Measures of Variation

Variance & Standard Deviation

- The **population variance** is the average of the squares of the distance each value is from the mean.
- The **standard deviation** is the square root of the variance.

Uses of the Variance and Standard Deviation

- To determine the spread of the data.
- To determine the consistency of a variable.
- To determine the number of data values that fall within a specified interval in a distribution.
- Used in inferential statistics.
- The **population variance** is

$$\sigma^2 = \frac{\sum (X - \mu)^2}{N}$$

- The **population standard deviation** is

$$\sigma = \sqrt{\frac{\sum (X - \mu)^2}{N}}$$

Step 1 Find the mean for the data. $\mu = \frac{\Sigma X}{N}$

Step 2 Find the Deviation for each data value. $X - \mu$

Step 3 Square each of the deviations. $(X - \mu)^2$

Step 4 Find the sum of the squares. $\Sigma(X - \mu)^2$

Outdoor Paint

Find the variance and standard deviation for the data set for Brand A paint. 10, 60, 50, 30, 40, 20

Months, X	μ	$X-\mu$	$(X-\mu)^2$
10	35	–25	625
60	35	25	625
50	35	15	225
30	35	–5	25
40	35	5	25
20	35	–15	225
			1750

$$\sigma^2 = \frac{\sum(X-\mu)^2}{n} = \frac{1750}{6} = \boxed{291.7}$$

$$\sigma = \sqrt{\frac{1750}{6}} = \boxed{17.1}$$

- The **sample variance** is

$$s^2 = \frac{\sum(X-\bar{X})^2}{n-1}$$

- The **sample standard deviation** is

$$s = \sqrt{\frac{\sum(X-\bar{X})^2}{n-1}}$$

Coefficient of Variation

The **coefficient of variation** is the standard deviation divided by the mean, expressed as a percentage.

$$CVAR = \frac{s}{\bar{X}} \cdot 100\%$$

Use *CVAR* to compare standard deviations when the units are different.

Example: Sales of Automobiles

The mean of the number of sales of cars over a 3-month period is 87, and the standard deviation is 5. The mean of the commissions is $5225, and the standard deviation is $773. Compare the variations of the two.

$$CVar = \frac{5}{87} \cdot 100\% = 5.7\% \qquad \text{Sales}$$

$$CVar = \frac{773}{5225} \cdot 100\% = 14.8\% \qquad \text{Commissions}$$

Commissions are more variable than sales.

CHAPTER FOUR

COUNTING TECHNIQUES

➢ Fundamental Counting Rule

The **fundamental counting rule** is also called the **multiplication of choices**.

In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 and the third has k_3, and so forth, the total number of possibilities of the sequence will be

$$k_1 \cdot k_2 \cdot k_3 \cdots k_n$$

Event		Event		Event
K1	.	K2	. . .	Kn

A paint manufacturer wishes to manufacture several different paints. The categories include

Color: red, blue, white, black, green, brown yellow

Type: latex, oil

Texture: flat, semi-gloss, high gloss

Use: outdoor, indoor

How many different kinds of paint can be made if you can select one color, one type, one texture, and one use?

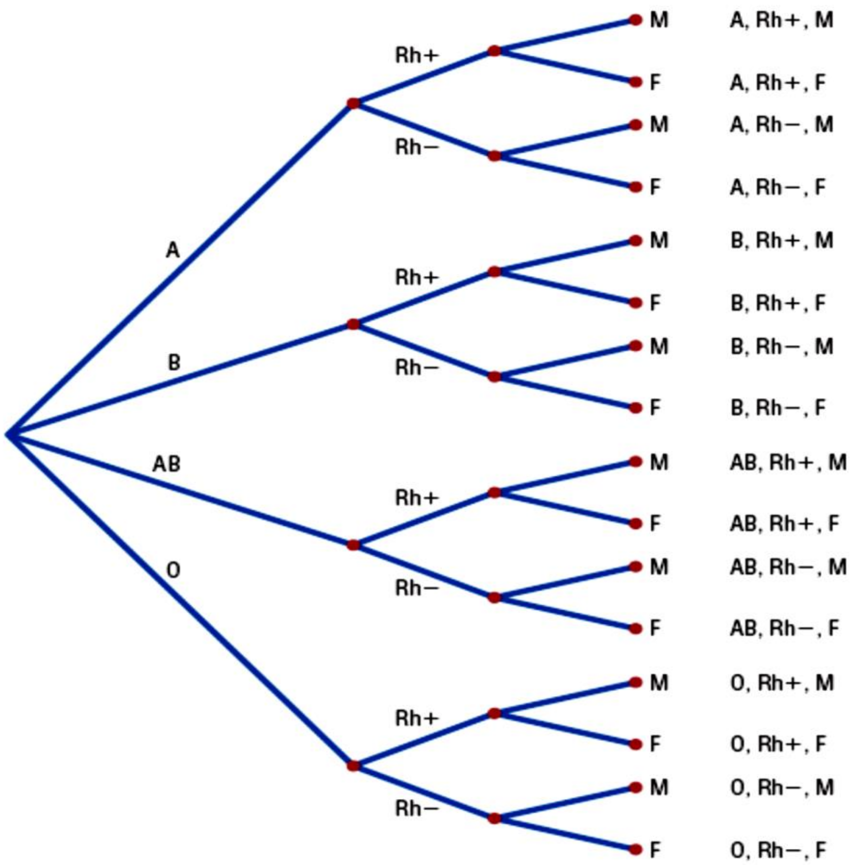
$$\begin{pmatrix}\text{\# of}\\ \text{colors}\end{pmatrix}\begin{pmatrix}\text{\# of}\\ \text{types}\end{pmatrix}\begin{pmatrix}\text{\# of}\\ \text{textures}\end{pmatrix}\begin{pmatrix}\text{\# of}\\ \text{uses}\end{pmatrix}$$

