of x , then :

$$15)\quad \frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad -1<u<1$$

$$16)\quad \frac{d}{dx}\cos^{-1}u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad -1<u<1$$

$$17)\quad \frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$$

$$18)\quad \frac{d}{dx}\cot^{-1}u = -\frac{1}{1+u^2}\frac{du}{dx}$$

$$19)\quad \frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad |u|>1$$

$$20)\quad \frac{d}{dx}\csc^{-1}u = -\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \qquad |u|>1$$

EX-11- Find $\frac{dy}{dx}$ in each of the following functions :

a) $y=\cot^{-1}\frac{2}{x}+\tan^{-1}\frac{x}{2}$ b) $y=\sin^{-1}\frac{x-1}{x+1}$

c) $y=x.\cos^{-1}2x-\frac{1}{2}\sqrt{1-4x^2}$ d) $y=\sec^{-1}5x$

e) $y=x.\ln(\sec^{-1}x)$ f) $y=3^{\sin^{-1}2x}$

Sol. –

$$a)\quad \frac{dy}{dx} = -\frac{1}{1+\left(\frac{2}{x}\right)^2}\,2.\left(-\frac{1}{x^2}\right)+\frac{1}{1+\left(\frac{x}{2}\right)^2}.\frac{1}{2}=\frac{4}{4+x^2}$$

$$b)\quad \frac{dy}{dx} = \frac{1}{\sqrt{1-\left(\frac{x-1}{x+1}\right)^2}}.\frac{(x+1).1-(x-1).1}{(x+1)^2}=\frac{1}{(x+1)\sqrt{x}}$$

$$c)\quad \frac{dy}{dx} = x\frac{-2}{\sqrt{1-4x^2}}+\cos^{-1}2x-\frac{1}{4}.\frac{-8x}{\sqrt{1-4x^2}}=\cos^{-1}2x$$

$$d)\quad \frac{dy}{dx} = \frac{5}{|5x|\sqrt{25x^2-1}}=\frac{1}{|x|\sqrt{25x^2-1}}$$

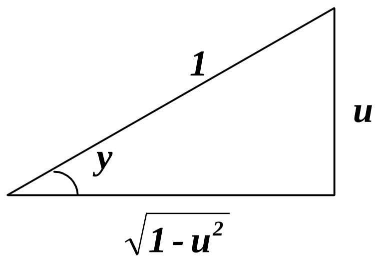
$$e)\quad \frac{dy}{dx}=\frac{x}{sec^{-1}x}\frac{1}{|x|\sqrt{x^2-1}}+ln(sec^{-1}x)=\frac{1}{\sqrt{x^2-1}.sec^{-1}x}+ln(sec^{-1}x)$$

$$f)\quad \frac{dy}{dx}=3^{sin^{-1}2x}.ln3.\frac{2}{\sqrt{1-4x^2}}$$

EX-12- **Prove that :**

$$a)\ \frac{d}{dx}sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad b)\ \frac{d}{dx}tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}$$

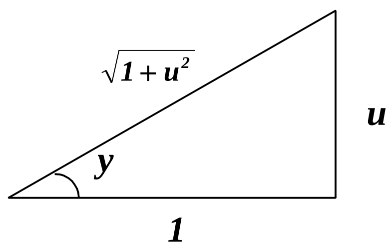
***Proof** : a)*



$$Let\ \ y=sin^{-1}u\Rightarrow u=sin\,y\Rightarrow\frac{du}{dx}=cos\,y.\frac{dy}{dx}=\sqrt{1-u^2}\frac{dy}{dx}$$

$$\Rightarrow\frac{dy}{dx}=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}\Rightarrow\frac{d}{dx}sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

b)



$$Let\ y=tan^{-1}u\Rightarrow u=tany\Rightarrow\frac{du}{dx}=sec^2y.\frac{du}{dx}=\left(\sqrt{1+u^2}\right)^2\frac{dy}{dx}$$

$$\Rightarrow\frac{dy}{dx}=\frac{1}{1+u^2}.\frac{du}{dx}\Rightarrow\frac{d}{dx}tan^{-1}u=\frac{1}{1+u^2}.\frac{du}{dx}$$

