

Introduction to Statics

1

CHAPTER OUTLINE

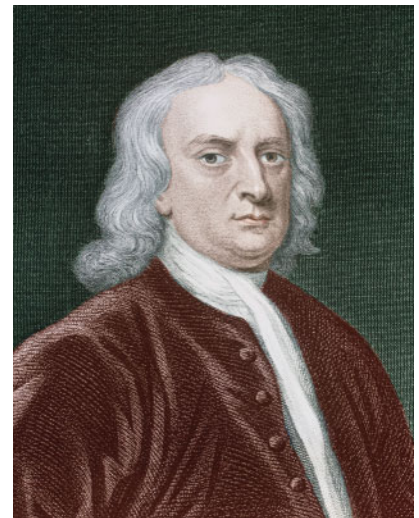
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| 1/1 Mechanics | 1/6 Law of Gravitation |
| 1/2 Basic Concepts | 1/7 Accuracy, Limits, and Approximations |
| 1/3 Scalars and Vectors | 1/8 Problem Solving in Statics |
| 1/4 Newton's Laws | 1/9 Chapter Review |
| 1/5 Units | |

1/1 Mechanics

Mechanics is the physical science which deals with the effects of forces on objects. No other subject plays a greater role in engineering analysis than mechanics. Although the principles of mechanics are few, they have wide application in engineering. The principles of mechanics are central to research and development in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and apparatus, and molecular, atomic, and subatomic behavior. A thorough understanding of this subject is an essential prerequisite for work in these and many other fields.

Mechanics is the oldest of the physical sciences. The early history of this subject is synonymous with the very beginnings of engineering. The earliest recorded writings in mechanics are those of Archimedes (287–212 B.C.) on the principle of the lever and the principle of buoyancy. Substantial progress came later with the formulation of the laws of vector combination of forces by Stevinus (1548–1620), who also formulated most of the principles of statics. The first investigation of a dynamics problem is credited to Galileo (1564–1642) for his experiments with falling stones. The accurate formulation of the laws of motion, as well as the law of gravitation, was made by Newton (1642–1727), who also conceived the idea of the infinitesimal in mathematical analysis. Substantial contributions to the development of mechanics were also made by da Vinci, Varignon, Euler, D'Alembert, Lagrange, Laplace, and others.

In this book we will be concerned with both the development of the principles of mechanics and their application. The principles of mechanics as a science are rigorously expressed by mathematics, and thus



Sir Isaac Newton

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mathematics plays an important role in the application of these principles to the solution of practical problems.

The subject of mechanics is logically divided into two parts: **statics**, which concerns the equilibrium of bodies under action of forces, and **dynamics**, which concerns the motion of bodies. *Engineering Mechanics* is divided into these two parts, *Vol. 1 Statics* and *Vol. 2 Dynamics*.

1/2 Basic Concepts

The following concepts and definitions are basic to the study of mechanics, and they should be understood at the outset.

Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*. Thus force is a vector quantity, and its properties are discussed in detail in Chapter 2.

A **particle** is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it.

Rigid body. A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand. For instance, the calculation of the tension in the cable which supports the boom of a mobile crane under load is essentially unaffected by the small internal deformations in the structural members of the boom. For the purpose, then, of determining the external forces which act on the boom, we may treat it as a rigid body. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Determination of the internal deformations belongs to the study of the mechanics of deformable bodies, which normally follows statics in the curriculum.

1/3 Scalars and Vectors

We use two kinds of quantities in mechanics—scalars and vectors. *Scalar quantities* are those with which only a magnitude is associated. Examples of scalar quantities are time, volume, density, speed, energy,

and mass. *Vector quantities*, on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition as described later in this article. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Speed is a scalar. It is the magnitude of velocity, which is a vector. Thus velocity is specified by a direction as well as a speed.

Vectors representing physical quantities can be classified as free, sliding, or fixed.

A **free vector** is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

A **sliding vector** has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole,* and thus it is a sliding vector.

A **fixed vector** is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.

Conventions for Equations and Diagrams

A vector quantity \mathbf{V} is represented by a line segment, Fig. 1/1, having the direction of the vector and having an arrowhead to indicate the sense. The length of the directed line segment represents to some convenient scale the magnitude $|\mathbf{V}|$ of the vector, which is printed with lightface italic type V . For example, we may choose a scale such that an arrow one inch long represents a force of twenty pounds.

In scalar equations, and frequently on diagrams where only the magnitude of a vector is labeled, the symbol will appear in lightface italic type. Boldface type is used for vector quantities whenever the directional aspect of the vector is a part of its mathematical representation. When writing vector equations, *always* be certain to preserve the mathematical distinction between vectors and scalars. In handwritten work, use a distinguishing mark for each vector quantity, such as an underline, \underline{V} , or an arrow over the symbol, \vec{V} , to take the place of boldface type in print.

Working with Vectors

The direction of the vector \mathbf{V} may be measured by an angle θ from some known reference direction as shown in Fig. 1/1. The negative of \mathbf{V} is a vector $-\mathbf{V}$ having the same magnitude as \mathbf{V} but directed in the sense opposite to \mathbf{V} , as shown in Fig. 1/1.

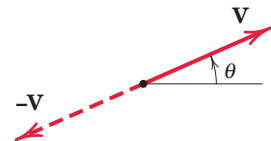


Figure 1/1

*This is the *principle of transmissibility*, which is discussed in Art. 2/2.

