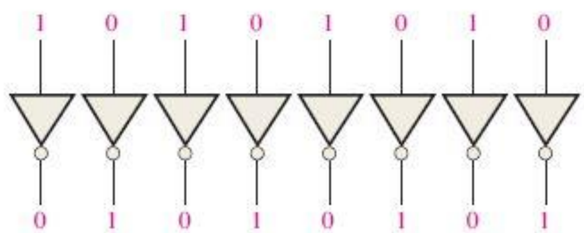
One's complement and two's complement:

Finding the 1's Complement

The 1's **complement** of a binary number is found by changing all 1s to 0s and all 0s to 1s, as illustrated below:

1 0 1 1 0 0 1 0	Binary number
↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓	
0 1 0 0 1 1 0 1	1's complement

The simplest way to obtain the 1's complement of a binary number with a digital circuit is to use parallel inverters (NOT circuits), as shown in Figure 2–2 for an 8-bit binary number.



Example of inverters used to obtain the 1's complement of a binary number.

1. 1's complement $0 \rightarrow 1$, $1 \rightarrow 0$

$1011101_2 \longrightarrow 0100010_2$ 1's complement

2. 2's complement = 1's complement +1

$1101010_2 \longrightarrow 0010101_2$

+ 1

0010110 2's complement

Note: 1's complement and 2's complement is use to represent negative number.

Why 1's complement and 2's complement?

All subtractions inside the CPU (Center Processing Unit) is done using complement.

b2	b1	b0	Unsigned	1’s complement	2’s complement
0	0	0	0	+0	0
0	0	1	1	+1	+1
0	1	0	2	+2	+2
0	1	1	3	+3	+3
1	0	0	4	-3	-4
1	0	1	5	-2	-3
1	1	0	6	-1	-2
1	1	1	7	-0	-1

Example1:

(010) unsigned =2

(010)1’s complement =+2   +ve.

(010)2’s complement =+2   +ve.

Example2:

(101) unsigned =5

(010)1’s complement = -2   - ve.

(011)2’s complement = -3   - ve.

Second: Subtraction using 1’s complement:

Example:

```
   00101111        47
 - 00011101     -  29
 ----------     -----
   00010010        18


   00101111
 + 11100010
 ----------
  100010001  means that the result is +ve.
  +       1
 ----------
 + 00010010
```

Third: subtraction in 2's complement

```
  00101111
+ 11100011
----------
 100010010  means that the result is +ve.
  00010010
```

C. Subtraction in hexadecimal system:

First: normal subtraction:

n1>n2, result is positive

```
 2F
-1D
----
+12
```

n1<n2, result is negative

```
 1D
-2F
----
-12
```

Second: Subtraction using 1's complement:

```
 2F
+E2
----
111    means that the result is +ve.
  1+
----
+12
```

```
 1D
+D0
----
 ED    -ve.
-12
```

Third: subtraction in 2's complement:

```
 2F
+E3
----
112    +ve.

+ 12
```

```
 1D
+D1
----
 EE

-12
```

Important note: most computers uses subtraction in 2's complement.

3. **Multiplication in Binary system**:

Example1:

```
   1011                    11     decimal
 × 1001                 ×   9     decimal
  ------                 ----
    1011                   99
   0000
  0000
 1011
 -------
 1100011         result in Dec. = 64+ 32 + 2+ 1
                                = 99
```

Example2:

```
     11.1      One digit after.      3.5
  × 101.01     Two digit after.    × 5.25
  --------                         ------
        111                        18.375
       000
      111
     000
    111
  ---------
   10010.011   Three digit after.
```

---

4. Division in Binary system:

Example1:

```
                        1011  ───────►  ($11_{10}$)
              ┌──────────────
   1001       │  1100011                        99/9=11
            -    1001
                 ------
                 001101
            -      1001
                   ------
                   01001
             -      1001
                    -----
                    0000
```

Example2:

```
101.010 | 10010. 0110          18.375/ 5.25=3.5

                11.1
101010 | 10010011
       -  101010
         ---------
          0111111
       -   101010
         ---------
           0101010
       -    101010
         ---------
            000000
```

Example2:

```
11. 1 | 10010.011          18.375/ 3.5=5.25

          101. 01
111   | 100100.11
         111
        --------
          1000
     -     111
        --------
           000111
              111
        --------
              000
```

Example:

A B C D	y0	y1	y2	y3	y4	y5	y6	y7
0 0 0 0	0	0	0	0	1	1	0	1
0 0 0 1	0	0	1	1	1	0	1	0
0 0 1 0	0	0	0	1	0	0	1	0
0 0 1 1	0	0	1	1	0	0	0	1
0 1 0 0	1	1	0	0	1	1	1	0
0 1 0 1	1	1	1	1	1	0	1	1
0 1 1 0	1	1	0	1	0	0	0	1
0 1 1 1	1	1	1	1	0	0	0	0
1 0 0 0	0	0	0	0	1	1	1	0
1 0 0 1	0	0	1	1	1	0	0	1
1 0 1 0	0	0	0	1	0	0	0	1
1 0 1 1	0	0	1	1	0	0	1	0
1 1 0 0	1	0	0	0	1	1	0	1
1 1 0 1	1	0	1	1	1	0	1	0
1 1 1 0	1	1	0	1	0	0	1	0
1 1 1 1	1	1	1	1	0	0	0	1