Hyperbolic functions **: If *u* is any differentiable function of *x* , then :**

$$21)\quad \frac{d}{dx}\sinh u = \cosh u.\frac{du}{dx}$$

$$22)\quad \frac{d}{dx}\cosh u = \sinh u.\frac{du}{dx}$$

$$23)\quad \frac{d}{dx}\tanh u = \operatorname{sech}^2 u.\frac{du}{dx}$$

$$24)\quad \frac{d}{dx}\coth u = -\operatorname{csch}^2 u.\frac{du}{dx}$$

$$25)\quad \frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$$

$$26)\quad \frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u.\coth u.\frac{du}{dx}$$

EX-13 **- Find $\frac{dy}{dx}$ for the following functions :**

a) $y = \coth(\tan x)$ b) $y = \sin^{-1}(\tanh x)$

c) $y = \ln\left|\tanh\frac{x}{2}\right|$ d) $y = x.\sinh 2x - \frac{1}{2}.\cosh 2x$

e) $y = \operatorname{sech}^3 x$ f) $y = \operatorname{csch}^2 x$

Sol. **-**

a) $\frac{dy}{dx} = -\operatorname{csch}^2(\tan x).\sec^2 x$

b) $\frac{dy}{dx} = \frac{\operatorname{sech}^2 x}{\sqrt{1-\tanh^2 x}} = \frac{\operatorname{sech}^2 x}{\sqrt{\operatorname{sech}^2 x}} = \operatorname{sech} x$

c) $\frac{dy}{dx} = \frac{1}{\tanh\frac{x}{2}}\operatorname{sech}^2\frac{x}{2}.\frac{1}{2} = \frac{\frac{1}{\cosh^2\frac{x}{2}}}{2.\frac{\sinh\frac{x}{2}}{\cosh\frac{x}{2}}}$

$$= \frac{1}{2\sinh\frac{x}{2}.\cosh\frac{x}{2}} = \frac{1}{\sinh x} = \operatorname{csch} x$$

$$d)\ \frac{dy}{dx} = x\cosh 2x.2 + \sinh 2x - \frac{1}{2}\sinh 2x.2 = 2x\cosh 2x$$

$$e)\ \frac{dy}{dx} = 3\,\operatorname{sech}^2 x(-\operatorname{sech} x.\tanh x) = -3\,\operatorname{sech}^3 x.\tanh x$$

$$f)\ \frac{dy}{dx} = 2\,\operatorname{csch} x(-\operatorname{csch} x.\coth x) = -2\,\operatorname{csch}^2 x.\coth x$$

EX-14- **Show that the functions :**

$$x = -\frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} \quad and \quad y = \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \cosh\frac{t}{\sqrt{3}}$$

Taken together , satisfy the differential equations :

$$i)\ \frac{dx}{dt} + 2\frac{dy}{dt} + x = 0 \quad and \quad ii)\ \frac{dx}{dt} - \frac{dy}{dt} + y = 0$$

Proof -

$$x = -\frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} \Rightarrow \frac{dx}{dt} = -\frac{2}{3}\cosh\frac{t}{\sqrt{3}}$$

$$y = \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \cosh\frac{t}{\sqrt{3}} \Rightarrow \frac{dy}{dt} = \frac{1}{3}\cosh\frac{t}{\sqrt{3}} + \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}}$$

$$i)\ \frac{dx}{dt} + 2\frac{dy}{dt} + x = -\frac{2}{3}\cosh\frac{t}{\sqrt{3}} + \frac{2}{3}\cosh\frac{t}{\sqrt{3}} + \frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} - \frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} = 0$$

$$ii)\ \frac{dx}{dt} - \frac{dy}{dt} + y = -\frac{2}{3}\cosh\frac{t}{\sqrt{3}} - \frac{1}{3}\cosh\frac{t}{\sqrt{3}} - \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \cosh\frac{t}{\sqrt{3}} = 0$$

