Al-Mustaqbal University

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جامعة المستقبل

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LECTURE: (2)

Subject: Cramer's Rule

Level: First

Lecturer: Dr. Mustafa Talal

9 – $(A\,B)^T = B^T A^T$

10 – $A^{-1} A = A.A^{-1} = I$

1.12 Cramer's Rule

Let the system of linear question as

$$\left.\begin{aligned} a_{11}\,\chi_1 + a_{12}\,\chi_2 &= b1 \\ a_{21}\,\chi_1 + a_{22}\,\chi_2 &= b2 \end{aligned}\right\} \rightarrow (i)$$

The system (i) can put in the form:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{11} & a_{22} \end{pmatrix}\begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \rightarrow (ii)$$

If D = $\begin{vmatrix} a11 & a12 \\ a21 & a22 \end{vmatrix} \neq 0$

Then the system (ii) has a unique solution, and Cramer's rule state that it may be found from the formulas:

$$\chi_1 = \frac{\begin{vmatrix} b1 & a12 \\ b2 & a22 \end{vmatrix}}{D}, \; X_2 = \frac{\begin{vmatrix} a11 & b1 \\ a21 & b2 \end{vmatrix}}{D}$$

Example: solve the system

$3X_1 - \chi_2 = 9$

$X_1 + 2X_2 = -4$

So, the system can put in the form

$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}, \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, = \begin{pmatrix} 9 \\ -4 \end{pmatrix}$$

$$D = \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = 7 \;,\; \chi_1 = \frac{\begin{vmatrix} 9 & -1 \\ -4 & 2 \end{vmatrix}}{D} = \frac{14}{7} = 2$$

$$\chi_2 = \frac{\begin{vmatrix} 3 & 9 \\ 1 & -4 \end{vmatrix}}{D} = \frac{21}{7} = 3$$

Let the following system in the unknowns:

$$\begin{aligned} a11\ \chi_1 + a11\ \chi_2 + a11\,3X_3 &= b1 \\ a21\ \chi_1 + a22\ \chi_2 + a23X_3 &= b2 \\ a31\ \chi_1 + a32X_2 + a33\ \chi_3 &= b3 \end{aligned}$$

The system (I) can be put in the form:

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} b1 \\ b2 \\ b3 \end{pmatrix} \quad \text{(II)}$$

$$\text{If } D = \begin{vmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{vmatrix} \neq 0$$

The system has a unique solution, given by Cramer's rule:

$$\chi_1 = \frac{1}{D}\begin{vmatrix} b1 & a12 & a13 \\ b2 & a22 & a23 \\ b3 & a32 & a33 \end{vmatrix} ,\; X_2 = \frac{1}{D}\begin{vmatrix} a11 & b1 & a13 \\ a21 & b2 & a23 \\ a31 & b3 & a33 \end{vmatrix} \; X_3 = \frac{1}{D}\begin{vmatrix} a11 & a12 & b1 \\ a21 & a22 & b2 \\ a31 & a32 & b3 \end{vmatrix}$$

Example: solve the system

$$\begin{aligned} X_1 + 3X_2 - 2X_3 &= 11 \\ 4X_1 - 2X_2 + X_3 &= -15 \\ 3X_1 + 4X_2 - X_3 &= 3 \end{aligned}$$

By cramer's rule.

The system (1) become $\begin{pmatrix} 1 & 3 & -2 \\ 4 & -2 & 1 \\ 3 & 4 & -1 \end{pmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{pmatrix} 11 \\ -15 \\ 3 \end{pmatrix}$

Since D = det = $\begin{vmatrix} 1 & 3 & -2 \\ 4 & -2 & 1 \\ 3 & 4 & -1 \end{vmatrix} = -25$

Cramer's rule gives the solution:

$$\chi_1 = \frac{\begin{vmatrix} 11 & 3 & -2 \\ -15 & -2 & 1 \\ 3 & 4 & -1 \end{vmatrix}}{-25} = \frac{50}{-25} = -2$$

$$\chi_2 = \frac{\begin{vmatrix} 1 & 11 & -2 \\ 4 & -15 & 1 \\ 3 & 3 & -1 \end{vmatrix}}{-25} = \frac{-25}{-25} = 1$$

$$\chi_3 = \frac{\begin{vmatrix} 1 & 3 & 11 \\ 4 & -2 & -15 \end{vmatrix}}{-25} = \frac{125}{-25} = -5$$

Chapter Two

Function

Numbers:

