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Department of Biology

Bio Physics

First lecture

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First Stage

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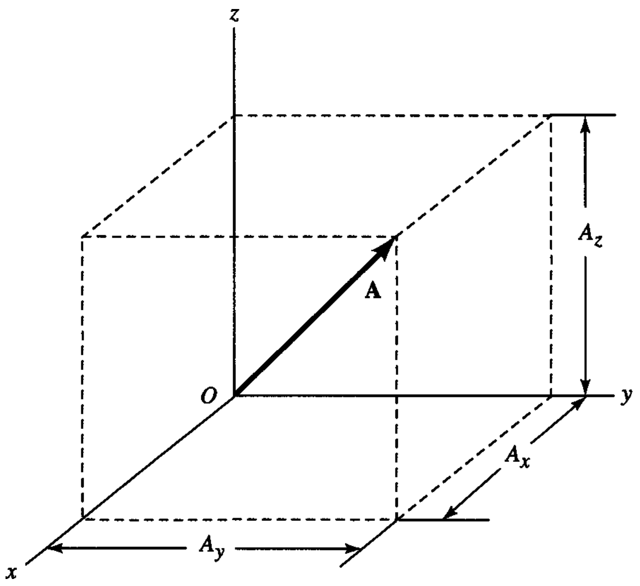
1.1 Physical Quantities

The motion of dynamical systems is typically described in terms of two basic quantities: scalars and vectors.

1. **A Scalar Quantities** is a physical quantity that has magnitude only, such as the mass of an object.

2. **Vector Quantitie** has both magnitude and direction, such as the displacement, velocity, acceleration, and force.

The scalar quantity is represented by the symbol (A), we denote vector quantities simply by ($\vec{A}$), a given vector ($\vec{A}$), is specified by stating its magnitude and its direction relative to some arbitrarily chosen coordinate system. It is represented diagrammatically as a directed line segment, as shown in three-dimensional space in Figure 1.



The Vector $\vec{A} = iA_x + jA_y + kA_z$ means that there vector (A) is expressed on the right in terms of its components in a particular Coordinate system.

1.2 The Definitions and Rules

1. Unit Vector

The unit vector ($\hat{u}_{\vec{A}}$) in the direction of the vector ($\vec{A}$) is defined as follows:

$$\hat{u}_{\vec{A}} = \frac{\vec{A}}{|\vec{A}|} \quad ....(1-1)$$

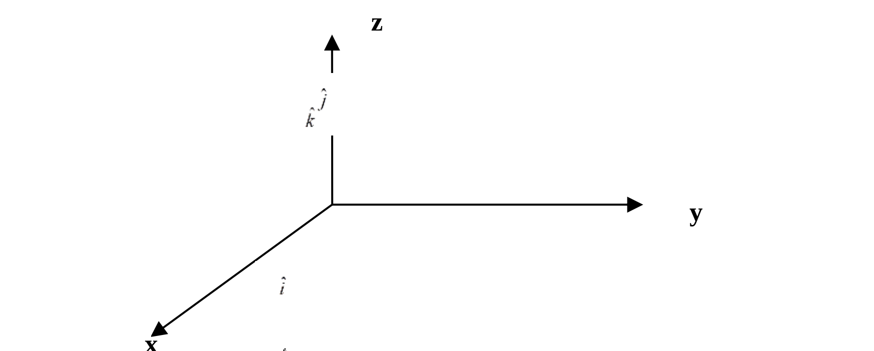
$\hat{u}_{\vec{A}}$:The unit vector is in the direction of vector **(** $\vec{A}$ **)**

$\vec{A}$ **:** The vector

$|\vec{A}|$ **:** The magnitude of the vector

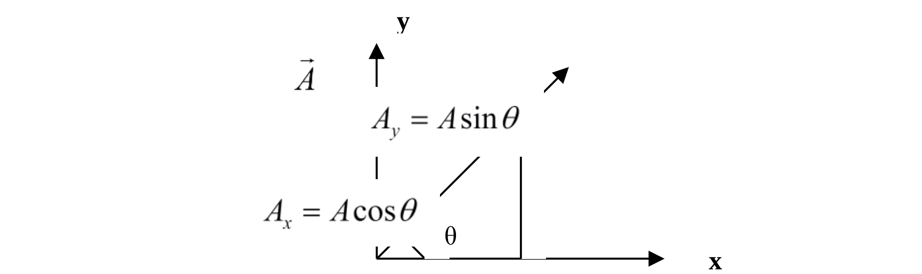
2. (Basic Unit Vectors) $\hat{i}, \hat{j}, \hat{k}$

They are vectors of unit magnitude and work in the positive directions of the axes (x, y, z), respectively, as in Figure (1-1). Therefore, these three vectors are perpendicular.



