(b) Without "don't cares" $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With "don't cares" $Y = A + BCD$

The truth table in above: (a) describes a logic function that has a 1 output only when the BCD code for 7,8, or 9 is present on the inputs. If the "don't cares" are used as 1s, the resulting expression for the function is A + BCD, as indicated in part (b). If the "don't cares" are not used as 1s, the resulting expression is ABC + ABCD: so you can see the advantage of using "don't care" terms to get the simplest expression.