Al-Mustaqbal University
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Computer Techniques Engineering Department

Digital Communication

Lecture 7

Source Coding Techniques

Calculations, and Practical Examples of PCM

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Aims of this Lecture

- **Calculate** critical parameters like code word length, bandwidth, and bit rate for PCM systems.
- **Solve** practical examples step-by-step to reinforce understanding of PCM concepts.
- **Identify** the advantages and limitations of PCM in communication systems.

Example 1: PCM Calculations

Problem: A television signal with a bandwidth of 4.2 MHz4.2 \, \text{MHz}4.2MHz is transmitted using binary PCM. The number of quantization levels is 512. Calculate:

1. Code word length
2. Transmission bandwidth
3. Final bit rate
4. Output signal-to-quantization noise ratio

Example 1

Step 1: Code Word Length (v)

The formula for the number of quantization levels is:

$$q = 2^v$$

Taking the logarithm base 2 of both sides to solve for v:

$$v = \log_2(q)$$

Substitute $q = 512$:

$$v = \log_2(512) = \log_{10}(512)/\log_{10}(2)$$

From logarithmic values:

$$\log_{10}(512) = 2.709 \quad \text{and} \quad \log_{10}(2) = 0.301$$

So:

$$v = \frac{2.709}{0.301} = 9\,\text{bits}$$

Answer: Code word length = **9 bits**

Example 1

Step 2: Transmission Bandwidth (B_T)

The formula for PCM transmission bandwidth is:

$$B_T \geq v \cdot W$$

Substitute $v = 9\,\text{bits}$ and $W = 4.2\,\text{MHz}$:

$$B_T \geq 9 \times 4.2 = 37.8\,\text{MHz}$$

Answer: Transmission bandwidth = **37.8 MHz**

Step 3: Final Bit Rate (r)

The formula for signaling rate is:

$$r = v \cdot f_s$$

The sampling frequency is related to the bandwidth (W):

$$f_s = 2W = 2 \times 4.2 = 8.4\,\text{MHz}$$

Substitute $v = 9\,\text{bits}$ and $f_s = 8.4\,\text{MHz}$:

$$r = 9 \times 8.4 = 75.6\,\text{Mbps}$$

Answer: Final bit rate = **75.6 Mbps**

Example 1

Step 4: Signal-to-Noise Ratio (S/N)

The formula for S/N in PCM is:

$$S/N = 4.8 + 6v \, (\text{in dB})$$

Substitute $v = 9 \, \text{bits}$:

$$S/N = 4.8 + 6 \times 9 = 4.8 + 54 = 58.8 \, \text{dB}$$

Answer: $S/N = 58.8 \, \text{dB}$

Example 2

Example:

A signal input to PCM has a bandwidth of $W = 4\,\mathrm{kHz}$. The input varies between $-3.8\,\mathrm{V}$ and $+3.8\,\mathrm{V}$ with an average power of $30\,\mathrm{mW}$. The SNR is $20\,\mathrm{dB}$. Calculate:

1. **Number of bits per sample**
2. **Transmission bandwidth** if 20 PCM coders are multiplexed.

Example 2

Step 1: Signal-to-Noise Ratio (S/N)

The formula for S/N is given in decibels:

$$S/N = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right)$$

From the problem, $S/N = 20\,\text{dB}$:

$$20 = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right)$$

Simplify:

$$\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right) = 2$$

So:

$$\frac{\text{Signal Power}}{\text{Noise Power}} = 10^2 = 100$$

Example 2

Step 2: Number of Bits per Sample (v)

The SNR for PCM is also given by:

$$S/N = 2^{2v}$$

Substitute $S/N = 100$:

$$2^{2v} = 100$$

Take the logarithm base 2:

$$2v = \log_2(100) = \frac{\log_{10}(100)}{\log_{10}(2)}$$

From logarithmic values:

$$\log_{10}(100) = 2, \quad \log_{10}(2) = 0.301$$

$$2v = \frac{2}{0.301} = 6.644$$

Divide by 2:

$$v = \frac{6.644}{2} = 3.322\,\text{bits}$$

Round up:

$$v = 7\,\text{bits}$$

Answer: $v = 7\,\text{bits}$

Example 2

Step 3: Transmission Bandwidth (B_T)

For PCM, the bandwidth is:

$$B_T \geq v \cdot W$$

Substitute $v = 7$ and $W = 4\,\mathrm{kHz}$:

$$B_T = 7 \cdot 4 = 28\,\mathrm{kHz}$$

If 20 PCM coders are multiplexed:

$$B_T = 20 \cdot 28 = 560\,\mathrm{kHz}$$

Answer: Transmission bandwidth = **560 kHz**

Example 3: PCM System Requirements

Problem Statement:

The information in an analog signal voltage waveform is to be transmitted over a PCM system with the following specifications:

- Accuracy: $\pm 0.1\%$ (full scale).
- Bandwidth: $W = 100\,\mathrm{Hz}$.
- Amplitude range: $-10\,\mathrm{V}$ to $+10\,\mathrm{V}$.