$$7 \cdot 2 \cdot 3 \cdot 2$$

84 different kinds of paint

Example: Distribution of Blood Types

There are four blood types, A, B, AB, and O. Blood can also be Rh+ and Rh-. Finally, a blood donor can be classified as either male or female. How many different ways can a donor have his or her blood labeled?



Solution

Since there are 4 possibilities for blood type, 2 possibilities for Rh factor, and 2 possibilities for the gender of the donor, there are 4 _ 2 _ 2, or 16, different classification categories, as shown.

Blood type	Rh	Gender	
4 •	2 •	2	= 16

Discuss how many ways 5-Digit password can be done?

- **Factorial** is the product of all the positive numbers from 1 to a number.

$$n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$$
$$0! = 1$$

- **Permutation** is an arrangement of objects in a specific order. Order matters. $${}_nP_r = \frac{n!}{(n-r)!} = \underbrace{n(n-1)(n-2)\cdots(n-r+1)}_{r \text{ items}}$$

- **Combination** is a grouping of objects. Order does not matter.

$${}_nC_r = \frac{n!}{(n-r)!r!} = \frac{{}_nP_r}{r!}$$

Factorial Example: Suppose a business owner has a choice of 5 locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the 5 locations?

Solution

There are

$$\begin{pmatrix}\text{first}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{second}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{third}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{fourth}\\\text{choice}\end{pmatrix}\begin{pmatrix}\text{fifth}\\\text{choice}\end{pmatrix}$$

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

5! = 5. 4. 3. 2. 1 = 120

120 different ways to rank the locations

different possible rankings. The reason is that she has 5 choices for the first location, 4 choices for the second location, 3 choices for the third location, etc.

Permutation Example: The advertising director for a television show has 7 ads to use on the program. If she selects 1 of them for the opening of the show, 1 for the middle of the show, and 1 for the ending of the show, how many possible ways can this be accomplished?

Solution

Since order is important, the solution is

$$_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$$

Hence, there would be 210 ways to show 3 ads.

Permutation Example: A school musical director can select 2 musical plays to present next year. One will be presented in the fall, and one will be presented in the spring. If she has 9 to pick from, how many different possibilities are there?

Solution

Order is important since one play can be presented in the fall and the other play in the spring.

$$_9P_2 = \frac{9!}{(9-2)!} = \frac{9!}{7!} = \frac{9 \cdot 8 \cdot 7!}{7!} = 72$$

There are 72 different possibilities.

Combination Example: Given the letters A, B, C, and D, list the permutations and combinations for selecting two letters.

Solution

The permutations are

AB	BA	CA	DA
AC	BC	CB	DB
AD	BD	CD	DC

In permutations, AB is different from BA. But in combinations, AB is the same as BA since the order of the objects does not matter in combinations. Therefore, if duplicates are removed from a list of permutations, what is left is a list of combinations, as shown.

AB	~~BA~~	~~CA~~	~~DA~~
AC	BC	~~CB~~	~~DB~~
AD	BD	CD	~~DC~~

Hence the combinations of A, B, C, and D are AB, AC, AD, BC, BD, and CD. (Alternatively, BA could be listed and AB crossed out, etc.) The combinations have been listed alphabetically for convenience, but this is not a requirement.

Combination Example: How many combinations of 4 objects are there, taken 2 at a time?

Solution

Since this is a combination problem, the answer is

$$_4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2!}}{\cancel{2} \cdot 1 \cdot \cancel{2!}} = 6$$

Combination Example: A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

Solution

$$_8C_3 = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

There are 56 possibilities.

- In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?

Solution :

There are not separate roles listed for each committee member, so order does not matter. We will use combinations.

$$\text{Women: } {}_7C_3 = \frac{7!}{4!3!} = 35, \quad \text{Men: } {}_5C_2 = \frac{5!}{3!2!} = 10$$

There are $35 \cdot 10 = 350$ different possibilities.

Rule	Definition	Formula
Fundamental counting rule	The number of ways a sequence of n events can occur if the first event can occur in k_1 ways, the second event can occur in k_2 ways, etc.	$k_1 \cdot k_2 \cdot k_3 \cdots k_n$
Permutation rule	The number of permutations of n objects taking r objects at a time (order is important)	${}_nP_r = \frac{n!}{(n-r)!}$
Combination rule	The number of combinations of r objects taken from n objects (order is not important)	${}_nC_r = \frac{n!}{(n-r)!r!}$