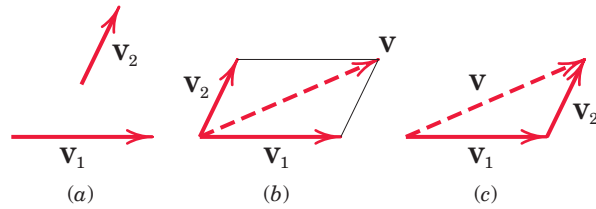


Figure 1/2

Vectors must obey the parallelogram law of combination. This law states that two vectors \mathbf{V}_1 and \mathbf{V}_2 , treated as free vectors, Fig. 1/2a, may be replaced by their equivalent vector \mathbf{V} , which is the diagonal of the parallelogram formed by \mathbf{V}_1 and \mathbf{V}_2 as its two sides, as shown in Fig. 1/2b. This combination is called the *vector sum*, and is represented by the vector equation

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$$

where the plus sign, when used with the vector quantities (in boldface type), means *vector* and not *scalar* addition. The scalar sum of the magnitudes of the two vectors is written in the usual way as $V_1 + V_2$. The geometry of the parallelogram shows that $V \neq V_1 + V_2$.

The two vectors \mathbf{V}_1 and \mathbf{V}_2 , again treated as free vectors, may also be added head-to-tail by the triangle law, as shown in Fig. 1/2c, to obtain the identical vector sum \mathbf{V} . We see from the diagram that the order of addition of the vectors does not affect their sum, so that $\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1$.

The difference $\mathbf{V}_1 - \mathbf{V}_2$ between the two vectors is easily obtained by adding $-\mathbf{V}_2$ to \mathbf{V}_1 as shown in Fig. 1/3, where either the triangle or parallelogram procedure may be used. The difference \mathbf{V}' between the two vectors is expressed by the vector equation

$$\mathbf{V}' = \mathbf{V}_1 - \mathbf{V}_2$$

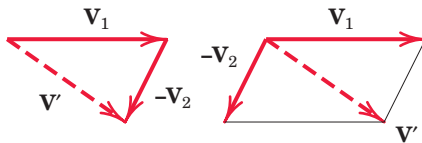


Figure 1/3

where the minus sign denotes *vector subtraction*.

Any two or more vectors whose sum equals a certain vector \mathbf{V} are said to be the *components* of that vector. Thus, the vectors \mathbf{V}_1 and \mathbf{V}_2 in Fig. 1/4a are the components of \mathbf{V} in the directions 1 and 2, respectively. It is usually most convenient to deal with vector components which are mutually perpendicular; these are called *rectangular components*. The

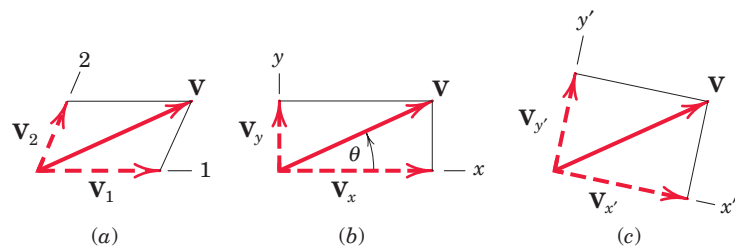


Figure 1/4

vectors \mathbf{V}_x and \mathbf{V}_y in Fig. 1/4b are the x - and y -components, respectively, of \mathbf{V} . Likewise, in Fig. 1/4c, $\mathbf{V}_{x'}$ and $\mathbf{V}_{y'}$ are the x' - and y' -components of \mathbf{V} . When expressed in rectangular components, the direction of the vector with respect to, say, the x -axis is clearly specified by the angle θ , where

$$\theta = \tan^{-1} \frac{V_y}{V_x}$$

A vector \mathbf{V} may be expressed mathematically by multiplying its magnitude V by a vector \mathbf{n} whose magnitude is one and whose direction coincides with that of \mathbf{V} . The vector \mathbf{n} is called a *unit vector*. Thus,

$$\mathbf{V} = V\mathbf{n}$$

In this way both the magnitude and direction of the vector are conveniently contained in one mathematical expression. In many problems, particularly three-dimensional ones, it is convenient to express the rectangular components of \mathbf{V} , Fig. 1/5, in terms of unit vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} , which are vectors in the x -, y -, and z -directions, respectively, with unit magnitudes. Because the vector \mathbf{V} is the vector sum of the components in the x -, y -, and z -directions, we can express \mathbf{V} as follows:

$$\mathbf{V} = V_x\mathbf{i} + V_y\mathbf{j} + V_z\mathbf{k}$$

We now make use of the *direction cosines* l , m , and n of \mathbf{V} , which are defined by

$$l = \cos \theta_x \quad m = \cos \theta_y \quad n = \cos \theta_z$$

Thus, we may write the magnitudes of the components of \mathbf{V} as

$$V_x = lV \quad V_y = mV \quad V_z = nV$$

where, from the Pythagorean theorem,

$$V^2 = V_x^2 + V_y^2 + V_z^2$$

Note that this relation implies that $l^2 + m^2 + n^2 = 1$.

1/4 Newton's Laws

Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity.* Slightly reworded with modern terminology, these laws are:

Law I. A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.

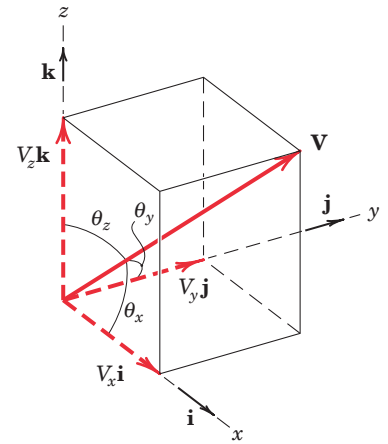


Figure 1/5

*Newton's original formulations may be found in the translation of his *Principia* (1687) revised by F. Cajori, University of California Press, 1934.

