1.8. Differentiation (definition and rules)

- Definition of derivatives

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Example 1: if y=x^2 $find\ \frac{dy}{dx}\ \ by\ using\ \ Definition$

$of\ derivatives$.

Solution/ /

$$\bar{y} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\bar{y} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$\bar{y} = \lim_{\Delta x \to 0} \frac{(x^2 + 2x\Delta x + \Delta x^2) - x^2}{\Delta x}$$

$$\bar{y} = 2x$$

Exercise 1:if y=x^3 $find\ \frac{dy}{dx}\ \ by\ using$

$Definition\ of\ derivatives$.

Rules of derivatives

$$\bar{y} = \frac{dy}{dx}. \ \bar{\bar{y}} = \frac{d^2y}{dx^2}$$

Assume U and V are differentiable functions of (x).

1. Constant: $\frac{d}{dx}(c) = 0$
2. Sum: $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
3. Difference: $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$
4. Constant Multiple: $\frac{d}{dx}(C.u) = C.\frac{du}{dx}$
5. Product: $\frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v.\frac{du}{dx}$
6. Quotient: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{du}{dx}}{v^2}$
7. Power: $\frac{d}{dx}x^n = n.x^{n-1}$

8. Chain Rule: if y=f(u) , u=g(x)

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$$

9.Parametric Equation: if X=f(t) , y=f(t)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

10. Implicit Differentiation

$$x^2y - xy^2 = y \qquad\qquad Implicit$$

Y=$x^2 + 2x + 2$ Explicit

1.9 Derivatives of trigonometric function

1. $\frac{d}{dx}$(sinu)=cos u.$\frac{du}{dx}$

2. $\frac{d}{dx}$(cosu)=-sin u.$\frac{du}{dx}$

3. $\frac{d}{dx}$(tanu)=sec^2u .$\frac{du}{dx}$

4. $\frac{d}{dx}$(cot u)=-csc^2u .$\frac{du}{dx}$

5. . $\frac{d}{dx}$(sec u)=sec $u.\tan u$.$\frac{du}{dx}$

6. $\frac{d}{dx}$(csc u)=-csc $u.\cot\ u$.$\frac{du}{dx}$

Example 1: Find $\frac{dy}{dx}. y = 2x^4 + 5x^2 + x + 19$

Solution//

$$\frac{dy}{dx} = 8x^3 + 10x + 1$$

Example 2: Find $\frac{dy}{dx}. y = (x^2 + 1)^5$

Solution//

$$\frac{dy}{dx} = 5(x^2+1)^4 * 2x = 10x(x^2+1)^4$$

Example 3: Find $\frac{dy}{dx}.\ y = [(5-x)(4-2x)]^2$

Solution/

$$\frac{dy}{dx} = 2[(5-x)(4-2x)]^1((5-x)*(-2)+(4-2\text{x})*(-1)$$

Example 4: Find $\frac{d^4y}{dx^4}.\ y = 3x^4+2x+19$

Solution/

$$\frac{dy}{dx} = 12x^3+2$$

$$\frac{d^2y}{dx^2} = 36x^2$$

$$\frac{d^3y}{dx^3} = 72x$$

$$\frac{d^4y}{dx^4} = 72$$

Example 5: Find $\frac{dy}{dx}.\ if\ y = u^2+2u+1\ and\ u = \sqrt{x^2+1}$

Solution//

$$\frac{dy}{du} = 2u+2\ . \qquad \frac{du}{dx} = \frac{1}{2\sqrt{x^2+1}} * 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = (2u + 2) * \frac{1}{2\sqrt{x^2 + 1}} * (2x)$$

$$\frac{\mathrm{dy}}{\mathrm{dx}} = (2\sqrt{x^2 + 1} + 2)\frac{x}{\sqrt{x^2 + 1}}$$

Example 6: Find $\frac{dy}{dx}. if\ y = 2t^3 - 6t\ and\ x = t^2$+2t

Solution//

$$\frac{dy}{dt} = 6t^2 - 6 \quad .\frac{dx}{dt} = 2t + 2$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t^2 - 6}{2t + 2} = \frac{3(t - 1)(t + 1)}{(t + 1)} = 3(t - 1)$$

Example 7: Find $\frac{dy}{dx}. given\ \ x^2y - xy^2 + x^2 + y^2 = 0$

Solution//

$$x^2\frac{\mathrm{dy}}{\mathrm{dx}} + y(2x) - x2y\frac{dy}{dx} - y^2 + 2x + 2y\frac{dy}{dx} = 0$$

$$\frac{\mathrm{dy}}{\mathrm{dx}}[x^2 - x2y + 2y] = -2yx + y^2 - 2x$$

$$\frac{dy}{dx} = \frac{-2yx + y^2 - 2x}{x^2 - x2y + 2y}$$

Example 8: Find $\frac{d^2y}{dx^2}. y = x^2 \sin x$

Solution//

$$\frac{dy}{dx} = x^2 \cos x + \sin x * 2x$$

$$\frac{d^2y}{dx^2} = x^2 * (-\sin x) + \cos x * 2x + \sin x * 2 + 2x * \cos x$$

Example 9: Find $\frac{dy}{dx} . y = \csc^{\frac{-2}{3}} \sqrt{5x}$

Solution//

$$\frac{dy}{dx} = -\frac{2}{3} \csc^{\frac{-5}{3}} \sqrt{5x} * -\csc\sqrt{5x} * \cot\sqrt{5x} * \frac{5}{2\sqrt{5x}}$$

Example 10: Find $\frac{dy}{dx} . given \;\; \sin xy = tan^2 x^2 - \sin(x + y) + 3\pi$

Solution/

$$\cos\ xy * (x\frac{dy}{dx} + y * 1) = 2\tan x^2 * \sec^2 x^2 * 2x - \cos(x+y) * (1 + \frac{dy}{dx})$$

$$x \cos\ xy * \frac{dy}{dx} + y * \cos xy) = 4x \tan x^2 * \sec^2 x^2 - \cos(x+y) - \cos(x+y)\frac{dy}{dx})$$

$$\frac{dy}{dx}[x \cos\ xy + \cos(x+y)] = 4x \tan x^2 * \sec^2 x^2 - \cos(x+y) - y \cos xy$$

$$\frac{\mathbf{dy}}{\mathbf{dx}} = \frac{4x\tan x^2 * sec^2 x^2 - cos(x+y) - y\cos xy}{x\cos\ xy + cos(x+y)}$$

Example 11: Find y= $\tan x\ prove\ that\ \frac{dy}{dx} = sec^2 x$

Solution//

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x * \cos x - \sin x * -\sin x}{cos^2 x} = \frac{cos^2 x + sin^2 x}{cos^2 x}$$

$$\frac{dy}{dx} = \frac{1}{cos^2 x} = sec^2 x$$

Example 12: $if\ y = \sin x\ \ proof\ that\ \frac{dy}{dx} = cos\ x$

Solution//

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\mathrm{f}(\,x + \Delta x) - f(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sin(\,x + \Delta x) - sin\ (x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sin(\,x) * cos\ (\Delta x) + sin\ (\Delta x) * cos(x) - sin\ x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{-\sin(\,x)(1 - cos\ (\Delta x)) + sin\ (\Delta x) * cos(x)}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0}\left(\frac{-\sin(x)\,(1-\cos(\Delta x)}{\Delta x} + \lim_{\Delta x \to 0}\left(\frac{\sin(\Delta x)\cos(x)}{\Delta x}\right.\right.$$

$$= \cos x$$