شكل (1-1) : متجهات الوحدة الأساسية

To find the magnitude of the vector in the case of a two-dimensional vector, as in Figure (1-2)



شكل (2-1) : متجه ذات بعدين

From Figure (2-1) it is clear to us that:

$$\left|\vec{A}\right| = \vec{A} = \sqrt{A_x{}^2 + A_y{}^2} \ldots(2-1)$$

$A_x = A\cos\theta$

$A_y = A\sin\theta$

θ= It is the angle that the resultant makes with the positive (x) axis, and is calculated from the following equation:

$$\theta = \tan^{-1}\frac{A_y}{A_x} \ldots(3-1)$$

Now this can be generalized to a vector in space (with three dimensions) as follows:

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2} \ldots(4-1)$$

Example: If $\vec{A} = 3\hat{i} + 4\hat{j}$

1- Calculate the magnitude of the vector ($\vec{A}$)?

2- What is the unit vector in the directio ($\vec{A}$)

Solution

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2} \ldots(2-1)$$

$$A_x = 3$$

$$A_y = 4$$

$$\left|\vec{A}\right| = \sqrt{(3)^2 + (4)^2}$$

$$\left|\vec{A}\right| = \sqrt{9 + 16}$$

$\boxed{\text{units } \left|\vec{A}\right| = 5}$ ($\vec{A}$) Vector magnitude

$$\hat{u}_{\vec{A}} = \frac{\vec{A}}{|\vec{A}|} \text{....}(1-1)$$

$$\hat{u}_{\vec{A}} = \frac{1}{5}(3\hat{i} + 4\hat{j})$$

$$\hat{u}_{\vec{A}} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

$\boxed{\hat{u}_{\vec{A}} = 0.6\hat{i} + 0.8\hat{j}}$ ($\vec{A}$) Unit vector in one direction

3. Addition and Subtraction of Vectors

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} \text{...}(5-1a)}$$

with three dimensions, the vector ($\vec{A}$) can be written in the following form:

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \text{...}(5-1b)}$$

For vector ($\vec{B}$)

$$\boxed{\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k} \text{...}(6-1)}$$

Therefore, from equations (1-5 b) and (1-6), the equation for summing the vectors ($\vec{A}$) and ($\vec{B}$) can be written as follows:

$$\boxed{\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k} \text{...}(7-1)}$$

The equation for subtracting the vectors ($\vec{A}$) and ($\vec{B}$) is as follows:

$$\boxed{\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j} + (A_z - B_z)\hat{k} \text{...}(8-1)}$$

4. **Equality of Vectors**

$$\vec{A} = \vec{B}$$

$$A_x = B_x$$

$$A_y = B_y$$

$$A_z = B_z$$

5.The null vector

The vector 0 = (0, 0, 0) is called the null vector. The direction of the null vector is undefined. From (4) it follows that A - A =0. Because there can be no confusion when the null vector is denoted by a zero, we shall hereafter use the notation 0=0.

6. Multiplication of Vectors

$$\vec{A}.\vec{B} = |A||B|\cos\theta ...(9-1)$$

$|A|$: Vector magnitude. $\vec{A}$

$|B|$: Vector magnitude. $\vec{B}$

θ: The smallest angle between the two vectors $\vec{A}$ and $\vec{B}$ their extension is

$$\theta = \cos^{-1}\frac{\vec{A}.\vec{B}}{|A||B|} ...(10-1)$$

$$\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\vec{A}.\vec{B} = A_xB_x\hat{i}.\hat{i} + A_xB_y\hat{i}.\hat{j} + A_xB_z\hat{i}.\hat{k} + A_yB_x\hat{j}.\hat{i} + A_yB_y\hat{j}.\hat{j} + A_yB_z\hat{j}.\hat{k} + A_zB_x\hat{k}.\hat{i} + A_zB_y\hat{k}.\hat{j} + A_zB_z\hat{k}.\hat{k} ...(11-1)$$

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$

$$\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = \hat{k}.\hat{j} = 0$$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z ...(12-1)$$

Example/ $\vec{A} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{B} = 3\hat{i} - 4\hat{k}$

Calculate:

1- The angle between the two vectors $\vec{A}$ and $\vec{B}$?

2- The angle between the vector $\vec{A}$ and the positive (x) axis?