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

The correctness of these laws has been verified by innumerable accurate physical measurements. Newton's second law forms the basis for most of the analysis in dynamics. As applied to a particle of mass m , it may be stated as

$$\mathbf{F} = m\mathbf{a} \quad (1/1)$$

where \mathbf{F} is the vector sum of forces acting on the particle and \mathbf{a} is the resulting acceleration. This equation is a *vector* equation because the direction of \mathbf{F} must agree with the direction of \mathbf{a} , and the magnitudes of \mathbf{F} and $m\mathbf{a}$ must be equal.

Newton's first law contains the principle of the equilibrium of forces, which is the main topic of concern in statics. This law is actually a consequence of the second law, since there is no acceleration when the force is zero, and the particle either is at rest or is moving with a uniform velocity. The first law adds nothing new to the description of motion but is included here because it was part of Newton's classical statements.

The third law is basic to our understanding of force. It states that forces always occur in pairs of equal and opposite forces. Thus, the downward force exerted on the desk by the pencil is accompanied by an upward force of equal magnitude exerted on the pencil by the desk. This principle holds for all forces, variable or constant, regardless of their source, and holds at every instant of time during which the forces are applied. Lack of careful attention to this basic law is the cause of frequent error by the beginner.

In the analysis of bodies under the action of forces, it is absolutely necessary to be clear about which force of each action–reaction pair is being considered. It is necessary first of all to *isolate* the body under consideration and then to consider only the one force of the pair which acts *on* the body in question.

1/5 Units

In mechanics we use four fundamental quantities called *dimensions*. These are length, mass, force, and time. The units used to measure these quantities cannot all be chosen independently because they must be consistent with Newton's second law, Eq. 1/1. Although there are a number of different systems of units, only the two systems most commonly used in science and technology will be used in this text. The four fundamental dimensions and their units and symbols in the two systems are summarized in the following table.

QUANTITY	DIMENSIONAL SYMBOL	SI UNITS		U.S. CUSTOMARY UNITS			
		UNIT	SYMBOL	UNIT	SYMBOL		
Mass	M	Base units	kilogram	kg	slug	—	
Length	L		meter	m	Base units	foot	ft
Time	T		second	s		second	sec
Force	F		newton	N		pound	lb

SI Units

The International System of Units, abbreviated SI (from the French, *Système International d'Unités*), is accepted in the United States and throughout the world, and is a modern version of the metric system. By international agreement, SI units will in time replace other systems. As shown in the table, in SI, the units kilogram (kg) for mass, meter (m) for length, and second (s) for time are selected as the base units, and the newton (N) for force is derived from the preceding three by Eq. 1/1. Thus, force (N) = mass (kg) \times acceleration (m/s^2) or

$$N = \text{kg} \cdot \text{m/s}^2$$

Thus, 1 newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2 .

Consider a body of mass m which is allowed to fall freely near the surface of the earth. With only the force of gravitation acting on the body, it falls with an acceleration g toward the center of the earth. This gravitational force is the *weight* W of the body, and is found from Eq. 1/1:

$$W (\text{N}) = m (\text{kg}) \times g (\text{m/s}^2)$$

U.S. Customary Units

The U.S. customary, or British system of units, also called the foot-pound-second (FPS) system, has been the common system in business and industry in English-speaking countries. Although this system will in time be replaced by SI units, for many more years engineers must be able to work with both SI units and FPS units, and both systems are used freely in *Engineering Mechanics*.

As shown in the table, in the U.S. or FPS system, the units of feet (ft) for length, seconds (sec) for time, and pounds (lb) for force are selected as base units, and the slug for mass is derived from Eq. 1/1. Thus, force (lb) = mass (slugs) \times acceleration (ft/sec^2), or

$$\text{slug} = \frac{\text{lb} \cdot \text{sec}^2}{\text{ft}}$$

Therefore, 1 slug is the mass which is given an acceleration of 1 ft/sec^2 when acted on by a force of 1 lb. If W is the gravitational force or weight and g is the acceleration due to gravity, Eq. 1/1 gives

$$m (\text{slugs}) = \frac{W (\text{lb})}{g (\text{ft/sec}^2)}$$

Note that seconds is abbreviated as s in SI units, and as *sec* in FPS units.

In U.S. units the pound is also used on occasion as a unit of mass, especially to specify thermal properties of liquids and gases. When distinction between the two units is necessary, the force unit is frequently written as lbf and the mass unit as lbm. In this book we use almost exclusively the force unit, which is written simply as lb. Other common units of force in the U.S. system are the *kilopound* (kip), which equals 1000 lb, and the *ton*, which equals 2000 lb.

The International System of Units (SI) is termed an *absolute* system because the measurement of the base quantity mass is independent of its environment. On the other hand, the U.S. system (FPS) is termed a *gravitational* system because its base quantity force is defined as the gravitational attraction (weight) acting on a standard mass under specified conditions (sea level and 45° latitude). A standard pound is also the force required to give a one-pound mass an acceleration of 32.1740 ft/sec².

In SI units the kilogram is used *exclusively* as a unit of mass—*never* force. In the MKS (meter, kilogram, second) gravitational system, which has been used for many years in non-English-speaking countries, the kilogram, like the pound, has been used both as a unit of force and as a unit of mass.

Primary Standards

Primary standards for the measurements of mass, length, and time have been established by international agreement and are as follows:

Mass. The kilogram is defined as the mass of a specific platinum–iridium cylinder which is kept at the International Bureau of Weights and Measures near Paris, France. An accurate copy of this cylinder is kept in the United States at the National Institute of Standards and Technology (NIST), formerly the National Bureau of Standards, and serves as the standard of mass for the United States.

