1.12 Integration

$$\text{Integration}\begin{cases} indefinite\ integration\ \int f(x)dx \\ definite\ integration\ \int_a^b f(x)dx \end{cases}$$

Properties of Integration:

1. $\int dx = x + c$

2. $\int f(x).dx = F(x) + c$

3. $\int a.f(x).dx = a.\int f(x).dx = a.F(x) + c$

4. $\int [f(x) \pm g(x)].dx = \int f(x).dx \pm \int g(x).dx$

5. $\int x^n.dx = \frac{x^{n+1}}{n+1} + c$

6. $\int [f(x)]^n.\bar{f}(x)dx = \frac{[f(x)]^{n+1}}{n+1}$

Example 1: Find $\int x\sqrt{2x^2 - 1}\, dx$

Solution //

$= \int x(2x^2 - 1)^{\frac{1}{2}}\, dx = \frac{1}{4} * \frac{2}{3}(2x^2 - 1)^{\frac{3}{2}} + c = \frac{1}{6}(2x^2 - 1)^{\frac{3}{2}} + c$

Example 2:Find $\int \sqrt{x^2 - x^4}\, dx$

Solution //

$$= \int \sqrt{x^2(1 - x^2)}\, dx$$

$$= \int x(1 - x^2)^{\frac{1}{2}} dx = \frac{-1}{2}\frac{2}{3}(1 - x^2)^{\frac{3}{2}} + c$$

$$= \frac{-1}{3}(1 - x^2)^{\frac{3}{2}} + c$$

Example 3:Find $\int (\frac{1}{x^2} + x)dx$

Solution //

$$= \int x^{-2}\, dx + \int x\, dx = -x^{-1} + \frac{x^2}{2} + c$$

Exercises:

$$\int \left(\sqrt{x} - 3\sqrt[3]{x^2} + 5\,\frac{1}{\sqrt{x}} \right). dx$$

$$\int \frac{x.dx}{\sqrt{4-x^2}}$$

$$\int \frac{dx}{\sqrt{x}.(1+ \sqrt{x})^2}$$

Integration of Trigonometric Function

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \sec^2 x \, dx = \tan x + c$$

$$\int \csc^2 x \, dx = -\cot x + c$$

$$\int \sec x \tan x \, dx = \sec x + c$$

$$\int \csc x \cot x \, dx = -\csc x + c$$

Example 1:Find $\int \cos(7x + 1)dx$

Solution //

$$= \frac{1}{7}\sin(7x + 1) + c$$

Example 2:Find $\int \cos^5 x \, dx$

Solution //

$$= \int (1 - sin^2 x)(1 - sin^2 x) \cos x \, dx$$
$$== \int (1 - 2sin^2 x + sin^4 x) \cos x \, dx$$

$$= \int \left(\cos x \right) dx - \int (2 \sin^2 x. \cos x) \, dx$$

$$+ \int \left(\sin^4 x. \cos x \right) dx =$$

$$= \sin x - \frac{2}{3} \sin^3 x + \frac{\sin^5 x}{5} + c$$

Example 3:Find $\int \sec^2 x. \tan x \, dx$

Solution //

$$= \int \sec x . \sec x. \ \tan x \, dx = \frac{\sec^2 x}{2} + c$$

Example 4:Find $\int \cos x (\sin x)^2 \, dx$

Solution //

$$= \frac{(\sin x)^3}{3} + c$$