EX-15 - **Prove that :**

$$a)\ \frac{d}{dx}\tanh u = \operatorname{sech}^2 u.\frac{du}{dx} \quad and \quad b)\ \frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$$

Proof-

$$a)\ \frac{d}{dx}\tanh u = \frac{d}{dx}\left(\frac{\sinh u}{\cosh u}\right) = \frac{\cosh u.\cosh u.\frac{du}{dx} - \sinh u.\sinh u.\frac{du}{dx}}{\cosh^2 u}$$

$$= \frac{(\cosh^2 u - \sinh^2 u)\frac{du}{dx}}{\cosh^2 u} = \frac{1}{\cosh^2 u}.\frac{du}{dx} = \operatorname{sech}^2 u.\frac{du}{dx}$$

$$b)\ \frac{d}{dx}\frac{1}{\cosh u} = -\frac{1}{\cosh^2 u}.\sinh u.\frac{du}{dx} = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$$

The inverse hyperbolic functions **: If *u* is any differentiable function of *x* , then :**

27) $$\frac{d}{dx}\sinh^{-1}u=\frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$$

28) $$\frac{d}{dx}\cosh^{-1}u=\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$$

29) $$\frac{d}{dx}\tanh^{-1}u=\frac{1}{1-u^2}\frac{du}{dx} \qquad |u|<1$$

30) $$\frac{d}{dx}\coth^{-1}u=\frac{1}{1-u^2}\frac{du}{dx} \qquad |u|>1$$

31) $$\frac{d}{dx}\operatorname{sech}^{-1}u=-\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$$

32) $$\frac{d}{dx}\operatorname{csch}^{-1}u=-\frac{1}{|u|\sqrt{1+u^2}}\frac{du}{dx}$$

EX-16 **- Find $\frac{dy}{dx}$ for the following functions :**

a) $y=\cosh^{-1}(\sec x)$ b) $y=\tanh^{-1}(\cos x)$

c) $y=\coth^{-1}(\sec x)$ d) $y=\operatorname{sech}^{-1}(\sin 2x)$

Sol.-

a) $$\frac{dy}{dx}=\frac{\sec x.\tan x}{\sqrt{\sec^2 x-1}}=\frac{\sec x.\tan x}{\sqrt{\tan^2 x}}=\sec x \qquad where\ \ \tan x>0$$

b) $$\frac{dy}{dx}=\frac{-\sin x}{1-\cos^2 x}=\frac{-\sin x}{\sin^2 x}=-\csc x$$

c) $$\frac{dy}{dx}=\frac{\sec x.\tan x}{1-\sec^2 x}=\frac{\sec x.\tan x}{-\tan^2 x}=-\csc x$$

d) $$\frac{dy}{dx}=-\frac{2.\cos 2x}{\sin 2x.\sqrt{1-\sin^2 2x}}=-2\csc 2x \qquad where\ \ \cos 2x>0$$

EX-17 **– Verify the following formulas :**

a) $$\frac{d}{dx}\cosh^{-1}u=\frac{1}{\sqrt{u^2-1}}.\frac{du}{dx}$$

b) $$\frac{d}{dx}\tanh^{-1}u=\frac{1}{1-u^2}.\frac{du}{dx} \qquad |u|<1$$

Proof

a) Let $y = cosh^{-1}u \Rightarrow u = cosh\, y$

$$\frac{du}{dx} = sinh\, y.\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{sinh\, y}.\frac{du}{dx}$$

$$cosh^2 y - sinh^2 y = 1 \Rightarrow u^2 - sinh^2 y = 1 \Rightarrow sinh\, y = \sqrt{u^2 - 1}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{u^2 - 1}}.\frac{du}{dx} \Rightarrow \frac{d}{dx}cosh^{-1} u = \frac{1}{\sqrt{u^2 - 1}}.\frac{du}{dx}$$