Length. The meter, originally defined as one ten-millionth of the distance from the pole to the equator along the meridian through Paris, was later defined as the length of a specific platinum–iridium bar kept at the International Bureau of Weights and Measures. The difficulty of accessing the bar and reproducing accurate measurements prompted the adoption of a more accurate and reproducible standard of length for the meter, which is now defined as 1 650 763.73 wavelengths of a specific radiation of the krypton-86 atom.

Time. The second was originally defined as the fraction 1/(86 400) of the mean solar day. However, irregularities in the earth’s rotation led to difficulties with this definition, and a more accurate and reproducible standard has been adopted. The second is now defined as the duration of 9 192 631 770 periods of the radiation of a specific state of the cesium-133 atom.

For most engineering work, and for our purpose in studying mechanics, the accuracy of these standards is considerably beyond our



The standard kilogram

needs. The standard value for gravitational acceleration g is its value at sea level and at a 45° latitude. In the two systems these values are

$$\begin{array}{ll} \text{SI units} & g = 9.806\,65 \text{ m/s}^2 \\ \text{U.S. units} & g = 32.1740 \text{ ft/sec}^2 \end{array}$$

The approximate values of 9.81 m/s^2 and 32.2 ft/sec^2 , respectively, are sufficiently accurate for the vast majority of engineering calculations.

Unit Conversions

The characteristics of SI units are shown inside the front cover of this book, along with the numerical conversions between U.S. customary and SI units. In addition, charts giving the approximate conversions between selected quantities in the two systems appear inside the back cover for convenient reference. Although these charts are useful for obtaining a feel for the relative size of SI and U.S. units, in time engineers will find it essential to think directly in terms of SI units without converting from U.S. units. In statics we are primarily concerned with the units of length and force, with mass needed only when we compute gravitational force, as explained previously.

Figure 1/6 depicts examples of force, mass, and length in the two systems of units, to aid in visualizing their relative magnitudes.

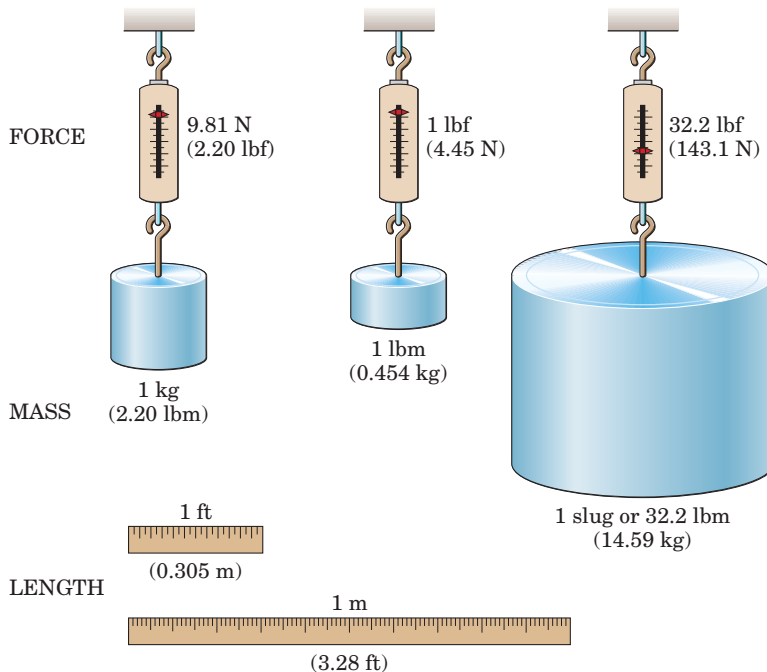


Figure 1/6

1/6 Law of Gravitation

In statics as well as dynamics we often need to compute the weight of a body, which is the gravitational force acting on it. This computation depends on the *law of gravitation*, which was also formulated by Newton. The law of gravitation is expressed by the equation

$$F = G \frac{m_1 m_2}{r^2} \quad (1/2)$$

where F = the mutual force of attraction between two particles

G = a universal constant known as the *constant of gravitation*

m_1, m_2 = the masses of the two particles

r = the distance between the centers of the particles

The mutual forces F obey the law of action and reaction, since they are equal and opposite and are directed along the line joining the centers of the particles, as shown in Fig. 1/7. By experiment the gravitational constant is found to be $G = 6.673(10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$.

Gravitational Attraction of the Earth

Gravitational forces exist between every pair of bodies. On the surface of the earth the only gravitational force of appreciable magnitude is the force due to the attraction of the earth. For example, each of two iron spheres 100 mm in diameter is attracted to the earth with a gravitational force of 37.1 N, which is its weight. On the other hand, the force of mutual attraction between the spheres if they are just touching is 0.000 000 095 1 N. This force is clearly negligible compared with the earth's attraction of 37.1 N. Consequently the gravitational attraction of the earth is the only gravitational force we need to consider for most engineering applications on the earth's surface.

The gravitational attraction of the earth on a body (its weight) exists whether the body is at rest or in motion. Because this attraction is a force, the weight of a body should be expressed in newtons (N) in SI units and in pounds (lb) in U.S. customary units. Unfortunately in common practice the mass unit kilogram (kg) has been frequently used as a measure of weight. This usage should disappear in time as SI units become more widely used, because in SI units the kilogram is used exclusively for mass and the newton is used for force, including weight.

