



(LIMITS) الغاية

Properties of limits

1- If $f(x)=k$ then $\lim_{x \rightarrow a} f(x) = k$

2- If $\lim_{x \rightarrow a} f_1(x) = L_1$ $\lim_{x \rightarrow a} f_2(x) = L_2$

a) Sum rule: $\lim_{x \rightarrow a} [f_1(x) + f_2(x)] = L_1 + L_2$

b) Difference rule: $\lim_{x \rightarrow a} [f_1(x) - f_2(x)] = L_1 - L_2$

c) Product rule : $\lim_{x \rightarrow a} [f_1(x) * f_2(x)] = L_1 * L_2$

d) Quotient rule: $\lim_{x \rightarrow a} \left[\frac{f_1(x)}{f_2(x)} \right] = \frac{L_1}{L_2}$

3- Polynomial $\lim_{x \rightarrow a} (c_0 + c_1x + c_2x^2 + \dots + c_nx^n) = c_0 + c_1a + c_2a^2 + \dots + c_na^n$

Example: Find the limits of the following:

1- $\lim_{x \rightarrow 2} (x^2 - 4x) = 2^2 - 4 * 2 = -4$

2- $\lim_{x \rightarrow 1} (x^3 - 2x^2) = 1^3 - 2 * 1^2 = -1$

3- $\lim_{x \rightarrow 1} \left[\frac{(3x-1)^2}{(x+1)^3} \right] = \frac{(3*1-1)^2}{(1+1)^3} = \frac{(2)^2}{(2)^3} = \frac{4}{8}$

4- $\lim_{x \rightarrow 2} \left[\frac{(x^2-4)}{x-2} \right] = \frac{0}{0}$ (Indeterminate quantities كمية غير محددة)

So $\lim_{x \rightarrow 2} \left[\frac{(x-2)(x+2)}{x-2} \right] = \lim_{x \rightarrow 2} (x + 2) = 2 + 2 = 4$

5- $\lim_{x \rightarrow 2} \left[\frac{(x^2-4)}{x^2-5x+6} \right] = \frac{0}{0}$ (Indeterminate quantities كمية غير محددة)

So $\lim_{x \rightarrow 2} \left[\frac{(x-2)(x+2)}{(x-2)(x-3)} \right] = \lim_{x \rightarrow 2} \left[\frac{(x+2)}{(x-3)} \right] = \frac{4}{-1} = -4.$



الدوال المستمرة (Continuous Functions) :

The Continuity Test:

The function $y = f(x)$ is continuous at $x=c$ if and only if the following statements are true:

- 1- $f(c)$ exists
- 2- $\lim_{x \rightarrow c} f(x)$ exists
- 3- $f(c) = \lim_{x \rightarrow c} f(x)$

Example: did the function $f(x) = 8 - x^3 - 2x^2$ is continuous at the $x=2$?

Sol:

$$f(2) = 8 - 2^3 - 2 * (2)^2 = 8$$

$$\lim_{x \rightarrow 2} [8 - x^3 - 2x^2] = 8 - 2^3 - 2 * (2)^2 = 8$$

$$f(2) = \lim_{x \rightarrow 2} f(x).$$

So the function is continuous at $x=2$.

Example: did the function $f(x) = \frac{(x^2-4)}{x-2}$ is continuous at the $x=2$?

Sol:

$$f(2) = \frac{(2^2-4)}{2-2} = \frac{0}{0} \text{ not exists}$$

So the function is not continuous at $x=2$.



H.W:

1- did the function $f(x) = \frac{(x^2-9)}{x-3}$ is continuous at the $x=3$?

2- Find the limit of the function $f(x) = \frac{(x^2-1)}{x-\sqrt{1}}$ is continuous at the $x=\sqrt{1}$?