Al-Mustaqbal University
College of Sciences
Intelligent Medical Systems Department

جامـــعـة المـــســتـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العـــــلوم
قـســـــم الامن السيبراني

LECTURE: (2)

Subject: FUNCTIONS
Level: First
Lecturer: Dr. Mustafa Talal

Chapter Two

Function

Numbers:

1 – N = set of natural numbers

N = {1, 2, 3, 4……}

2 – I = set of integers

= {……., -3, -2, -1, 0, 1, 2, 3…}

Note that: NCI

3 – A = set of rational numbers

$$=\left(\chi:\chi=\frac{\rho}{q}\ \rho\ and\ q\ are\ integers\ q\neq 03\right]$$

Ex: $\frac{3}{2},-\frac{4}{5},\frac{3}{1},\frac{-7}{1}$

Note that: ICA

4 – B = set of irrational numbers

= $\{X : X\ is\ not\ arational\ number\}$

Ex: $\sqrt{2}$, $\sqrt{3}$, $-\sqrt{7}$

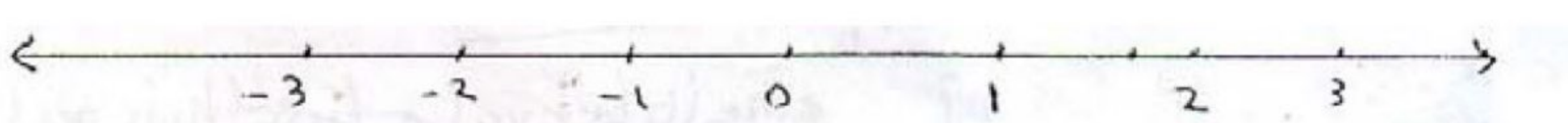
5 – R: set of real numbers

= set of all rational and irrational numbers

Note that

R = AUB

Note: the set of real numbers is represented by a line called a line of numbers:



(ii) NCR, ICR, ACR, BCR

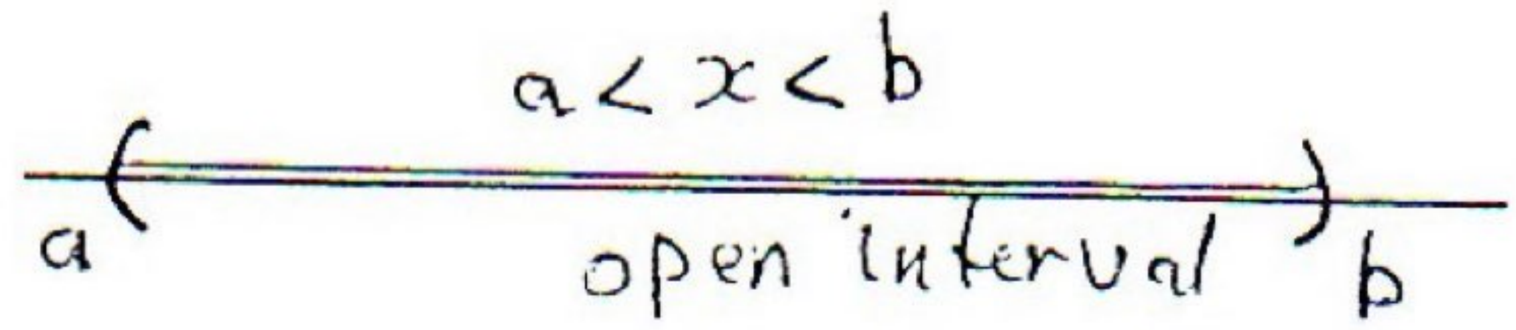
Intervals

The set of values that a variable χ may take on is called the domain of χ. The domains of the variables in many applications of calculus are intervals like those shows below.