Solution/

$$\theta = \cos^{-1}\frac{\vec{A}.\vec{B}}{|A||B|} ...(10-1)$$

$$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z \ldots (12-1)$$

$$\vec{A}.\vec{B} = (1)(3) + (2)(0) + (-2)(-4)$$

$$\boxed{\vec{A}.\vec{B} = 11}$$

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$$

$$\left|\vec{A}\right| = \sqrt{(1)^2 + (2)^2 + (-2)^2}$$

$$\boxed{\left|\vec{A}\right| = 3units}$$

$$\left|\vec{B}\right| = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2}$$

$$\left|\vec{B}\right| = \sqrt{(3)^2 + (0)^2 + (-4)^2}$$

$$\boxed{\left|\vec{B}\right| = 5units}$$

$$\boxed{\therefore \left|\vec{A}\right|\left|\vec{B}\right| = 15units}$$

$$\theta = \cos^{-1} \frac{11}{15}$$

$$\boxed{\theta = 42.8^\circ}$$

2 -

$$\vec{A}.\hat{i} = |A|.\left|\hat{i}\right| \cos\theta$$

$$\theta = \cos^{-1} \frac{\vec{A}.\hat{i}}{|A|.\left|\hat{i}\right|}$$

$$\vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z \ldots (12-1)$$

$$\vec{A}.i = (1)(1) + (2)(0) + (-2)(0)$$

$$\boxed{\vec{A}.\hat{i} = 1}$$

$$\boxed{\left|\vec{A}\right| = 3}$$

$$\theta = \cos^{-1} \frac{1}{3} \qquad \boxed{\theta = 70.5^\circ}$$

7. Vector product

$$\boxed{\vec{A}x\vec{B} = |A||B|\sin\theta ...(13-1)}$$

$|A|$: the magnitude of the vector $\vec{A}$

$|B|$: the magnitude of the vector $\vec{B}$

θ: The smallest angle between two vectors $\vec{A}$ and $\vec{B}$ their extension

$$\vec{A}x\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})x(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$\boxed{\vec{A}x\vec{B} = A_xB_x\hat{i}x\hat{i} + A_xB_y\hat{i}x\hat{j} + A_xB_z\hat{i}x\hat{k} + A_yB_x\hat{j}x\hat{i} + A_yB_y\hat{j}x\hat{j} + A_yB_z\hat{j}x\hat{k} + A_zB_x\hat{k}x\hat{i} + A_zB_y\hat{k}x\hat{j} + A_zB_z\hat{k}x\hat{k} ...(14-1)}$$

$$\hat{i}x\hat{i} = \hat{j}x\hat{j} = \hat{k}x\hat{k} = 0$$

while

$$\hat{k}x\hat{i} = \hat{j} \quad \hat{j}x\hat{k} = \hat{i} \quad \hat{i}x\hat{j} = \hat{k}$$

and

$$\hat{j}x\hat{i} = -\hat{k} \quad \hat{k}x\hat{j} = -\hat{i} \quad \hat{i}x\hat{k} = -\hat{j}$$

$$\boxed{\vec{A}x\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k} ...(15-1)}$$

Note: It becomes clear to us that in the case of cross multiplication:

$$\boxed{\vec{A}x\vec{B} = -\vec{B}x\vec{A}}$$

Example

$$\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{B} = \hat{i} - 2\hat{j} + 2\hat{k}$$

Calculate all of the following:

1 $2\vec{A} - 3\vec{B}$ ؟

$$2\vec{A} - 3\vec{B} = 2(2\hat{i} + 3\hat{j} + \hat{k}) - 3(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$2\vec{A} - 3\vec{B} = 4\hat{i} + 6\hat{j} + 2\hat{k} - 3\hat{i} + 6\hat{j} - 6\hat{k}$$

$$\boxed{2\vec{A} - 3\vec{B} = \hat{i} + 12\hat{j} - 4\hat{k}}$$

2 $\vec{A}$ and $\vec{B}$ ؟

$$\left|\vec{A}\right| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$$

$$\left|\vec{A}\right| = \sqrt{(2)^2 + (3)^2 + (1)^2}$$

$$\boxed{\therefore \left|\vec{A}\right| = \sqrt{14} units}$$

$$\left|\vec{B}\right| = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2}$$

$$\left|\vec{B}\right| = \sqrt{(1)^2 + (-2)^2 + (2)^2}$$

$$\boxed{\therefore \left|\vec{B}\right| = 3 units}$$

3 The angle between the two vectors $\vec{A}$ و $\vec{B}$ ؟

$$\theta = \cos^{-1} \frac{\vec{A}.\vec{B}}{|A||B|} ...(10-1)$$

$$\theta = \cos^{-1} \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$$

$$\theta = \cos^{-1} \frac{(2)(1) + (3)(-2) + (1)(2)}{3\sqrt{14}}$$

$$\theta = \cos^{-1} \frac{-2}{3\sqrt{14}}$$

$$\boxed{\therefore \theta = 100.3^\circ}$$

4 $\vec{A}x\vec{B}$ ؟

$$\vec{A}x\vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k} ...(15-1)$$

$$\vec{A}x\vec{B} = \left[(3)(2) - (1)(-2)\right]\hat{i} + \left[(1)(1) - (2)(2)\right]\hat{j} + \left[(2)(-2) - (3)(1)\right]\hat{k}$$

$$\boxed{\vec{A}x\vec{B} = 8\hat{i} - 3\hat{j} - 7\hat{k}}$$

5 Unit vector in direction $\vec{A}x\vec{B}$ ؟

$$\hat{u}_{\vec{A}x\vec{B}} = \frac{\vec{A}x\vec{B}}{\left|\vec{A}x\vec{B}\right|}$$

$$\hat{u}_{\vec{A}x\vec{B}} = \frac{8\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{(8)^2 + (-3)^2 + (-7)^2}}$$

$$\boxed{\hat{u}_{\vec{A}x\vec{B}} = \frac{8\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{122}}}$$

