Digital Logic

Numbers systems:

	Base	Probabilities	Range
1. Decimal Numbers	10	10	0 - 9
2. Binary Numbers	2	2	0 - 1
3. Octal Numbers	8	8	0 - 7
4. Hexadecimal Numbers	16	16	0 - F

1. Decimal Numbers

Decimal number system has ten symbols for each digit: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. it uses *positional notation*. That is, the least-significant digit (right-most digit) is of the order of 10^0 (units or ones), the second right-most digit is of the order of 10^1 (tens), the third right-most digit is of the order of 10^2 (hundreds), and so on, where ^ denotes exponent. For example $(23)_{10}$

$$735 = 700 + 30 + 5 = 7\times10^2 + 3\times10^1 + 5\times10^0$$

The position of each digit in a decimal number indicates the magnitude of the quantity represented and can be assigned a weight. The weights for whole numbers are positive powers of ten that increase from right to left, beginning with $10^0 = 1$.

$$\ldots\ 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0$$

For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1}.

$$10^2\ 10^1\ 10^0.10^{-1}\ 10^{-2}\ 10^{-3}\ \ldots$$

Decimal point

EXAMPLE 2–1

Express the decimal number 47 as a sum of the values of each digit.

Solution

The digit 4 has a weight of 10, which is 10^1, as indicated by its position. The digit 7 has a weight of 1, which is 10^0, as indicated by its position.

$$47 = (4 \times 10^1) + (7 \times 10^0)$$
$$= (4 \times 10) + (7 \times 1) = \mathbf{40 + 7}$$

Related Problem*

Determine the value of each digit in 939.

*Answers are at the end of the chapter.

EXAMPLE 2–2

Express the decimal number 568.23 as a sum of the values of each digit.

Solution

The whole number digit 5 has a weight of 100, which is 10^2, the digit 6 has a weight of 10, which is 10^1, the digit 8 has a weight of 1, which is 10^0, the fractional digit 2 has a weight of 0.1, which is 10^{-1}, and the fractional digit 3 has a weight of 0.01, which is 10^{-2}.

$$568.23 = (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2})$$
$$= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01)$$
$$= \mathbf{500 + 60 + 8 + 0.2 + 0.03}$$

Related Problem

Determine the value of each digit in 67.924.

2. Binary Numbers

The binary number system is another way to represent quantities. It is less complicated than the decimal system because the binary system has only two values in each digit. The decimal system with its ten values is a base-ten system; the binary system with its two values is a base-two system.

The two binary values in each binary digit (bits) are 0 and 1. The position of 0 or 1 in a binary number indicates its weight, or value within the number, just as

the position of a decimal digit determines the value of that digit. The weights in a binary number are based on powers of two. The weight structure of a binary number is

$$2^{n-1} \ldots 2^3\, 2^2\, 2^1\, 2^0 \,.\, 2^{-1}\, 2^{-2} \ldots 2^{-n}$$

Binary point

Binary weights.

Positive Powers of Two (Whole Numbers)									Negative Powers of Two (Fractional Number)					
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
256	128	64	32	16	8	4	2	1	1/2 0.5	1/4 0.25	1/8 0.125	1/16 0.625	1/32 0.03125	1/64 0.015625

3. Octal Numbers

The octal number system is composed of eight values, which are

0, 1, 2, 3, 4, 5, 6, 7

4. Hexadecimal Numbers

The hexadecimal number system has sixteen characters

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Where the character A=10, B=11, C=12, D=13, E=14, F=15 in decimal number

Important Table:

Binary b3	b2	b1	b0	Decimal	Octal	Hex
0	0	0	0	0	0	0
0	0	0	1	1	1	1
0	0	1	0	2	2	2
0	0	1	1	3	3	3
0	1	0	0	4	4	4
0	1	0	1	5	5	5
0	1	1	0	6	6	6
0	1	1	1	7	7	7
1	0	0	0	8	10	8
1	0	0	1	9	11	9
1	0	1	0	10	12	A
1	0	1	1	11	13	B
1	1	0	0	12	14	C
1	1	0	1	13	15	D
1	1	1	0	14	16	E
1	1	1	1	15	17	F

Numbers base Conversion:

