Al-Mustaqbal University
College of Sciences
Intelligent Medical Systems Department

جامـــعة المـــستـقبـل
AL MUSTAQBAL UNIVERSITY

كلية العـــــلوم
قـســـــم الامن السيبراني

LECTURE: (5)

Subject: Limits
Level: First
Lecturer: Dr. Mustafa Talal

Limits:

We say that L is a right hand limit for $f(\chi)$ when X approaches C for the right, written

$$\lim_{X \to C^{+}} f(\chi) = L \quad .$$

Similary, L is the left – hand limit for $f(\chi)$ when X approaches C for the left, written

$$\lim_{X \to C^{-}} = f(\chi) = L,$$

Then $\lim_{X \to C} = f(\chi) = L,$

If and.only if $\lim_{\chi \to C^{+}} f(X) = \lim_{\chi \to C^{-}} f(x)$

Example:

$$\lim_{\chi \to 1} \frac{\chi \pm 1}{\chi - 1} = \lim_{\chi \to 1} \frac{(\chi - 1)(\chi + 1)}{(\chi - 1)}$$

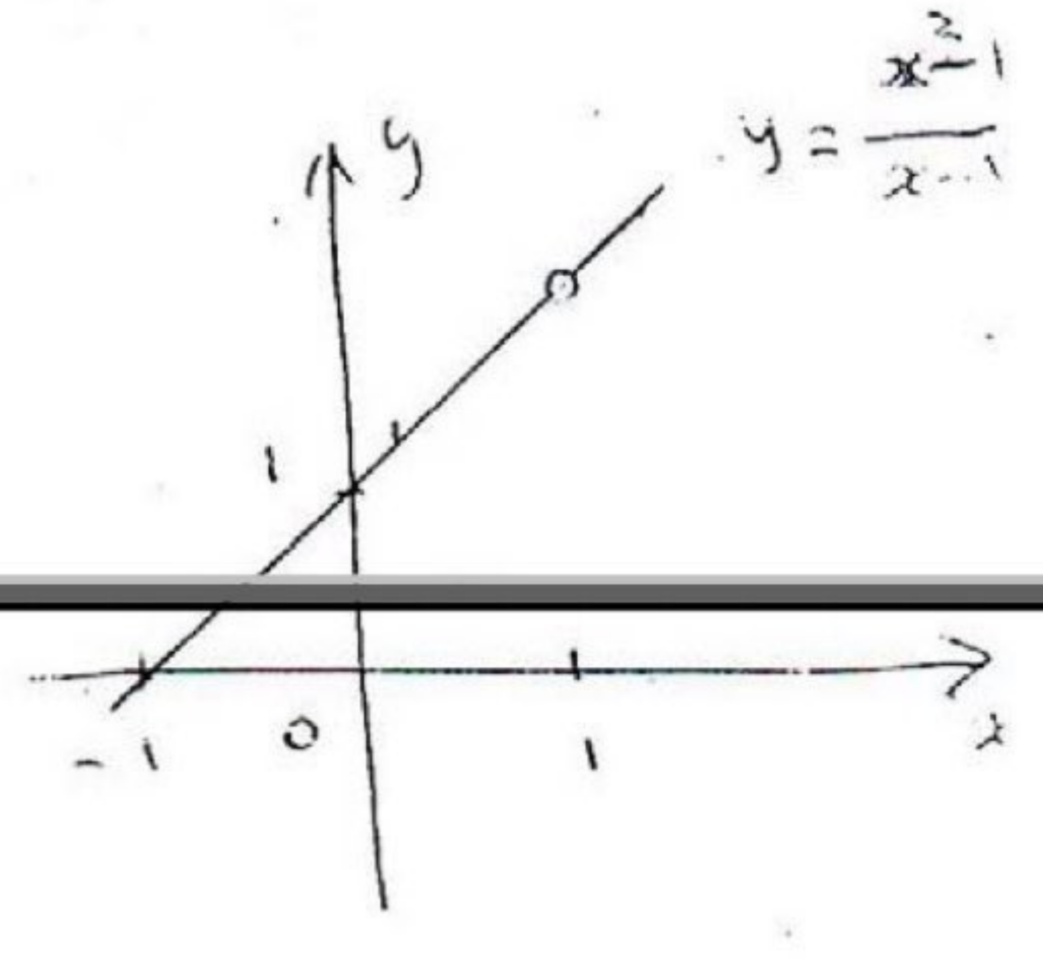
$$\lim_{\chi \to 1} \times + 1 = 1 + 1 = 2$$