- **open intervals**

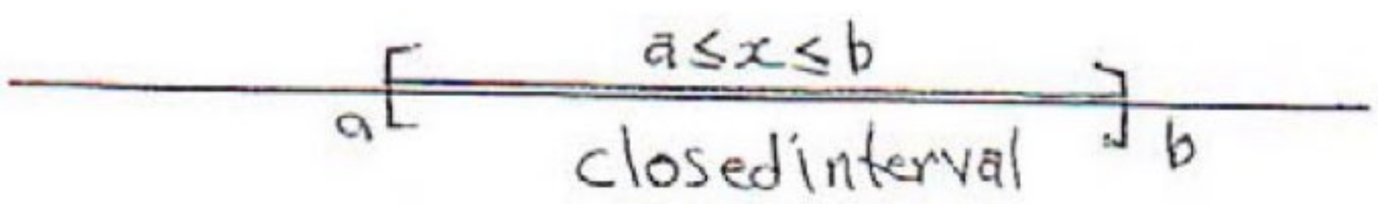
is the set of all real numbers that lie strictly between two fixed numbers a and b:

In symbols	**In words**
$a\langle \chi \langle b\ or\ (q,b)$	The open interval a b



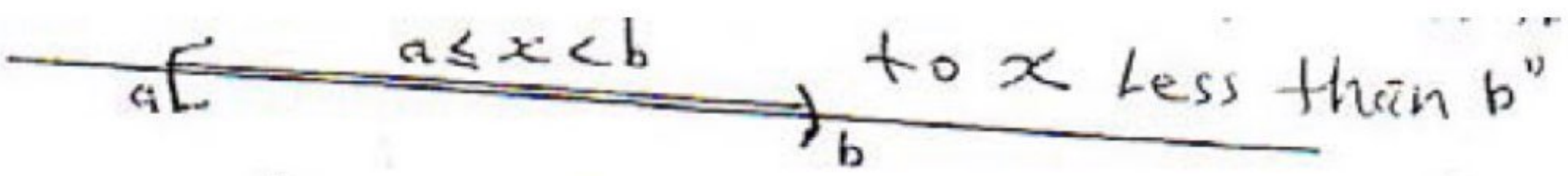
- Closed Intervals contain both endpoints:

In symbols	**In words**
$a \le \chi \le b \quad or \quad [a,b]$	the closed interval a b

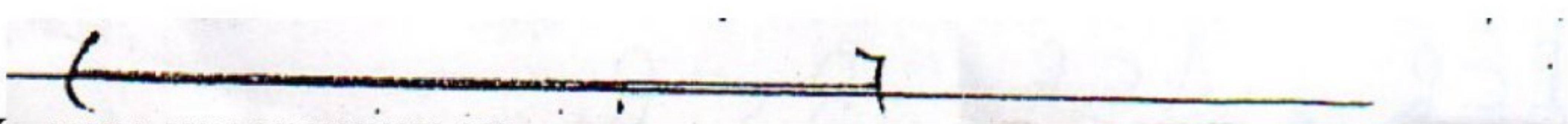


- Half – open intervals contain one but not both end points:

In symbols:	**in wards**
$a \le \chi \langle b$ or [a,b]	‘ the interval a less than or equal
$a \le \chi c b$	To χ less than b