1. Decimal to Binary conversion:

$(18.75)_{10}$

÷2 *2

		Reminder	
2	18		
2	9	0	LSB
2	4	1	
2	2	0	
2	1	0	
	0	1	MSB

0.75 * 2 = 1.5

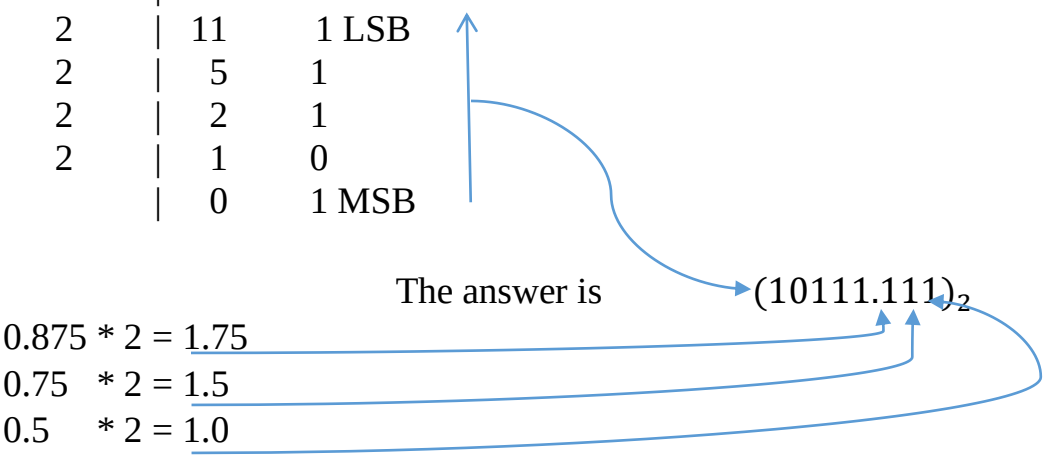
0.5 * 2 = 1.0

The answer is $(10010.11)_2$

$(23.875)_{10}$

÷2 *2

2	23	Reminder
2	11	1 LSB
2	5	1
2	2	1
2	1	0
	0	1 MSB



0.875 * 2 = 1.75

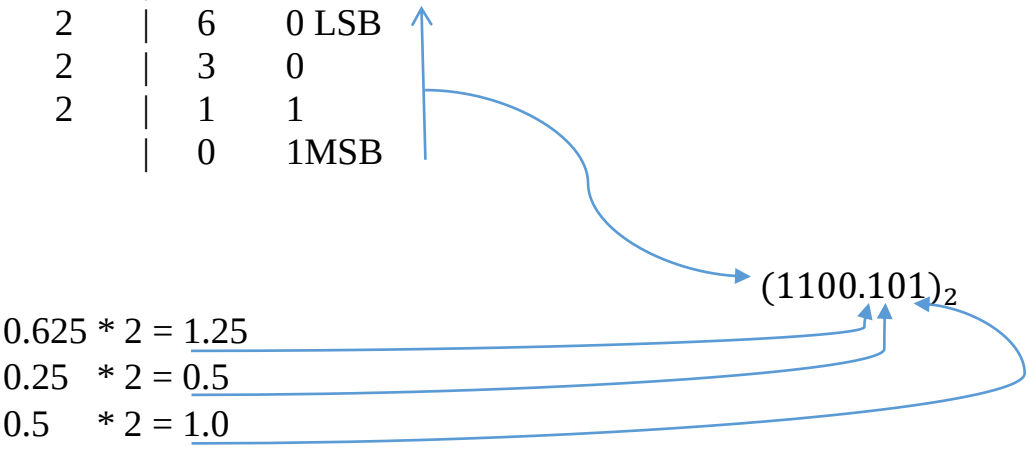
0.75 * 2 = 1.5

0.5 * 2 = 1.0

$(12.625)_{10}$

÷2 *2

2	12	Reminder
2	6	0 LSB
2	3	0
2	1	1
	0	1MSB



0.625 * 2 = 1.25

0.25 * 2 = 0.5

0.5 * 2 = 1.0

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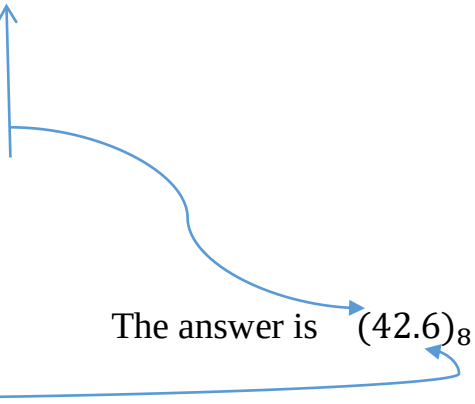
2. Decimal to Octal conversion:

$(34.75)_{10}$

÷8 *8

8	|	34	Reminder
8	|	4	2
	|	0	4



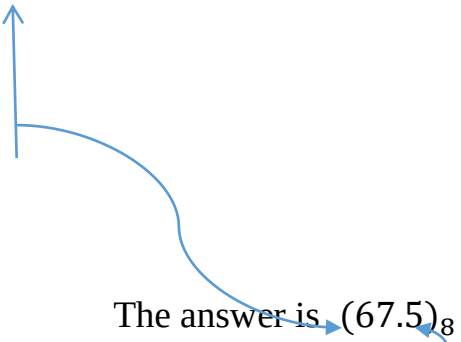
The answer is $(42.6)_8$

0. 75 * 8 = 6.0

$(55.625)_{10}$

÷8 *8

8	|	55	Reminder
8	|	6	7
	|	0	6



The answer is $(67.5)_8$

0. 625 * 8 = 5.0

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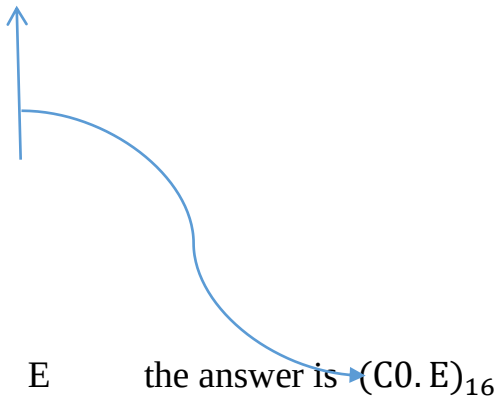
3. Decimal to Hex conversion:

$(192.875)_{10}$

÷16 *16

16	192	Reminder	
16	12	0	
	0	12 ⇨	C

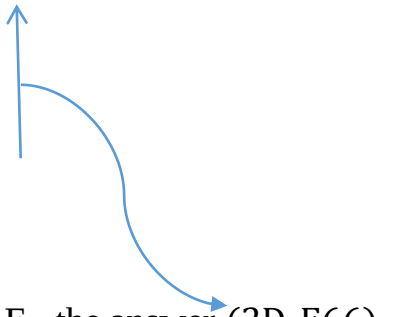


0. 875 * 16 = 14.0 ⇨ E the answer is $(C0.E)_{16}$

$(45.9)_{10}$

÷16 *16

16	45	Reminder	
16	2	13 ⇨	D
	0	2	



14 ⇨ E the answer $(2D.E66)_{16}$

0.9 * 16 = 14.4

0.4 * 16 = 6.4

0.4 * 16 = 6.4

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4. Binary to Decimal conversion:

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2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}

128 64 32 16 8 4 2 1 . 1/ 2 1/ 4 1/ 8 1/16 1/ 32 1/64 1/ 128

128 64 32 16 8 4 2 1 . 0.5 0.25 0.125 0.0625

Ex: $(1011.101)_2$ $(?)_{10}$

8	4	2	1	0.5	0.25	0.125
1	0	1	1	1	0	1

No. = 1*8 + 0*4 + 1*2 +1*1 + 1*0.5 + 0*0.25 + 1*0.125
=$(11.625)_{10}$

Ex: $(11101.1101)_2$ $(?)_{10}$

16	8	4	2	1	0.5	0.25	0.125	0.0625
1	1	1	0	1	1	1	0	1

No. = 1*16 + 1*8 + 1*4 + 0*2 +1*1 + 1*0.5 + 1*0.25 + 0*0.125 +1*0.0625
=$(29.8125)_{10}$

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5. Octal to Decimal conversion:

$8^3 \quad 8^2 \quad 8^3 \quad 8^1 \quad 8^0 \quad 8^{-1} \quad 8^{-2} \quad 8^{-3}$

$(75.6)_8$

$8^1 \quad 8^0 \quad 8^{-1}$

N0. = $7*8^1 + 5*8^0 + 6*8^{-1}$ → =56 + 5 + 0.75

The answer is =$(61.75)_{10}$

Ex:

$(\ 36\ .7\ )_8$

$8^1 \quad 8^0 \quad 8^{-1}$

N0. = $3*8^1 + 6*8^0 + 7*8^{-1}$ → =24 + 6 + 0.875

The answer is = $(\ 30\ .875\ )_{10}$

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6. Hexadecimal to Decimal conversions:

$16^4 \quad 16^3 \quad 16^2 \quad 16^1 \quad 16^0 . \ 16^{-1} \quad 16^{-2} \quad 16^{-3}$

Ex: $(\ C3\ .C\ )_{16}$

$16^1 \quad 16^0 \quad 16^{-1}$

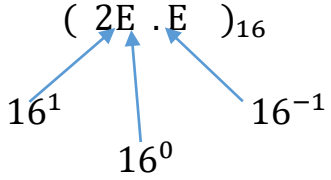
N0. = $C*16^1 + 3*16^0 + C*16^{-1}$

=12 * 16 + 3*1 + 12*1/16

The answer is = $(\ 195.75 \ )_{10}$

Ex:

$(\ 2E \ . E \ )_{16}$



N0. = $2*16^1 + E*16^0 + E*16^{-1}$ ⟶ $=2*16 + 14*1 + 14*1/16$

$= 32 + 14 + 0.875$

The answer is = $(\ 46.875 \ )_{10}$

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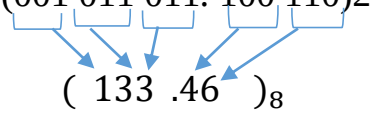
7. **Binary to Octal conversions**:

Binary ⟶ Decimal ⟶ Octal

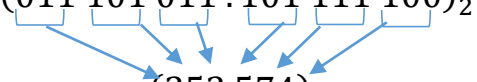
Or Binary ⟶ Octal

Ex: (001 011 011. 100 110)2



$(\ 133 \ .46 \ )_8$

Ex: $(011\ 101\ 011\ .101\ 111\ 100)_2$



$(353.574)_8$

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8. Octal to Binary conversion:

Octal → Decimal → Binary

Or Octal → Binary

$(365.72)_8$

$(011\ 110\ 101\ .\ 111\ 010)_2$

Ex:

$(62.10)_8$

$(110\ 010\ .001\ 000)_2$

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9. Binary to Hexa conversions:

Binary → Decimal → Hex

Or Binary → Hex

Ex:

$(1011\ 1011\ .1011\ 1110)_2$

$(BB.BE)_{16}$

Ex:

$(0010\ 0111\ 0111\ .\ 1000\ 1100\ 1000)_2$

$(277.8C8)_{16}$

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10. Hexa to Binary conversion:

Hexa → Decimal → Binary

Or Hexa → Binary

Ex: $(FC.ED)_{16}$

$(1111\ 1100\ .1110\ 1101)_2$

Ex:

$(C2.AB)_{16}$

$(1100\ 0010\ .1010\ 1011)_2$

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11. Octal to Hexa conversions:

Octal ⟶ Decimal ⟶ Hexa

Or Octal ⟶ Binary ⟶ Hexa

Ex:

Oct $(362.576)_8$

Bin $(011\ 110\ 010\ .101\ 111\ 110)_2$

$(01111\ 0010\ .1011\ 1111\ 0)_2$

Hex $(F\ 2\ .\ B\ F)_{16}$

Ex:

Oct $(1\ 2\ 0\ .\ 6\ 5\ 3)_8$

Bin $(001\ 010\ 000\ .110\ 101\ 011)_2 000$

Hex $(5\ 0\ .\ D\ 5\ 8)_{16}$

12.Hexa to Octal conversions:

Hexa → Decimal → Octal
Or Hexa → Binary → Octal

Ex: $(CF\ .\ E\ A\ B)_{16}$ $(\ ?\ )_8$

Hex $(CF\ .\ E\ A\ B)_{16}$

Bin $(1100\ 1111\ .\ 1110\ 1010\ 1011)_2$

$0\ (11\ 001\ 111\ .\ 111\ 010\ 101\ 011)_2$

Oct $(3\ 1\ 7\ .\ 7\ 2\ 5\ 3)_8$

Ex: $(AB\ .\ CD)_{16}$ $(?)_8$

Hex $(AB\ .\ CD)_{16}$

Bin $(1010\ 1011\ .\ 1100\ 1101)_2$

$0\ (10\ 101\ 011\ .\ 110\ 011\ 010\ )_2$

Oct $(2\ 5\ 3\ .\ 6\ 3\ 2)_8$