1 – N = set of natural numbers

N = {1, 2, 3, 4……}

2 – I = set of integers

= {……., -3, -2, -1, 0, 1, 2, 3…}

Note that: NCI

3 – A = set of rational numbers

$$=\left(\chi:\chi=\frac{\rho}{q}\ \rho\ and\ q\ are\ integers\ q\neq 03\right]$$

Ex: $\frac{3}{2},-\frac{4}{5},\frac{3}{1},\frac{-7}{1}$

Note that: ICA

4 – B = set of irrational numbers

= $\{X: X\ is\ not\ a rational\ number\}$

Ex: $\sqrt{2}\,,\ \sqrt{3}\,,\ -\sqrt{7}$

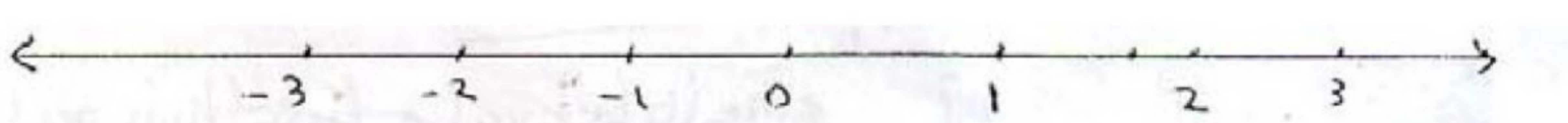
5 – R: set of real numbers

= set of all rational and irrational numbers

Note that

R = AUB

Note: the set of real numbers is represented by a line called a line of numbers:



(ii) NCR, ICR, ACR, BCR

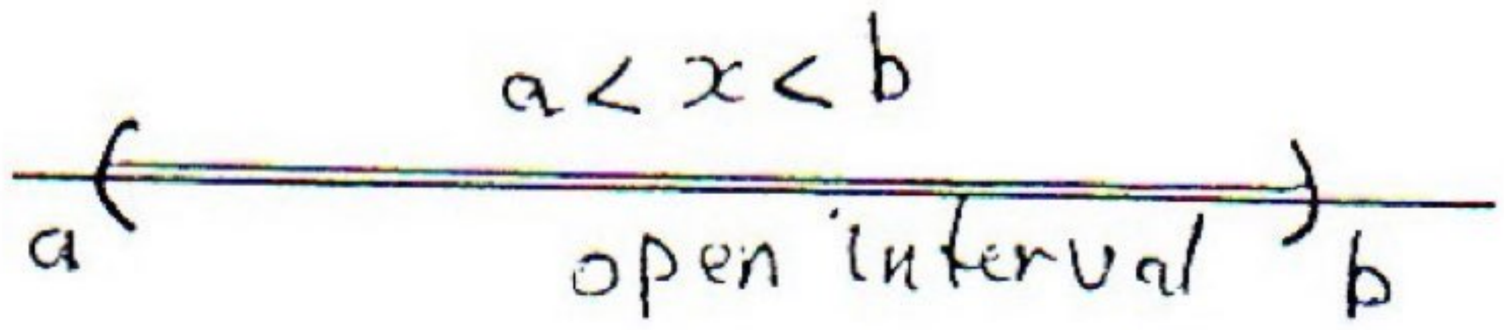
Intervals

The set of values that a variable χ may take on is called the domain of χ. The domains of the variables in many applications of calculus are intervals like those shows below.

- **open intervals**

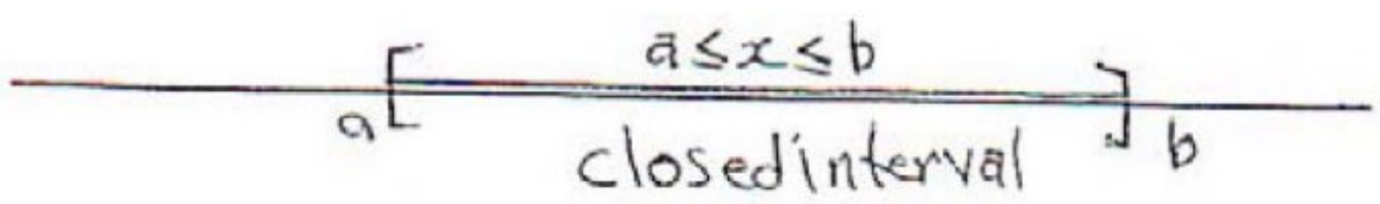
is the set of all real numbers that lie strictly between two fixed numbers a and b:

In symbols	**In words**
$a\langle \chi \langle b\ or\ (q,b)$	The open interval a b



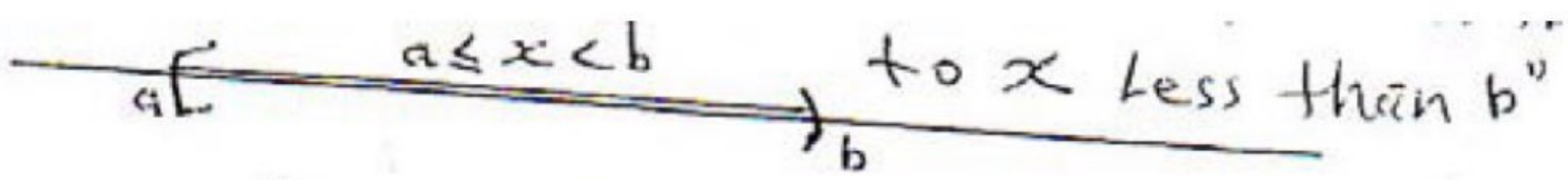
- Closed Intervals contain both endpoints:

In symbols	**In words**
$a \le \chi \le b$ or $[a,b]$	the closed interval a b

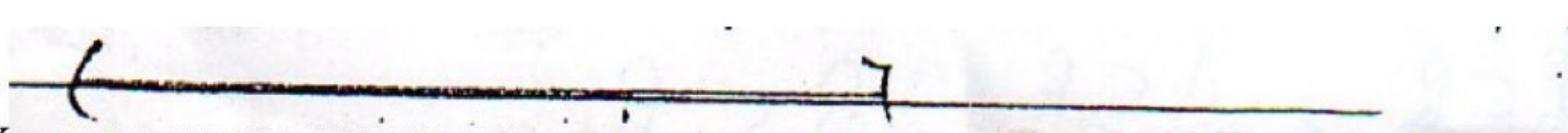


- Half – open intervals contain one but not both end points:

In symbols:	**in wards**
$a \le \chi \langle b$ or [a,b]	' the interval a less than or equal
$a \le \chi c b$	To χ less than b