b) Let $y = tanh^{-1} u \Rightarrow u = tanh\, y$

$$\frac{du}{dx} = sech^2 y.\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{sech^2 y}.\frac{du}{dx}$$

$$sech^2 y + tanh^2 y = 1 \Rightarrow sech^2 y + u^2 = 1 \Rightarrow sech^2 y = 1 - u^2$$

$$\frac{dy}{dx} = \frac{1}{1 - u^2}.\frac{du}{dx} \Rightarrow \frac{d}{dx}tanh^{-1} u = \frac{1}{1 - u^2}.\frac{du}{dx}$$

The derivatives of functions like u^v **:** **Where *u* and *v* are differentiable functions of x , are found by logarithmic differentiation :**

Let $y = u^v \Rightarrow ln\, y = v.ln\, u$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{v}{u}.\frac{du}{dx} + ln\, u.\frac{dv}{dx}$$

$$\frac{dy}{dx} = y\left[\frac{v}{u}.\frac{du}{dx} + ln\, u.\frac{dv}{dx}\right]$$

33) $$\frac{d}{dx}u^v = u^v.\left[\frac{v}{u}.\frac{du}{dx} + ln\, u.\frac{dv}{dx}\right]$$

EX-18- **Find $\frac{dy}{dx}$ for :**

a) $y = x^{cosx}$ b) $y = (lnx + x)^{tanx}$

Sol. -

a) $y = x^{cos\, x} \Rightarrow ln\, y = cos\, x.ln\, x \Rightarrow \frac{1}{y}.\frac{dy}{dx} = \frac{cos\, x}{x} + ln\, x.(-sin\, x)$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{cos\, x}{x} - sin\, x.ln\, x\right]$$

or by formula , where $u = x$ and $v = cosx$

$$\frac{dy}{dx} = y\left[\frac{cos\, x}{x} - sin\, x.ln\, x\right]$$

b) $y = (\ln x + x)^{\tan x} \Rightarrow \ln y = \tan x . \ln(\ln x + x)$

$$\Rightarrow \frac{1}{y} . \frac{dy}{dx} = \frac{\tan x}{\ln x + x} . (\frac{1}{x} + 1) + \ln(\ln x + x) . \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{(x+1) . \tan x}{x(\ln x + x)} + \ln(\ln x + x) . \sec^2 x \right]$$

or by formula, where $u = \ln x + x$ and $v = \tan x$

$$\frac{dy}{dx} = y . \left[\frac{\tan x}{\ln x + x} (\frac{1}{x} + 1) + \ln(\ln x + x) . \sec^2 x \right]$$

$$= y . \left[\frac{(x+1) . \tan x}{x(\ln x + x)} + \ln(\ln x + x) . \sec^2 x \right]$$

Problems -3

1. Find $\frac{dy}{dx}$ for the following functions :

1) $y=(x-3)(1-x)$ $\quad(ans.: 4-2x)$

2) $y=\frac{ax+b}{x}$ $\quad(ans.: -\frac{b}{x^2})$

3) $y=\frac{3x+4}{2x+3}$ $\quad(ans.: \frac{1}{(2x+3)^2})$

4) $y=3x^3-2\sqrt{x}+\frac{5}{x^2}$ $\quad(ans.: 9x^2-\frac{1}{\sqrt{x}}-\frac{10}{x^3})$

5) $y=\left(\sqrt{x^3}-\frac{1}{\sqrt{x^3}}\right)^2$ $\quad(ans.: \frac{3(x^6-1)}{x^4})$

6) $y=(2x-1)^2(3x+2)^3+\frac{1}{(x-2)^2}$ $\quad(ans.: (2x-1)(3x+2)^2(30x-1)-\frac{2}{(x-2)^3})$