Determine:

1. The **number of levels** required for such accuracy.
2. The **code word length**.
3. The **minimum bit rate** required.
4. The **bandwidth required for the PCM signal**.

Example 3

Solution

Step 1: Number of Levels (q)

The quantization step size (δ) is determined by the accuracy requirement:

$$\delta = \frac{\text{Full Scale Range}}{\text{Number of Levels}}$$

Rearranging for q:

$$q = \frac{\text{Full Scale Range}}{\delta}$$

- Full scale range = $10 - (-10) = 20\,\text{V}$.
- Accuracy requirement = $\pm 0.1\%$:

$$\delta = 0.001 \times 20 = 0.02\,\text{V}$$

Substitute $\delta = 0.02\,\text{V}$:

$$q = \frac{20}{0.02} = 1000\,\text{levels}$$

Answer: $q = 1000\,\text{levels}$

Example 3

Step 2: Code Word Length (v)

The code word length (v) is related to the number of levels (q) by:

$$q = 2^v$$

Take the logarithm base 2 of both sides:

$$v = \log_2(q)$$

Substitute $q = 1000$:

$$v = \log_2(1000) = \frac{\log_{10}(1000)}{\log_{10}(2)}$$

From logarithmic values:

$$\log_{10}(1000) = 3, \quad \log_{10}(2) = 0.301$$

$$v = \frac{3}{0.301} = 9.966$$

Since v must be an integer, round up to the nearest whole number:

$$v = 10\,\text{bits}$$

Answer: $v = 10\,\text{bits}$

Step 3: Minimum Bit Rate (r)

Example 3

The bit rate (r) is calculated as:

$$r = v \cdot f_s$$

The sampling frequency (f_s) must satisfy the Nyquist criterion:

$$f_s = 2W = 2 \cdot 100 = 200\,\text{Hz}$$

Substitute $v = 10\,\text{bits}$ and $f_s = 200\,\text{Hz}$:

$$r = 10 \cdot 200 = 2000\,\text{bps} = 2\,\text{kbps}$$

Answer: $r = 2\,\text{kbps}$

Step 4: Bandwidth Required (B_T)

The minimum bandwidth required for a PCM signal is:

$$B_T = \frac{r}{2}$$

Substitute $r = 2000\,\text{bps}$:

$$B_T = \frac{2000}{2} = 1000\,\text{Hz} = 1\,\text{kHz}$$

Answer: $B_T = 1\,\text{kHz}$

Homework

Q1/ The information in an analog waveform with maximum frequency $f_m = 3\ kHz$ is to be transmitted over 16- levels PCM system. The quantization distortion is specified not exceed 1% of peak to peak analog signal.

i- What is the number of bits per sample that should be used in this PCM?

ii- What is minimum bit transmission rate?

Q2 / A signal of bandwidth 3.5 kHz is sampled, quantized and coded by PCM system. The code signal is then transmitted over a transmission channel of supporting a transmission rate of 50 kbps. Calculate the maximum signal to noise that can obtained by this system. The input signal has peak to peak value of 4 volts and rms value of 0.2 V.

Q3 / Consider an audio signal comprised of the sinusoidal term $(t) = 3\cos(500\pi t)$.

i- Find the number of quantization level with an accuracy of 1%.

ii- Determine the signaling rate.

iii- The bandwidth of transmission channel.

Advantages of PCM

- Reduces the effect of channel noise and interference.
- Allows regeneration of signals along the transmission path, reducing errors.
- Enables easy multiplexing of multiple PCM signals.
- Supports encryption and decryption for secure communication.

Limitations of PCM

- PCM systems are more complex compared to analog pulse modulation systems.
- Requires higher channel bandwidth due to digital coding.

Modifications of PCM

1. **Delta Modulation**: Simplified implementation for specific applications.
2. **Wideband Communication Channels**: PCM can be adapted to support high-bandwidth applications.
3. **Data Compression**: Reduces redundancy, improving efficiency.

Thank you