For a body of mass m near the surface of the earth, the gravitational attraction F on the body is specified by Eq. 1/2. We usually denote the

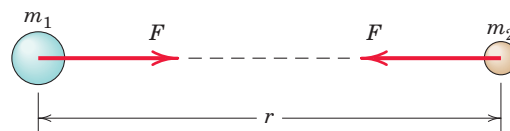


Figure 1/7

NASA Marshall Space Flight Center (NASA-MSFC)



The gravitational force which the earth exerts on the moon (foreground) is a key factor in the motion of the moon.

magnitude of this gravitational force or weight with the symbol W . Because the body falls with an acceleration g , Eq. 1/1 gives

$$W = mg \quad (1/3)$$

The weight W will be in newtons (N) when the mass m is in kilograms (kg) and the acceleration of gravity g is in meters per second squared (m/s^2). In U.S. customary units, the weight W will be in pounds (lb) when m is in slugs and g is in feet per second squared. The standard values for g of 9.81 m/s^2 and 32.2 ft/sec^2 will be sufficiently accurate for our calculations in statics.

The true weight (gravitational attraction) and the apparent weight (as measured by a spring scale) are slightly different. The difference, which is due to the rotation of the earth, is quite small and will be neglected. This effect will be discussed in *Vol. 2 Dynamics*.

1/7 Accuracy, Limits, and Approximations

The number of significant figures in an answer should be no greater than the number of figures justified by the accuracy of the given data. For example, suppose the 24-mm side of a square bar was measured to the nearest millimeter, so we know the side length to two significant figures. Squaring the side length gives an area of 576 mm^2 . However, according to our rule, we should write the area as 580 mm^2 , using only two significant figures.

When calculations involve small differences in large quantities, greater accuracy in the data is required to achieve a given accuracy in the results. Thus, for example, it is necessary to know the numbers 4.2503 and 4.2391 to an accuracy of five significant figures to express their difference 0.0112 to three-figure accuracy. It is often difficult in lengthy computations to know at the outset how many significant figures are needed in the original data to ensure a certain accuracy in the answer. Accuracy to three significant figures is considered satisfactory for most engineering calculations.

In this text, answers will generally be shown to three significant figures unless the answer begins with the digit 1, in which case the answer will be shown to four significant figures. For purposes of calculation, consider all data given in this book to be exact.

Differentials

The *order* of differential quantities frequently causes misunderstanding in the derivation of equations. Higher-order differentials may always be neglected compared with lower-order differentials when the mathematical limit is approached. For example, the element of volume ΔV of a right circular cone of altitude h and base radius r may be taken to be a circular slice a distance x from the vertex and of thickness Δx . The expression for the volume of the element is

$$\Delta V = \frac{\pi r^2}{h^2} [x^2 \Delta x + x(\Delta x)^2 + \frac{1}{3}(\Delta x)^3]$$

Note that, when passing to the limit in going from ΔV to dV and from Δx to dx , the terms containing $(\Delta x)^2$ and $(\Delta x)^3$ drop out, leaving merely

$$dV = \frac{\pi r^2}{h^2} x^2 dx$$

which gives an exact expression when integrated.

Small-Angle Approximations

When dealing with small angles, we can usually make use of simplifying approximations. Consider the right triangle of Fig. 1/8 where the angle θ , expressed in radians, is relatively small. If the hypotenuse is unity, we see from the geometry of the figure that the arc length $1 \times \theta$ and $\sin \theta$ are very nearly the same. Also $\cos \theta$ is close to unity. Furthermore, $\sin \theta$ and $\tan \theta$ have almost the same values. Thus, for small angles we may write

$$\sin \theta \cong \tan \theta \cong \theta \quad \cos \theta \cong 1$$

provided that the angles are expressed in radians. These approximations may be obtained by retaining only the first terms in the series expansions for these three functions. As an example of these approximations, for an angle of 1°

$$\begin{aligned} 1^\circ &= 0.017\,453 \text{ rad} & \tan 1^\circ &= 0.017\,455 \\ \sin 1^\circ &= 0.017\,452 & \cos 1^\circ &= 0.999\,848 \end{aligned}$$

If a more accurate approximation is desired, the first two terms may be retained, and they are

$$\sin \theta \cong \theta - \theta^3/6 \quad \tan \theta \cong \theta + \theta^3/3 \quad \cos \theta \cong 1 - \theta^2/2$$

where the angles must be expressed in radians. (To convert degrees to radians, multiply the angle in degrees by $\pi/180^\circ$.) The error in replacing the sine by the angle for 1° (0.0175 rad) is only 0.005 percent. For 5° (0.0873 rad) the error is 0.13 percent, and for 10° (0.1745 rad), the error is still only 0.51 percent. As the angle θ approaches zero, the following relations are true in the mathematical limit:

$$\sin d\theta = \tan d\theta = d\theta \quad \cos d\theta = 1$$

where the differential angle $d\theta$ must be expressed in radians.

1/8 Problem Solving in Statics

We study statics to obtain a quantitative description of forces which act on engineering structures in equilibrium. Mathematics establishes the relations between the various quantities involved and enables us to predict effects from these relations. We use a dual thought process in

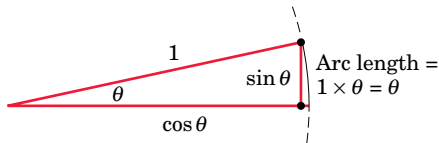


Figure 1/8

solving statics problems: We think about both the physical situation and the corresponding mathematical description. In the analysis of every problem, we make a transition between the physical and the mathematical. One of the most important goals for the student is to develop the ability to make this transition freely.

Making Appropriate Assumptions

We should recognize that the mathematical formulation of a physical problem represents an ideal description, or *model*, which approximates but never quite matches the actual physical situation. When we construct an idealized mathematical model for a given engineering problem, certain approximations will always be involved. Some of these approximations may be mathematical, whereas others will be physical.

For instance, it is often necessary to neglect small distances, angles, or forces compared with large distances, angles, or forces. Suppose a force is distributed over a small area of the body on which it acts. We may consider it to be a concentrated force if the dimensions of the area involved are small compared with other pertinent dimensions.