7) $y=\ln(\ln x)$ $\quad(ans.: \frac{1}{x.\ln x})$

8) $y=\ln(Cosx)$ $\quad(ans.: -\tan x)$

9) $y=Sinx^3$ $\quad(ans.: 3x^2.Cosx^3)$

10) $y=Cos^{-3}(5x^2+2)$ $\quad(ans.: \frac{30x.Sin(5x^2+4)}{Cos^4(5x^2+4)})$

11) $y=\tan x.\sin x$ $\quad(ans.: Sinx+\tan x.Secx)$

12) $y=\tan(Secx)$ $\quad(ans.: Sec^2(Secx).Secx.\tan x)$

13) $y=Cot^3\left(\frac{x+1}{x-1}\right)$ $\quad(ans.: \frac{6}{(x-1)^2}.Cot^2\left(\frac{x+1}{x-1}\right).Csc^2\left(\frac{x+1}{x-1}\right))$

14) $y=\frac{Cosx}{x}$ $\quad(ans.: -\frac{x.Sinx+Cosx}{x^2})$

15) $y=\sqrt{\tan\sqrt{2x+7}}$ $\quad(ans.: \frac{Sec^2\sqrt{2x+7}}{2\sqrt{2x+7}\sqrt{\tan\sqrt{2x+7}}})$

16) $y=x^2.Sinx$ $\quad(ans.: x^2.Cosx+2x.Sinx)$

17) $y=Csc^{-\frac{2}{3}}\sqrt{5x}$ $\quad(ans.: \frac{5}{3\sqrt{5x}}.\frac{Cot\sqrt{5x}}{Csc^{\frac{2}{3}}\sqrt{5x}})$

18) $y=x[Sin(\ln x)+Cos(\ln x)]$ $\quad(ans.: 2.Cos(\ln x))$

19) $y = Sin^{-1}(5x^2)$ $\quad (ans.: \dfrac{10x}{\sqrt{1-25x^4}})$

20) $y = Cot^{-1}\left(\dfrac{1+x}{1-x}\right)$ $\quad (ans.: -\dfrac{1}{1+x^2})$

21) $y = tan^{-1}\sqrt{4x^3-2}$ $\quad (ans.: \dfrac{6x^2}{(4x^3-1)\sqrt{4x^3-2}})$

22) $y = Sec^{-1}(3x^2+1)^3$ $\quad (ans.: \dfrac{18x}{|3x^2+1|\sqrt{(3x^2+1)^6-1}})$

23) $y = Sin^{-1}\dfrac{x^2}{2-x} + x^2.Sec^{-1}\dfrac{x}{2}$ $\quad (ans.: \dfrac{4x-x^2}{(2-x)\sqrt{(2-x)^2-x^4}} + \dfrac{2x}{\sqrt{x^2-4}} + 2x.Sec^{-1}\dfrac{x}{2})$

24) $y = Sin^{-1}2x.Cos^{-1}2x$ $\quad (ans.: \dfrac{2(Cos^{-1}2x - Sin^{-1}2x)}{\sqrt{1-4x^2}})$

25) $y = \sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$ $\quad (ans.: \dfrac{y}{3}\left[\dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x-2} - \dfrac{2x}{x^2+1} - \dfrac{2}{2x+3}\right])$

26) $y = tan^{-1}(\ln x)$ $\quad (ans.: \dfrac{1}{x(1+(\ln x)^2)})$

27) $y^{\frac{4}{3}} = \dfrac{\sqrt{\sin x.\cos x}}{1+2.\ln x}$ $\quad (ans.: \dfrac{3y}{4}(\dfrac{\cot x}{2} - \dfrac{\tan x}{2} - \dfrac{2}{x(1+2\ln x)}))$

28) $\sqrt{y} = \dfrac{x^5.tan^{-1}x}{(3-2x).\sqrt[3]{x}}$ $\quad (ans.: 2y(\dfrac{14}{3x} + \dfrac{1}{(1+x^2).tan^{-1}x} + \dfrac{2}{3-2x}))$

29) $y = sec^{-1}e^{2x}$ $\quad (ans.: \dfrac{2}{\sqrt{e^{4x}-1}})$

30) $y = (\cos x)^{\sqrt{x}}$ $\quad (ans.: \dfrac{y}{2\sqrt{x}}(\ln\cos x - 2x.\tan x))$