$a \langle \chi \le b$ or [a,b] the interval a less than χ less than or equal b



Ex. find the domain of

1 – Y = $\sqrt{1-X^2}$

The domain of χ is the closed interval

$1- \le \chi \le 1$

2 – Y = $\dfrac{1}{\sqrt{1-X^2}}$

The domain for χ is open interval

$-1 \langle \chi \langle 1$ because $\dfrac{1}{0}$ is not defined

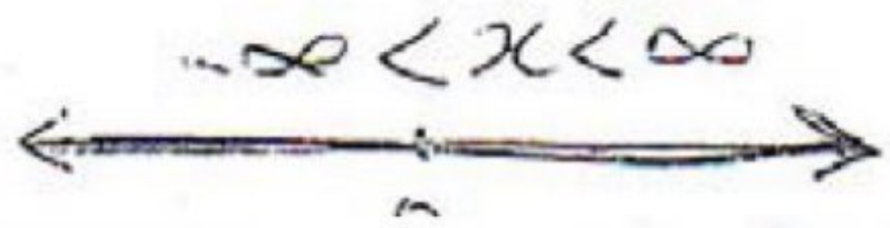
B - y = $\sqrt{\dfrac{1}{X}-1}$

$1/X - 1 \ge 0 \quad or \quad \dfrac{1}{X} \ge 1$

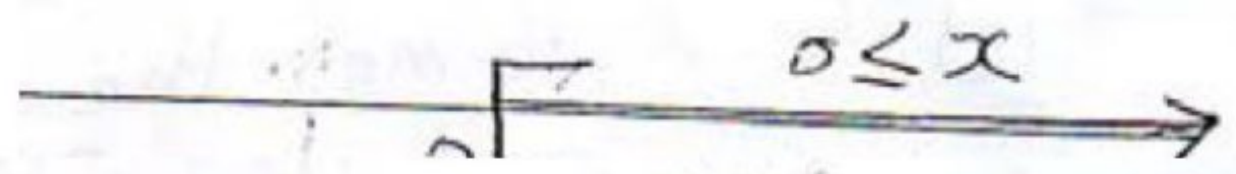
The domain for χ is the half – open $0 < \chi \leq 1$

