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LECTURE: (2)

**Subject:** MEASURES OF DISPERSION AND POSITION
**Level:** First
**Lecturer:** Dr. Mustafa Talal

# MEASURES OF DISPERSION AND POSITION

In statistics, to describe the data set accurately, statisticians must know more than the measures of central tendency. Two data sets with the same mean may have completely different variation or dispersion, so the measures that help us know about the spread data set are called **the measures of dispersion** such as:

1- Range.
2- Variance.
3- Standard deviation.

**1- Range:_** The range is the simplest of the three measures and is defined now. The range is the highest value minus the lowest value. The symbol R is used for the range.

**R = highest value - lowest value**

**Disadvantage of range**:

a- Based on two values only, largest and smallest.

b- Extremely large or extremely small data can significantly affect the range.

***Ex. (1):*** Calculate the range for the following data set: 5 -7 2 0 -9 16 10 7

***Sol:*** -9 -7 0 2 5 7 10 16

R = highest value - lowest value

R= 16 – (-9) = 25

**2- Variance and Standard Deviation.**

**a- Ungrouped data**

$$Population\ variance: \quad \sigma^2 = \frac{\sum(x-\mu)^2}{N}$$

$$Sample\ variance: \quad s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$$

$$Population\ standard\ deviation: \quad \sigma = \sqrt{\sigma^2}$$

$$Sample\ standard\ deviation: \quad s = \sqrt{s^2}$$

***Exp(2):_*** Find the sample variance , standard deviation and the range ,for the amount of European auto sales for a sample of 6 years shown. The data are in millions of dollars.

11.2, 11.9, 12.0, 12.8, 13.4, 14.3

***Sol:***

$1 - Variance: \qquad s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$

$\bar{x} = \sum x/n = 11.2 + 11.9 + 12.0 + 12.8 + 13.4 + 14.3/6 = 75.6/6 = 12.6$

$s^2 = (11.2-12.6)^2 + (11.9-12.6)^2 + (12.0-12.6)^2 + (12.8-12.6)^2 + (13.4-12.6)^2 + (14.3-12.6)^2 / 5 = 1.278$

2- Standard deviation:

$s = \sqrt{1.278} = 1.13$

3- The range (R) = 14.3 – 11.2 = 3.1

**b- Grouped data**

$Population\ variance: \qquad \sigma^2 = \frac{\sum f(x_m-\mu)^2}{N}$

$Sample\ variance: \qquad s^2 = \frac{\sum f(x_m-\bar{x})^2}{n-1}$

$Population\ standard\ deviation: \qquad \sigma = \sqrt{\sigma^2}$

$Sample\ standard\ deviation: \qquad s = \sqrt{s^2}$

***Ex(3):***

Find the variance and the standard deviation for the data in this frequency distribution table. The data represent the number of miles that 20 runners ran during one week.

| **Class boundaries** | 5.5–10.5 | 10.5–15.5 | 15.5-20.5 | 20.5–25.5 | 25.5–30.5 | 30.5–35.5 | 35.5-40.5 |
|---|---|---|---|---|---|---|---|
| **Freq. ($f_i$)** | 1 | 2 | 3 | 5 | 4 | 3 | 2 |

***Sol:***

| $i$ | Class boundaries | Freq. ($f_i$) | $xm$ | $xm.fi$ | $(xm - x-)2$ | $fi(xm - x-)2$ |
|---|---|---|---|---|---|---|
| 1 | 5.5–10.5 | 1 | 8 | 8 | 272.25 | 272.25 |
| 2 | 10.5–15.5 | 2 | 13 | 26 | 132.25 | 264.5 |
| 3 | 15.5–20.5 | 3 | 18 | 54 | 42.25 | 126.75 |
| 4 | 20.5–25.5 | 5 | 23 | 115 | 2.25 | 11.25 |
| 5 | 25.5–30.5 | 4 | 28 | 112 | 12.25 | 49.00 |
| 6 | 30.5–35.5 | 3 | 33 | 99 | 72.25 | 216.75 |
| 7 | 35.5–40.5 | 2 | 38 | 76 | 182.25 | 364.5 |
| | | | | ∑490 | | ∑1305 |

$$x^- = \frac{\sum f.x_m}{n} \quad , \quad x^- = \frac{490}{20} = 24.5$$

$$s^2 = \frac{\sum f(x_m - x^-)^2}{n-1} = \frac{1305}{19} = 68.68$$

s= 8.28

**<u>Coefficient of Variation</u>**

A statistic that allows you to compare standard deviations when the units are different, it denoted by $C_{Var}$, is the standard deviation divided by the mean. The result is expressed as a percentage.

For samples: $C_{Var} = (s/x^-) * 100$

For populations: $C_{Var} = (\sigma/\mu) * 100$

***Ex:*** the mean of the number of sales of cars over a 3-month period is 87, and the standard deviation is 5. The mean of the commissions is 5225 $, and the standard deviation is 773 $. Compare the variations of the two.

***Sol:***

The coefficients of variation are:

$C_{Var} = (s/x^-) * 100 = (5/87) * 100 = 5.75\%$ sales

$C_{Var} = (s/x^-) * 100 = (773/5225) * 100 = 14.8\ \%$ commissions

Since the coefficient of variation is larger for commissions, the commissions are more than the sales.