31) $y = (\sin x)^{\tan x}$ $\quad (ans.: y(1+sec^2 x.\ln\sin x))$

32) $y = \sqrt{2x^2+cosh^2(5x)}$ $\quad (ans.: \dfrac{2x+5cosh(5x).sinh(5x)}{\sqrt{2x^2+cosh^2(5x)}})$

33) $y = sinh(\cos 2x)$ $\quad (ans.: -2\sin 2x.cosh(\cos 2x))$

34) $y = csch\dfrac{1}{x}$ $\quad (ans.: \dfrac{1}{x^2}.csch\dfrac{1}{x}.coth\dfrac{1}{x})$

35) $y = x^{2.}.tanh^2\sqrt{x}$ $\quad (ans.: x.tanh\sqrt{x}(\sqrt{x}\,sech^2\sqrt{x} + 2tanh\sqrt{x}))$

36) $y = \ln \dfrac{\sin x.\cos x + \tan^3 x}{\sqrt{x}}$ $\quad$ $(ans.: \dfrac{\cos^2 x - \sin^2 x + 3\tan^2 x.\sec^2 x}{\sin x.\cos x + \tan^3 x} - \dfrac{1}{2x})$

37) $y = \log_4 \sin x$ $\quad$ $(ans.: \dfrac{\cot x}{\ln 4})$

38) $y = e^{(x^2 - e^{5x})}$ $\quad$ $(ans.: (2x - 5e^{5x})e^{(x^2 - e^{5x})})$

39) $y = e^{x^2 \tan x}$ $\quad$ $(ans.: (x^2 \sec^2 x + 2x \tan x)e^{x^2 \tan x})$

40) $y = 7^{\csc\sqrt{2x+3}}$ $\quad$ $(ans.: \dfrac{-7^{\csc\sqrt{2x+3}} \ln 7}{\sqrt{2x+3}} \csc\sqrt{2x+3}.\cot\sqrt{2x+3})$

41) $y = \left[\ln(x^2 + 2)^2\right] \cos x$ $\quad$ $(ans.: \dfrac{4x.\cos x}{x^2 + 2} - 2\ln(x^2 + 2)\sin x)$

42) $y = \sinh^{-1}(\tan x)$ $\quad$ $(ans.: |\sec x|)$

43) $y = \sqrt{1 + (\ln x)^2}$ $\quad$ $(ans.: \dfrac{\ln x}{x\sqrt{1 + (\ln x)^2}})$

44) $y = \dfrac{e^x}{\ln x}$ $\quad$ $(ans.: \dfrac{e^x(x \ln x - 1)}{x(\ln x)^2})$

45) $y = x^3 \log_2(3 - 2x)$ $\quad$ $(ans.: 3x^2 \log_2(3 - 2x) - \dfrac{2x^3}{(3 - 2x)\ln 2})$

46) $y = 2\cosh^{-1} \dfrac{x}{2} + \dfrac{x}{2}\sqrt{x^2 - 4}$ $\quad$ $(ans.: \dfrac{x^2}{\sqrt{x^2 - 4}})$

2. Verify the following derivatives :

a) $\dfrac{d}{dx}\left[5x + (\sqrt{x} + \dfrac{1}{\sqrt{x}})^2\right] = 6 - \dfrac{1}{x^2}$

b) $\dfrac{d}{dx}\left[\sqrt{x}(ax^2 + bx + c)\right] = \dfrac{1}{2\sqrt{x}}(5ax^2 + 3bx + c)$

3. Find the derivative of y with respect to x in the following functions :

a) $y = \dfrac{u^2}{u^2 + 1}$ and $u = 3x^3 - 2$ $\quad$ $(ans.: \dfrac{18x^2 y^2}{(3x^3 - 2)^3})$

b) $y = \sqrt{u} + 2u$ and $u = x^2 - 3$ $\quad$ $(ans.: \dfrac{x}{\sqrt{x^2 - 3}} + 4x)$