Ex: **the equation** **the domain**

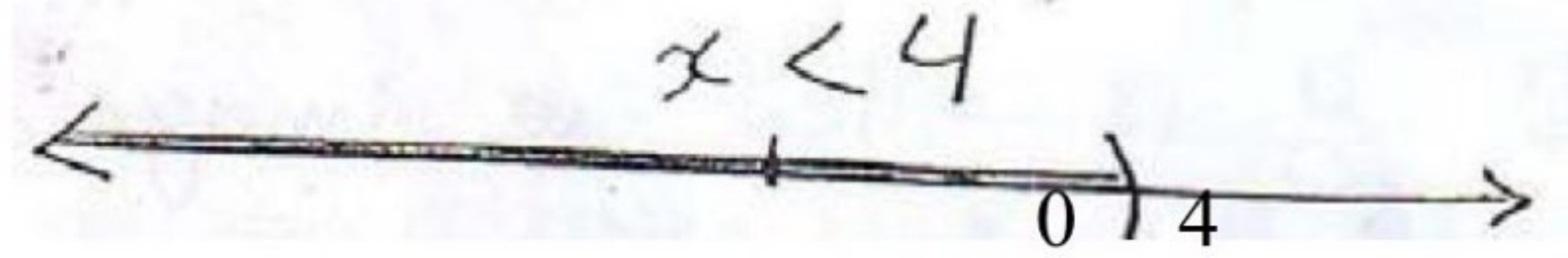
$Y = \chi^2$



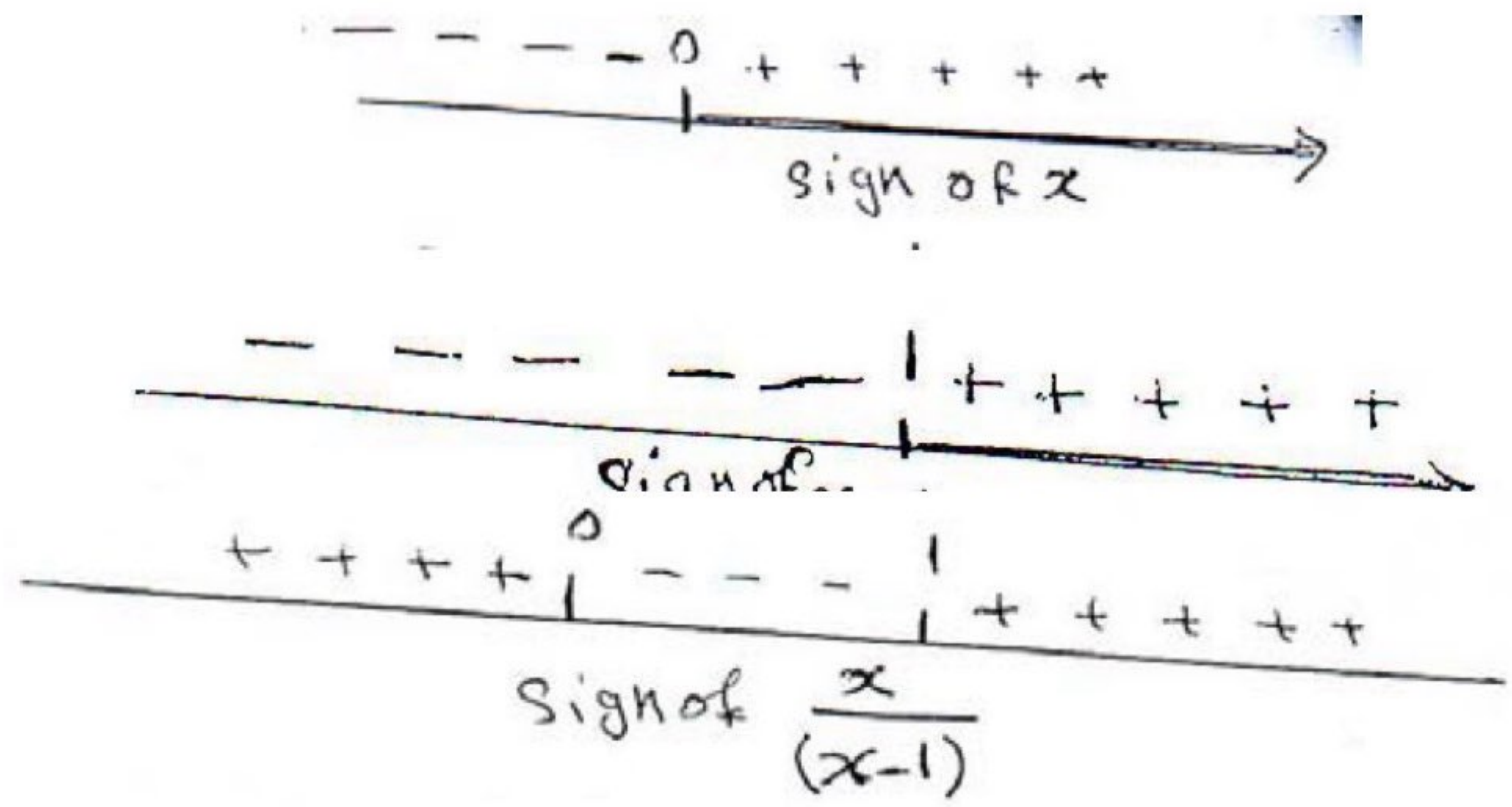
$Y = \sqrt{\chi}$



$Y = \frac{1}{\sqrt{4-X}}$



$y = \sqrt{\frac{\chi}{(\chi-1)}}$



The domain for χ is $X \leq 0 \cup X > 1$

Definition: A function, say f is a relation between the elements of two sets say A and B such that for every $\chi \in A$ there exists one and only one $Y \in B$ with Y = F(X).

The set A which contain the values of χ is called the domain of function F.

The set B which contains the values of Y corresponding to the values of χ is called the range of the function F. χ is called the independer variable of the function F, while Y is called the dependant variable of F.

Note:

1 – Some times the domain is denoted by DF and the range by RF.

2 – Y is called the image of χ.

Example: Let the domain of χ be the set $\{0,1,2,3,4\}$. Assign to each value of χ the number Y = χ^2. The function so defined is the set of pairs, { (0,0), (1,1), (2,4), (3,9), (4,16) }.

Example: Let the domain of χ be the closed interval $-2 \leq \chi \leq 2$. Assign to each value of χ the number y = χ^2.

The set of order pairs (χ, y) such that $-2 \leq \chi \leq 2$

And y = χ^2 is a function.