Theorem 1

If $\lim_{\chi \to c} f(\chi) = L_1, \lim_{\chi \to c} g(\chi) = L_2$

Then

1 - $\lim_{\chi \to c} [f(\chi) \pm g(\chi)] = L1 \pm L2$



$2 - \text{Lim}\ [f(\chi)\, g(\chi)] = L1 \cdot L2$

$3 - Lim \left[\frac{f(\chi)}{g(\chi)}\right] = \frac{L1}{L2}\ if\ L2 \neq 0$

$4 - \underset{\chi \to c}{Lim} \left[K\, f(\chi)\right] = KL_1$ Where K is a constant

Theorem 2

$1 - Lim\, K = K$, K is constant

$2 - \underset{\chi \to c}{Lim}[a0 + a1\chi_1 + a2\chi^2 + ... + an\,\chi^n] = a_0 + a_1 c + a_2 c^2 ....anc^n$

$3 - \underset{\chi \to 0}{Lim}\, smx = 0$

$4 - \underset{\chi \to 0}{Lim}\ Cos\ \chi = 1$

$5 - \underset{\chi \to 0}{Lim} \frac{Sin\ \chi}{X} = 1$

Example: Evaluate

$1 - \underset{\chi \to 2}{Lim}(4\chi^2) = 4 \underset{\chi \to 2}{Lim}\ \chi^2 = 4(2)^2 = 16$

$2 - \underset{x \to 2}{Lim}(\chi^2 - 9) = 4 - 9 = 5$

$3 - Lim \frac{\chi^3 - 8}{\chi^2 - 4} = \frac{(\chi - 2)(\chi^2 + 2\chi + 4)}{(\chi - 2)(\chi + 2)} = \frac{\chi^2 + 2\chi + 4}{\chi + 2}$

$= \frac{12}{4} = 3$

$4 - \underset{\chi \to 0}{Lim} \frac{Sin\chi}{\chi} = 3 \underset{3\chi \to 0}{Lim} \frac{Sin\, 3\chi}{3\chi} = 3(1) = 3$

$$5 - \lim_{\chi \to 0} \frac{\tan x}{\chi} = \frac{\frac{Sin\,\chi}{Cos\,\chi}}{\chi} = \lim_{X \to 0} \left[\frac{Sin\,\chi}{Cos} . \frac{1}{\chi} \right]$$

$$= \lim_{X \to 0} \left[\frac{Sin\,\chi}{\chi} . \frac{1}{Cos\,\chi} \right]$$

$$= \left[\lim_{\chi \to 0} \frac{Sin\,\chi}{\chi} \right] . \left[\lim_{\chi \to 0} \frac{1}{Cos\,\chi} \right] = (1)\ (1) = 1$$

Infinity as Limits

Evaluate:

1 - $\lim_{X \to 0^+} \frac{1}{X} = \infty$ (2) $\lim_{\chi \to 0^-} \frac{1}{X} = -\infty$

3 - $\lim_{x \to \infty} \frac{1}{X} = 0$

4 - $\lim_{x \to \infty} \frac{2\chi 2 - \chi + 3}{3\chi^2 - 5} = \lim_{x \to \infty} \frac{\frac{2x2}{X2}}{\frac{3\chi^2}{\chi^2} - \frac{3}{X^2}}$

Theorem

If $f(\chi) \le g(\chi) \le h(X)$ and $\lim f(\chi) = \lim h(X) = L$ then

L is the limit of g(x)

Example: Evaluate

1 - $\lim_{X \to \infty} \frac{Sin\,X}{X}$

2 - $\lim_{X \to \infty} \chi \, Sin(\frac{1}{X})$

Continuity

Definition: A function f is said to be continuous at $\chi = C$ provided the following conditions are satisfied:

1 $f(C)$ is defined

2 $\lim_{\chi \to C} f(x)$ exists

3 $\lim_{x \to C} f(x) = f(C)$

Theorem

Any Polynomial

1 $P(\chi) = a_0 + a_1\chi + a_2X^2 + ... + an\,X^4$ (an $\neq 0$)

Is continuous for all χ

2 $R(\chi) = \frac{a_0 + a1X + a_2 \; X^2 + ... + anX^4}{bo + b_1 \; X + b_2 \; X^2 .... + bnX^4}$ $(an \neq 0, bn \neq 0)$

Is continuous at every point of its domain of definition that is at every point where its denominator isd not zero

3 Each of the igonometric function SinX, CosX, tanX, CotX SecX, and CscX, is continuous at every point of its domain of definition.

Example1

$\lim_{X \to \pi} (Cos^2 X + Cos\,X + 1)$

Solution

$\lim (Cos^2 X + CosX + 1) = (Cos^2 \pi + Cos\,\pi + 1)$

$= (-1)^2 - 1 + 1) = 1$

Example2

$f(\chi) = \frac{|X|}{X}$ Discontinuous at $\chi = 0$

$Lim \frac{|X|}{X} = 1$
$X \rangle 0$

$Lim \frac{|X|}{X} \frac{+1}{-1} = -1$
$\chi \langle 0$

$Lim \frac{|X|}{X}$ does not exist

$f(\chi)$ discontinuous

Example 3: check the continuity of the function $\chi = 3$

$$f(\chi) = \begin{cases} \chi - 2 & \chi \neq 3 \\ 1 & \chi = 3 \end{cases}$$

SOL

f (3) = 1

$Lim (\chi - 2) = 3 - 2 = 1$
$\chi \to 3$

$f(3) = Lim f(\chi)$
$\chi \to 3$

The function continuous at $\chi = 3$

Problems

Q1// find Domain, range and sketch each of the following:

1 - $y = \chi^2$ 2 - $y = \sqrt{X}$

3 - $y = 1\chi 1$ 4- $y = |\chi + 2|$

5 - $y = \frac{|\chi|}{\chi}$ 6 - $y = \frac{1}{\chi}$

7 - $y = \frac{\chi + 1}{\chi - 1}$

8 - $y = 2\,Sin\,\chi$

9 - $y = -2\,Sin\,\chi$

10 - $y = 2 + Cos\chi$

Q2 // Evaluate each of the following limits:

1 - $\lim_{y \to 2} \frac{t+3}{t+2}$

2 - $\lim_{\chi \to 1} \frac{\chi^2 - 1}{\chi - 1}$

3 - $\lim_{y \to 2} \frac{y2 + 5y + b}{y+2}$

4 - $\lim_{y \to 2} \frac{y^2 - 5y + 6}{y - 2}$

5 - $\lim_{\chi \to -3} \frac{\chi 2 + 4\chi + 3}{\chi + 3}$

6 - $\lim_{t \to \infty} \frac{t+1}{t^2 + 1}$

7 - $\lim_{t \to \infty} \frac{t^2 - 2t}{2t^2 + 5t - 3}$

8 - $\lim_{\theta \to 0} \frac{\tan\theta}{\theta}$

9 - $\lim_{\theta \to 0} \frac{Sin\,2\theta}{\theta}$

10- $\lim_{\chi \to 0} \frac{Sin\chi}{3\chi}$

11 - $\lim_{\chi \to 0} \frac{Sin\,5\,\chi}{Sin\,3\,\chi}$

12 - $\lim_{\chi \to \infty} \chi\,Sin\frac{1}{\chi}$

13 - $\lim_{\chi \to 0} \frac{Sin^2\chi}{\chi}$

14 - $\lim_{\chi \to 0} \frac{Sin^2\chi}{2\chi^2 + \chi}$

15 - $\lim_{\chi \to 0} \tan 2\,\chi\,Csc\,4\,\chi$