4. Find the second derivative for the following functions :

a) $y = (x + \frac{1}{x})^3$ $\qquad$ $(ans.: 6x + \frac{6}{x^3} + \frac{12}{x^5})$

b) $f(x) = \sqrt{2x} + \frac{2\sqrt{2}}{\sqrt{x}}$ at $x = 2$ $\qquad$ $(ans.: \frac{1}{4})$

c) $x^2 - 2xy + y^2 - 16x = 0$ $\qquad$ $(ans.: \mp x^{-\frac{3}{2}})$

5. Find the third derivative of the function :

$y = \sqrt{x^3}$ $\qquad$ $(ans.: -\frac{3}{8y})$

6. Show for $y = \frac{u}{v}$ **that** $y'' = \frac{v(vu'' - uv'') - 2v'(vu' - uv')}{v^3}$.

7. Show for $y = u.v$ **that** $y''' = uv''' + 3u'v'' + 3u''v' + u'''v$.

8. Show that $y = 35x^4 - 30x^2 + 3$ **satisfies** $(1 - x^2)y'' - 2xy' + 20y = 0$.

9. Find $\frac{dy}{dx}$ **for the following implicit functions :**

a) $x^3 + 4x\sqrt{y} - \frac{5y^2}{x} = 3$ $\quad (ans.: \frac{3x^2 + 5y^2x^{-2} + 4\sqrt{y}}{10x^{-1}y - \frac{2x}{\sqrt{y}}})$

b) $\sqrt{xy} + 1 = y$ $\quad (ans.: \frac{y}{2\sqrt{xy} - x})$

c) $3xy = (x^3 + y^3)^{\frac{3}{2}}$ $\quad (ans.: \frac{3x^2\sqrt{x^3 + y^3} - 2y}{2x - 3y^2\sqrt{x^3 + y^3}})$

d) $x^3 + x.\tan^{-1} y = y$ $\quad (ans.: \frac{(1 + y^2)(3x^2 + \tan^{-1} y)}{1 + y^2 - x})$

e) $\sin^{-1}(xy) = \cos^{-1}(x - y)$ $\quad (ans.: \frac{y\sqrt{1-(x-y)^2} + \sqrt{1-(xy)^2}}{\sqrt{1-(xy)^2} - x\sqrt{1-(x-y)^2}})$

f) $y^2.\sin(xy) = \tan x$ $\quad (ans.: \frac{\sec^2 x - y^3.\cos(xy)}{2y.\sin(xy) + xy^2.\cos(xy)})$

g) $\sinh y = \tan^2 x$ $\quad (ans.: \frac{2.\tan x.\sec^2 x}{\cosh y})$

10. Prove the following formulas :

a) $\frac{d}{dx}\cot u = -\csc^2 u.\frac{du}{dx}$

b) $\frac{d}{dx}\csc u = -\csc u.\cot u.\frac{du}{dx}$

c) $\frac{d}{dx}\cos^{-1} u = -\frac{1}{\sqrt{1 - u^2}}.\frac{du}{dx}$

d) $\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2 - 1}}.\frac{du}{dx}$

e) $\frac{d}{dx}\sinh u = \cosh u.\frac{du}{dx}$

f) $\frac{d}{dx}\csc h\ u = -\csc h\ u.\coth u.\frac{du}{dx}$

g) $\frac{d}{dx}\sinh^{-1} u = \frac{1}{\sqrt{1 + u^2}}.\frac{du}{dx}$

h) $\frac{d}{dx}\sec h^{-1} u = -\frac{1}{|u|\sqrt{1 - u^2}}.\frac{du}{dx}$

11. Show that the tangent to the hyperbola $x^2 - y^2 = 1$ **at the point** ***P(coshu, sinhu)*** **, cuts the x-axis at the point** ***(sechu , 0)*** **and except when vertical , cuts the y-axis at the point** ***(0 , -cschu)*** **.**