We may neglect the weight of a steel cable if the tension in the cable is many times greater than its total weight. However, if we must calculate the deflection or sag of a suspended cable under the action of its weight, we may not ignore the cable weight.

Thus, what we may assume depends on what information is desired and on the accuracy required. We must be constantly alert to the various assumptions called for in the formulation of real problems. The ability to understand and make use of the appropriate assumptions in the formulation and solution of engineering problems is certainly one of the most important characteristics of a successful engineer. One of the major aims of this book is to provide many opportunities to develop this ability through the formulation and analysis of many practical problems involving the principles of statics.

Using Graphics

Graphics is an important analytical tool for three reasons:

1. We use graphics to represent a physical system on paper with a sketch or diagram. Representing a problem geometrically helps us with its physical interpretation, especially when we must visualize three-dimensional problems.
2. We can often obtain a graphical solution to problems more easily than with a direct mathematical solution. Graphical solutions are both a practical way to obtain results, and an aid in our thought processes. Because graphics represents the physical situation and its mathematical expression simultaneously, graphics helps us make the transition between the two.
3. Charts or graphs are valuable aids for representing results in a form which is easy to understand.



KEY CONCEPTS

Formulating Problems and Obtaining Solutions

In statics, as in all engineering problems, we need to use a precise and logical method for formulating problems and obtaining their solutions. We formulate each problem and develop its solution through the following sequence of steps.

1. Formulate the problem:
 - (a) State the given data.
 - (b) State the desired result.
 - (c) State your assumptions and approximations.
2. Develop the solution:
 - (a) Draw any diagrams you need to understand the relationships.
 - (b) State the governing principles to be applied to your solution.
 - (c) Make your calculations.
 - (d) Ensure that your calculations are consistent with the accuracy justified by the data.
 - (e) Be sure that you have used consistent units throughout your calculations.
 - (f) Ensure that your answers are reasonable in terms of magnitudes, directions, common sense, etc.
 - (g) Draw conclusions.

Keeping your work neat and orderly will help your thought process and enable others to understand your work. The discipline of doing orderly work will help you develop skill in formulation and analysis. Problems which seem complicated at first often become clear when you approach them with logic and discipline.

The Free-Body Diagram

The subject of statics is based on surprisingly few fundamental concepts and involves mainly the application of these basic relations to a variety of situations. In this application the *method* of analysis is all important. In solving a problem, it is essential that the laws which apply be carefully fixed in mind and that we apply these principles literally and exactly. In applying the principles of mechanics to analyze forces acting on a body, it is essential that we *isolate* the body in question from all other bodies so that a complete and accurate account of all forces acting on this body can be taken. This *isolation* should exist mentally and should be represented on paper. The diagram of such an isolated body with the representation of *all* external forces acting *on* it is called a *free-body diagram*.

The free-body-diagram method is the key to the understanding of mechanics. This is so because the *isolation* of a body is the tool by which

cause and *effect* are clearly separated, and by which our attention is clearly focused on the literal application of a principle of mechanics. The technique of drawing free-body diagrams is covered in Chapter 3, where they are first used.

Numerical Values versus Symbols

In applying the laws of statics, we may use numerical values to represent quantities, or we may use algebraic symbols, and leave the answer as a formula. When numerical values are used, the magnitude of each quantity expressed in its particular units is evident at each stage of the calculation. This is useful when we need to know the magnitude of each term.

The symbolic solution, however, has several advantages over the numerical solution. First, the use of symbols helps to focus our attention on the connection between the physical situation and its related mathematical description. Second, we can use a symbolic solution repeatedly for obtaining answers to the same type of problem, but having different units or numerical values. Third, a symbolic solution enables us to make a dimensional check at every step, which is more difficult to do when numerical values are used. In any equation representing a physical situation, the dimensions of every term on both sides of the equation must be the same. This property is called *dimensional homogeneity*.

Thus, facility with both numerical and symbolic forms of solution is essential.

Solution Methods

Solutions to the problems of statics may be obtained in one or more of the following ways.

1. Obtain mathematical solutions by hand, using either algebraic symbols or numerical values. We can solve most problems this way.
2. Obtain graphical solutions for certain problems.
3. Solve problems by computer. This is useful when a large number of equations must be solved, when a parameter variation must be studied, or when an intractable equation must be solved.

Many problems can be solved with two or more of these methods. The method utilized depends partly on the engineer's preference and partly on the type of problem to be solved. The choice of the most expedient method of solution is an important aspect of the experience to be gained from the problem work. There are a number of problems in *Vol. 1 Statics* which are designated as *Computer-Oriented Problems*. These problems appear at the end of the Review Problem sets and are selected to illustrate the type of problem for which solution by computer offers a distinct advantage.

1/9 Chapter Review

This chapter has introduced the concepts, definitions, and units used in statics, and has given an overview of the procedure used to formulate and solve problems in statics. Now that you have finished this chapter, you should be able to do the following:

1. Express vectors in terms of unit vectors and perpendicular components, and perform vector addition and subtraction.
2. State Newton's laws of motion.
3. Perform calculations using SI and U.S. customary units, using appropriate accuracy.
4. Express the law of gravitation and calculate the weight of an object.
5. Apply simplifications based on differential and small-angle approximations.
6. Describe the methodology used to formulate and solve statics problems.

SAMPLE PROBLEM 1/1

Determine the weight in newtons of a car whose mass is 1400 kg. Convert the mass of the car to slugs and then determine its weight in pounds.