Note: Now can describe function by two things:

1 – the domain of the first variable χ.

2 – the rule or condition that the pairs (χ, y) must satistfy to belong to the function.

Example:

The function that pairs with each value of χ diffrent from 2 the number

$$\frac{\chi}{\chi - 2}$$

$$y = f(\chi) = \frac{\chi}{\chi - 2} \quad \chi \neq 2$$

Note 2: Let $f(\chi)$ and $g(\chi)$ be two function.

1 - $(f \pm g)(\chi) = f(\chi) \pm g(\chi)$

2 - $(f.g)(\chi) = f(\chi) \,.\, g(\chi)$

3 - $(\frac{f}{g})\,(\chi) = \frac{f(\chi)}{g(\chi)}$ if g $(\chi) \neq 0$

Example: Let $f(\chi) = \chi + 2, g(\chi) = \sqrt{\chi - 3}$ evaluate

$$f \pm g \,,\, f.g \; and \; \frac{f}{g}$$

So: $(f \pm g)(\chi) = f(\chi) \pm g(\chi) = \chi + 2 \pm (\sqrt{\chi - 3}$

$(f.g)\ (\chi) = f(\chi)\ .\ g(\chi) = (\chi + 2)\ (\sqrt{\chi - 3}$

$(\frac{f}{g})\ (\chi) = \frac{f(\chi)}{g(\chi)} = \frac{\chi + 2}{\sqrt{\chi - 3}} \quad \{X : X \rangle\ 3\}$

Composition of Function:

Let $f(\chi)$ and $g(\chi)$ be two functions

We define: $(fog)\ (\chi) = f(g(\chi))$

Example: Let $f(\chi) = \chi^2,\ g(\chi) = \chi - 7$ evaluate fog and gof

So: $(fog)\ (\chi) = f\,[g(x)] = f\,(\chi - 7) = (\chi - 7)^2$

$(gof)\ (\chi) = g[f(\chi)] = g(\chi^2) = \chi^2 - 7$

$\therefore\ fog \neq gof$

Inverse Function

Given a function F with domain A and the range B.

The inverse function of f written f, is a function with domain B and range A such that for every $y \in B$ there exists only $\chi \in A$ with $\chi = f^{-1}(y)$.

Note that: $f^{-1} \neq \frac{1}{f}$

Polynomials: A polynomial of degree n with independent variable, written $\mathbf{f_n(x)}$ or simply $f(\chi)$ is an expression of the form:

$fn(\chi) = q_o + a_1\chi + a_2X^2 + \ldots\ldots\ldots + anX^n \ldots\ldots\ldots.(*)$

Where $q_0, a1, \ldots\ldots., an$ are constant (numbers).

The degree of polynomial in equation (*) is n (the highest power of equation)

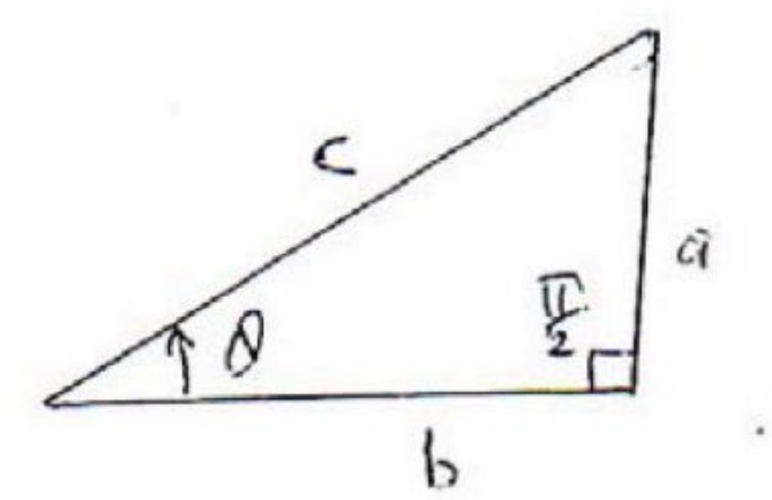
Example:

(i) f (χ) = 5X polynomial of degree one.