6 Unit vector in direction $\vec{B}x\vec{A}$ ؟

$$\vec{A}x\vec{B} = -\vec{B}x\vec{A}$$

$$\hat{u}_{\vec{A}x\vec{B}} = -\hat{u}_{\vec{B}x\vec{A}}$$

$$\boxed{\hat{u}_{\vec{B}x\vec{A}} = -\frac{8\hat{i} - 3\hat{j} - 7\hat{k}}{\sqrt{122}}}$$

(3)

Ex3:- Find $\vec{A}+\vec{B}$ if $\vec{A} = 2\hat{i}+2\hat{j}$
$\vec{B} = 2\hat{i}-4\hat{j}$
and then find the magnetud of $\vec{A}+\vec{B}$

Sol//

$$\vec{A}+\vec{B} = (2\hat{i}+2\hat{j}) + (2\hat{i}-4\hat{j})$$
$$= (2+2)\hat{i} + (2-4)\hat{j}$$
$$= \underset{A_x+B_x}{\underline{4}}\hat{i} - \underset{A_y+B_y}{\underline{2}}\hat{j}$$

لايجاد قيمة $\vec{A}+\vec{B}$

$$\vec{A}+\vec{B} = |\vec{A}+\vec{B}| = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2}$$
$$= \sqrt{(4)^2+(-2)^2} = \sqrt{16+4} = \sqrt{20}$$
$$= 4.5$$

Ex4:- If $\vec{A} = 2\hat{i}+3\hat{j}$ and $\vec{B} = -\hat{i}+2\hat{j}$
Find ① $\vec{A}\cdot\vec{B}$ ② angle θ between $\vec{A}$ and $\vec{B}$

Sol// ① $\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$

القانون الاول للضرب بعدم وجود زاوية

$$\vec{A}\cdot\vec{B} = (2\hat{i}+3\hat{j})\cdot(-\hat{i}+2\hat{j})$$

$\boxed{\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 \quad \hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0}$

$$= (2\hat{i}\cdot-\hat{i}) + (3\hat{j}\cdot2\hat{j})$$
$$= -2+6 = 4$$

$$\boxed{\vec{A}\cdot\vec{B} = 4}$$

④ لايجاد الزاوية بين المتجهين نستخدم قانون الضرب العددي الثاني ② EX4

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

قانون الضرب العددي بوجود زاوية

توجيد قيمة المتجه $\vec{A}$

$$\vec{A} = \underset{A_x}{2}\hat{i} + \underset{A_y}{3}\hat{j}$$

$$\vec{A} = |\vec{A}| = A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}$$

توجيد قيمة المتجه $\vec{B}$

$$\vec{B} = \underset{B_x}{-}\hat{i} + \underset{B_y}{2}\hat{j}$$

$$\vec{B} = |\vec{B}| = B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}$$

$$\vec{A}\cdot\vec{B} = 4$$

من المطلب الاول

$$\because \vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} \Rightarrow \cos\theta = \frac{4}{\sqrt{13}\sqrt{5}}$$

$$\cos\theta = \frac{4}{8.06} \Rightarrow \cos\theta = 0.49$$

$$\theta = \cos^{-1} 0.49$$

$$\therefore \theta = 60.6$$

(5)

Ex5:- If $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = -\hat{i} + 2\hat{j}$

Find $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} \\ A_x & A_y \\ B_x & B_y \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} \\ 2 & 3 \\ -1 & 2 \end{vmatrix}$$

$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$$\vec{A} \times \vec{B} = \hat{k}(A_x B_y - A_y B_x)$$

$$= \hat{k}(2 \times 2 - (3 \times -1)) = \hat{k}(4 + 3)$$

$$= 7\hat{k}$$

$$-\vec{B} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} \\ -1\ (B_x) & 2\ (B_y) \\ 2\ (A_x) & 3\ (A_y) \end{vmatrix} = \hat{k}(B_x A_y - B_y A_x)$$

$$= \hat{k}((-1 \times 3) - 2 \times 2)$$

$$= k(-3 - 4$$

$$= -7\hat{k}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$7\hat{k} = -7\hat{k}$$