$a \langle \chi \le b$ or [a,b] the interval a less than χ less than or equal b



Ex. find the domain of

1 – Y = $\sqrt{1-X^2}$

The domain of χ is the closed interval

$1- \le \chi \le 1$

2 – Y = $\dfrac{1}{\sqrt{1-X^2}}$

The domain for χ is open interval

$-1 \langle \chi \langle 1$ because $\dfrac{1}{0}$ is not defined

B - y = $\sqrt{\dfrac{1}{X}-1}$

$1/X - 1 \ge 0$ or $\dfrac{1}{X} \ge 1$

The domain for χ is the half – open $0 \langle \chi \leq 1$

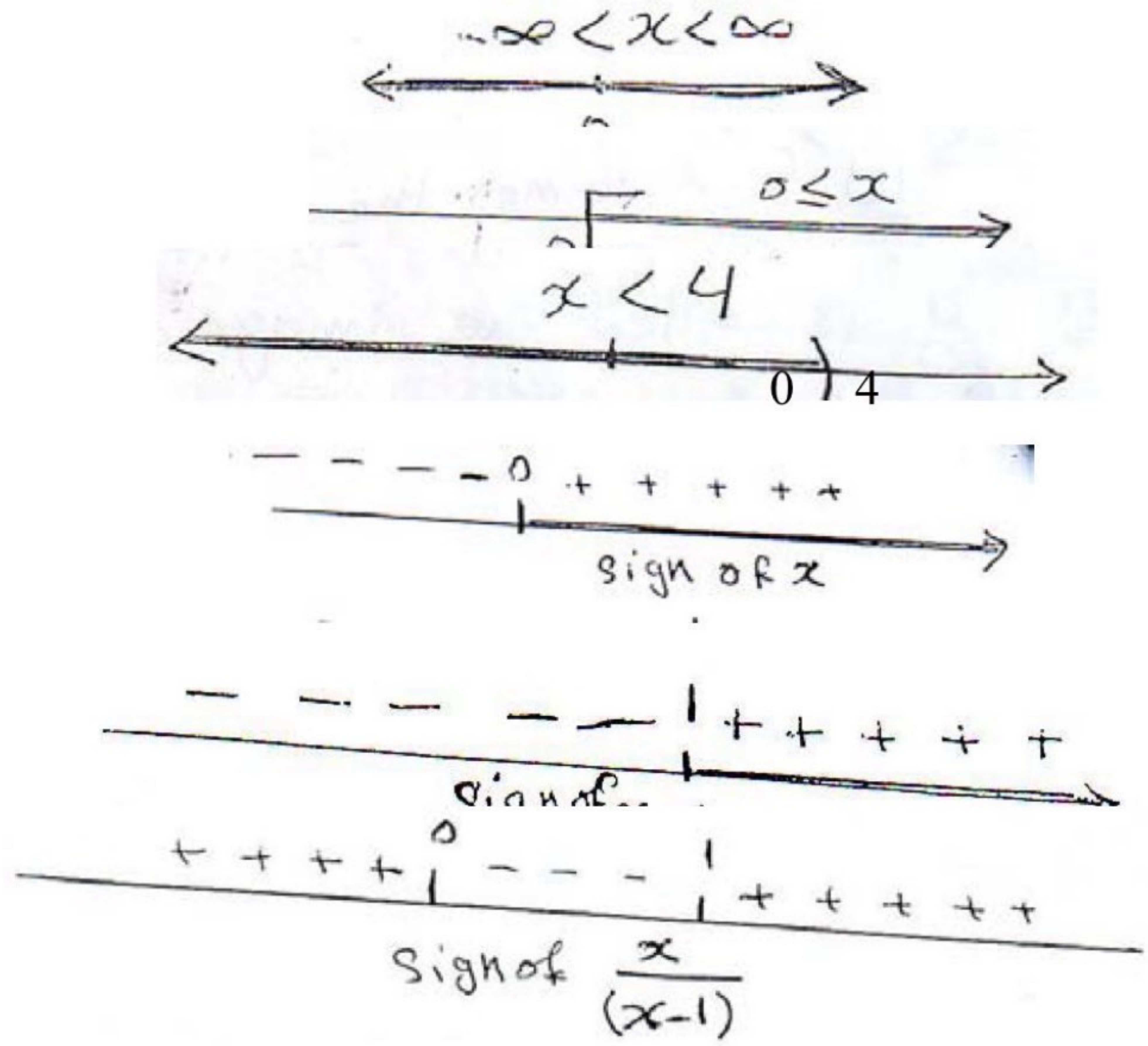
Ex: **the equation** **the domain**

$Y = \chi^2$

$Y = \sqrt{\chi}$

$Y = \dfrac{1}{\sqrt{4-X}}$

$y = \sqrt{\dfrac{\chi}{(\chi - 1)}}$



The domain for χ is $X \leq 0 \cup X \rangle 1$

Definition: A function, say f is a relation between the elements of two sets say A and B such that for every $\chi \in A$ there exists one and only one $Y \in B$ with Y = F(X).

The set A which contain the values of χ is called the domain of function F.

The set B which contains the values of Y corresponding to the values of χ is called the range of the function F. χ is called the independer variable of the function F, while Y is called the dependant variable of F.

Note:

1 – Some times the domain is denoted by DF and the range by RF.

2 – Y is called the image of χ.

Example: Let the domain of χ be the set $\{0,1,2,3,4\}$. Assign to each value of χ the number Y = χ^2. The function so defined is the set of pairs, { (0,0), (1,1), (2,4), (3,9), (4,16) }.

Example: Let the domain of χ be the closed interval $-2 \leq \chi \leq 2$. Assign to each value of χ the number y = χ^2.

The set of order pairs (χ, y) such that $-2 \leq \chi \leq 2$

And y = χ^2 is a function.

Note: Now can describe function by two things:

1 – the domain of the first variable χ.

2 – the rule or condition that the pairs (χ, y) must satistfy to belong to the function.

Example:

The function that pairs with each value of χ diffrent from 2 the number

$$\frac{\chi}{\chi - 2}$$

$$y = f(\chi) = \frac{\chi}{\chi - 2} \quad \chi \neq 2$$

Note 2: Let $f(\chi)$ and $g(\chi)$ be two function.

1 - $(f \pm g)(\chi) = f(\chi) \pm g(\chi)$

2 - $(f.g)(\chi) = f(\chi) \, . \, g(\chi)$

3 - $(\frac{f}{g})(\chi) = \frac{f(\chi)}{g(\chi)}$ if g $(\chi) \neq 0$

Example: Let $f(\chi) = \chi + 2, g(\chi) = \sqrt{\chi - 3}$ evaluate

$f \pm g \, , f.g \; and \; \frac{f}{g}$