$2 - \text{Cos} \quad \varphi = \dfrac{b}{c}$

$3 - \tan \quad \varphi = \dfrac{a}{b}$

$4 - \text{Cotan}\ \varphi = \dfrac{1}{\tan\varphi} = \dfrac{b}{a}$

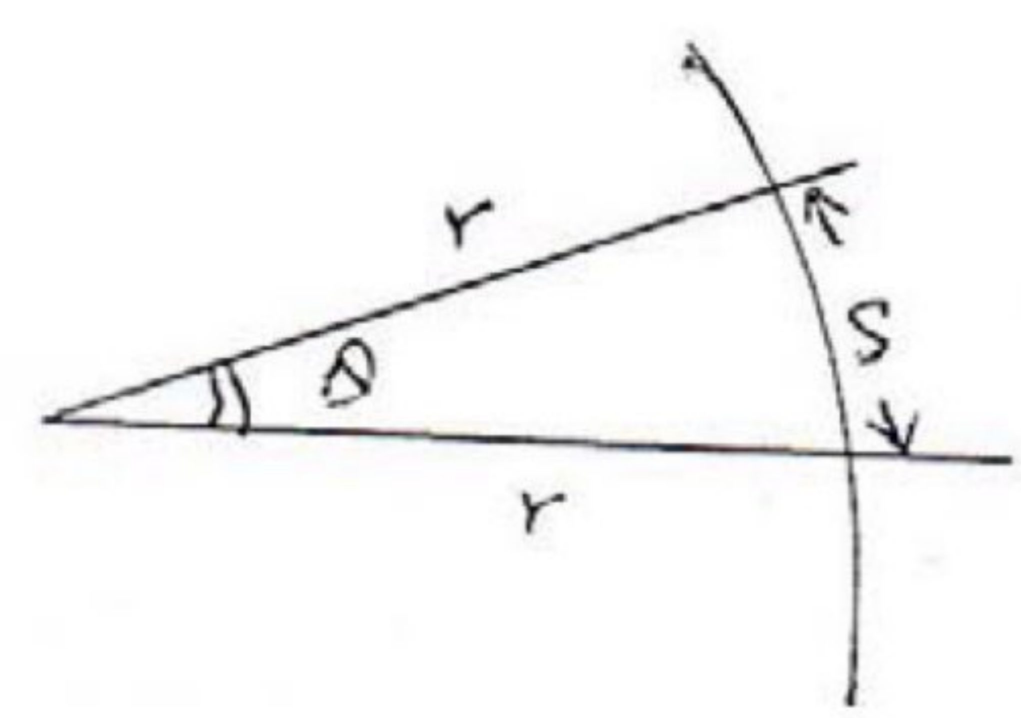


$5 - \text{Sec} \quad \varphi = \dfrac{1}{\cos\varphi} = \dfrac{c}{b}$

$6\text{- CSC} \qquad \varphi = \dfrac{1}{Sin\,\varphi} = \dfrac{c}{a}$

Relation ships between degrees and radians

φ In radius $= \dfrac{s}{r}$



$$360^{o} = \frac{2\pi r}{r} = 2\pi\, radius$$

$1^{o} = \dfrac{\pi}{180}$ radius = 0.0174 radian

1 radian $= \dfrac{180}{\pi}$ $\deg ree = 57.29578^{o}$

$\left(\dfrac{360}{2\pi}\right) = 1 radian = 57^{o}.18$

$180^{o} = \pi$ radians = 3.14159 – radians

$1^{o} = \dfrac{2\pi}{360} = \dfrac{\pi}{180} \approx 0.001754$ radians

$\tan\chi = \dfrac{\sin\chi}{\cos\chi}$

$Cot\chi = \dfrac{Cos\chi}{Sin\chi} = \dfrac{1}{\tan\chi}$