Solution. From relationship 1/3, we have

$$1 \quad W = mg = 1400(9.81) = 13\,730 \text{ N} \quad \text{Ans.}$$

From the table of conversion factors inside the front cover of the textbook, we see that 1 slug is equal to 14.594 kg. Thus, the mass of the car in slugs is

$$2 \quad m = 1400 \text{ kg} \left[\frac{1 \text{ slug}}{14.594 \text{ kg}} \right] = 95.9 \text{ slugs} \quad \text{Ans.}$$

Finally, its weight in pounds is

$$3 \quad W = mg = (95.9)(32.2) = 3090 \text{ lb} \quad \text{Ans.}$$

As another route to the last result, we can convert from kg to lbm. Again using the table inside the front cover, we have

$$m = 1400 \text{ kg} \left[\frac{1 \text{ lbm}}{0.45359 \text{ kg}} \right] = 3090 \text{ lbm}$$

The weight in pounds associated with the mass of 3090 lbm is 3090 lb, as calculated above. We recall that 1 lbm is the amount of mass which under standard conditions has a weight of 1 lb of force. We rarely refer to the U.S. mass unit lbm in this textbook series, but rather use the slug for mass. The sole use of slug, rather than the unnecessary use of two units for mass, will prove to be powerful and simple—especially in dynamics.

- 3 Note that we are using a previously calculated result (95.9 slugs). We must be sure that when a calculated number is needed in subsequent calculations, it is retained in the calculator to its full accuracy, (95.929834 . . .) until it is needed. This may require storing it in a register upon its initial calculation and recalling it later. We must not merely punch 95.9 into our calculator and proceed to multiply by 32.2—this practice will result in loss of numerical accuracy. Some individuals like to place a small indication of the storage register used in the right margin of the work paper, directly beside the number stored.

SAMPLE PROBLEM 1/2

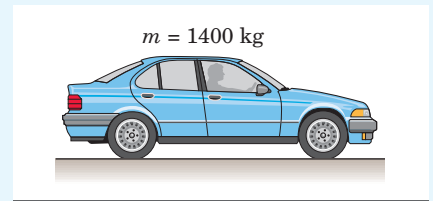
Use Newton's law of universal gravitation to calculate the weight of a 70-kg person standing on the surface of the earth. Then repeat the calculation by using $W = mg$ and compare your two results. Use Table D/2 as needed.

Solution. The two results are

$$1 \quad W = \frac{Gm_e m}{R^2} = \frac{(6.673 \cdot 10^{-11})(5.976 \cdot 10^{24})(70)}{[6371 \cdot 10^3]^2} = 688 \text{ N} \quad \text{Ans.}$$

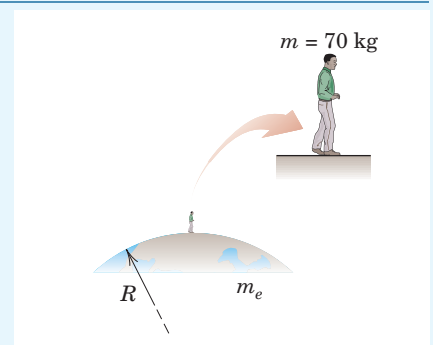
$$W = mg = 70(9.81) = 687 \text{ N} \quad \text{Ans.}$$

The discrepancy is due to the fact that Newton's universal gravitational law does not take into account the rotation of the earth. On the other hand, the value $g = 9.81 \text{ m/s}^2$ used in the second equation does account for the earth's rotation. Note that had we used the more accurate value $g = 9.80665 \text{ m/s}^2$ (which likewise accounts for the earth's rotation) in the second equation, the discrepancy would have been larger (686 N would have been the result).



Helpful Hints

- 1 Our calculator indicates a result of 13 734 N. Using the rules of significant-figure display used in this textbook, we round the written result to four significant figures, or 13 730 N. Had the number begun with any digit other than 1, we would have rounded to three significant figures.
- 2 A good practice with unit conversion is to multiply by a factor such as $\left[\frac{1 \text{ slug}}{14.594 \text{ kg}} \right]$, which has a value of 1, because the numerator and the denominator are equivalent. Make sure that cancellation of the units leaves the units desired; here the units of kg cancel, leaving the desired units of slug.



Helpful Hint

- 1 The effective distance between the mass centers of the two bodies involved is the radius of the earth.

SAMPLE PROBLEM 1/3

For the vectors \mathbf{V}_1 and \mathbf{V}_2 shown in the figure,

- determine the magnitude S of their vector sum $\mathbf{S} = \mathbf{V}_1 + \mathbf{V}_2$
- determine the angle α between \mathbf{S} and the positive x -axis
- write \mathbf{S} as a vector in terms of the unit vectors \mathbf{i} and \mathbf{j} and then write a unit vector \mathbf{n} along the vector sum \mathbf{S}
- determine the vector difference $\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2$

Solution (a) We construct to scale the parallelogram shown in Fig. *a* for adding \mathbf{V}_1 and \mathbf{V}_2 . Using the law of cosines, we have

$$S^2 = 3^2 + 4^2 - 2(3)(4) \cos 105^\circ$$

$$S = 5.59 \text{ units}$$

- 1 (b) Using the law of sines for the lower triangle, we have

$$\frac{\sin 105^\circ}{5.59} = \frac{\sin(\alpha + 30^\circ)}{4}$$

$$\sin(\alpha + 30^\circ) = 0.692$$

$$(\alpha + 30^\circ) = 43.8^\circ \quad \alpha = 13.76^\circ$$

(c) With knowledge of both S and α , we can write the vector \mathbf{S} as

$$\mathbf{S} = S[\mathbf{i} \cos \alpha + \mathbf{j} \sin \alpha]$$

$$= 5.59[\mathbf{i} \cos 13.76^\circ + \mathbf{j} \sin 13.76^\circ] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units}$$

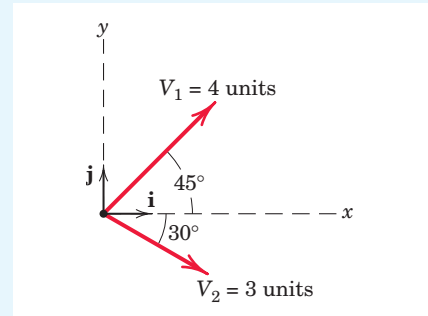
- 2 Then $\mathbf{n} = \frac{\mathbf{S}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j}$

(d) The vector difference \mathbf{D} is

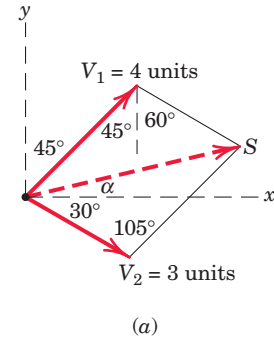
$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i} \cos 45^\circ + \mathbf{j} \sin 45^\circ) - 3(\mathbf{i} \cos 30^\circ - \mathbf{j} \sin 30^\circ)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units}$$

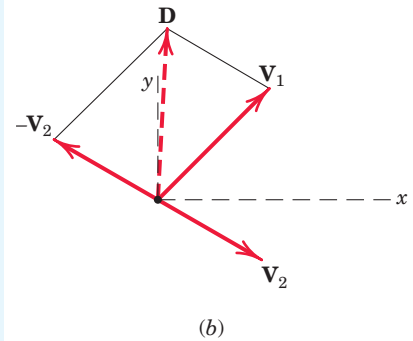
The vector \mathbf{D} is shown in Fig. *b* as $\mathbf{D} = \mathbf{V}_1 + (-\mathbf{V}_2)$.



Ans.



Ans.



Ans.

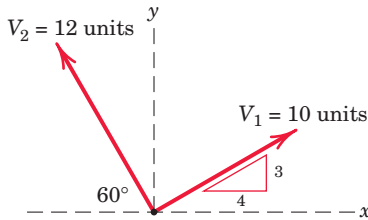
Ans.

Helpful Hints

- 1 You will frequently use the laws of cosines and sines in mechanics. See Art. C/6 of Appendix C for a review of these important geometric principles.
- 2 A unit vector may always be formed by dividing a vector by its magnitude. Note that a unit vector is dimensionless.

PROBLEMS

- 1/1** Determine the angles made by the vector $\mathbf{V} = 40\mathbf{i} - 30\mathbf{j}$ with the positive x - and y -axes. Write the unit vector \mathbf{n} in the direction of \mathbf{V} .
- 1/2** Determine the magnitude of the vector sum $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ and the angle θ_x which \mathbf{V} makes with the positive x -axis. Complete both graphical and algebraic solutions.

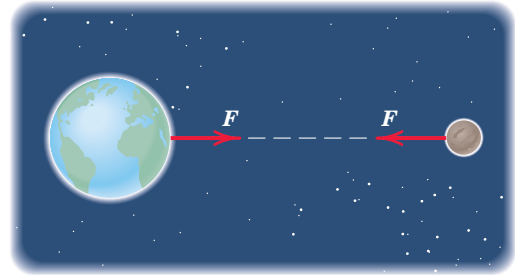


Problem 1/2

- 1/3** For the given vectors \mathbf{V}_1 and \mathbf{V}_2 of Prob. 1/2, determine the magnitude of the vector difference $\mathbf{V}' = \mathbf{V}_2 - \mathbf{V}_1$ and the angle θ_x which \mathbf{V}' makes with the positive x -axis. Complete both graphical and algebraic solutions.
- 1/4** A force is specified by the vector $\mathbf{F} = 120\mathbf{i} - 160\mathbf{j} + 80\mathbf{k}$ lb. Calculate the angles made by \mathbf{F} with the positive x -, y -, and z -axes.
- 1/5** What is the mass in both slugs and kilograms of a 3000-lb car?
- 1/6** From the gravitational law calculate the weight W (gravitational force with respect to the earth) of a 90-kg man in a spacecraft traveling in a circular orbit 250 km above the earth's surface. Express W in both newtons and pounds.
- 1/7** Determine the weight in newtons of a woman whose weight in pounds is 130. Also, find her mass in slugs and in kilograms. Determine your own weight in newtons.

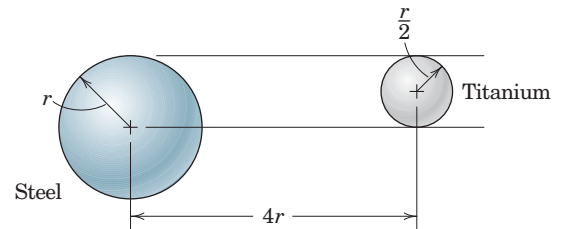
- 1/8** Suppose that two nondimensional quantities are exactly $A = 6.67$ and $B = 1.726$. Using the rules for significant figures as stated in this chapter, express the four quantities $(A + B)$, $(A - B)$, (AB) , and (A/B) .

- 1/9** Compute the magnitude F of the force which the earth exerts on the moon. Perform the calculation first in newtons and then convert your result to pounds. Refer to Table D/2 for necessary physical quantities.



Problem 1/9

- 1/10** The uniform steel and titanium spheres are positioned as shown. Determine the magnitude of the small gravitational force of mutual attraction if $r = 50$ mm.



Problem 1/10

- 1/11** Determine the percent error n in replacing the sine and the tangent of an angle by the value of the angle in radians for angle values of 5° , 10° , and 20° . Explain the qualitative difference between the sine and tangent results.