$y0 = \sum m\,(4, 5, 6, 7, 12, 13, 14, 15)$

$\overline{y0} = \Pi M\,(0, 1, 2, 3, 8, 9, 10, 11)$

$= \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

y0 =B

B ———————————— y0

$y1 = \sum m\,(4, 5, 6, 7, 14, 15)$

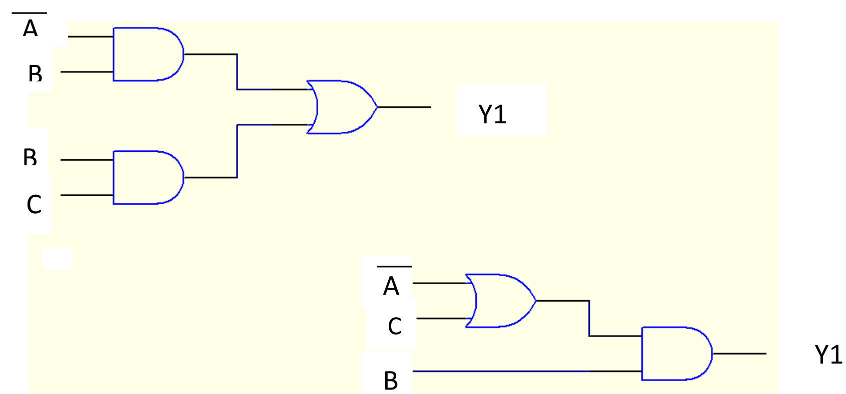
$\overline{y1} = \Pi M\,(0, 1, 2, 3, 8, 9, 10, 11, 12, 13)$

$= \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + ABC\bar{D} + ABCD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	0	0	1	1
$A\bar{B}$	0	0	0	0

$y1 = \bar{A}B + BC$

$= B(\bar{A} + C)$



$y2 = \sum m\,(1, 3, 5, 7, 9, 11, 13, 15)$

$\overline{y2} = \Pi M\,(0, 2, 4, 6, 8, 10, 12, 14)$

$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}CD + AB\bar{C}D + ABCD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	1	0
$\bar{A}B$	0	1	1	0
AB	0	1	1	0
$A\bar{B}$	0	1	1	0

D ——————— Y

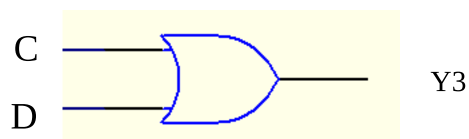
$y3 = \sum m\ (1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15)$

$\overline{y3} = \Pi M\ (0, 4, 8, 12)$

$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}D +$
$A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}D + ABC\bar{D} + ABCD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	1	1
$\bar{A}B$	0	1	1	1
AB	0	1	1	1
$A\bar{B}$	0	1	1	1

$y3 = C + D$



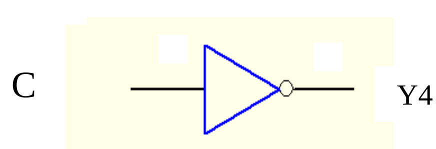
$y4 = \sum m\ (0, 1, 4, 5, 8, 9, 12, 13)$

$\overline{y4} = \Pi M\ (2, 3, 6, 7, 10, 11, 14, 15)$

$= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	1	1	0	0
AB	1	1	0	0
$A\bar{B}$	1	1	0	0

$y4 = \bar{C}$



$y5 = \sum m\,(0, 4, 8, 12)$

$\overline{y5} = \Pi M\,(1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15)$

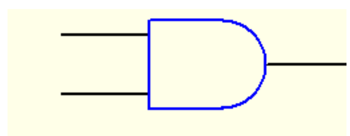
$= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D}$

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	0	0	0
$\overline{A}B$	1	0	0	0
AB	1	0	0	0
$A\overline{B}$	1	0	0	0

$y5 = \overline{C}\,\overline{D}$

$\overline{C}$

$\overline{D}$



Y5

$y6 = \sum m\,(1, 2, 4, 7, 8, 11, 13, 14)$

$\overline{y6} = \Pi M\,(0, 3, 5, 6, 9, 10, 12, 15)$

$= \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\overline{D} + \overline{A}B\overline{C}\,\overline{D} + \overline{A}BCD + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD + AB\overline{C}D + ABC\overline{D}$

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	0	1	0	1
$\overline{A}B$	1	0	1	0
AB	0	1	0	1
$A\overline{B}$	1	0	1	0

$y6 = \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\overline{D} + \overline{A}B\overline{C}\,\overline{D} + \overline{A}BCD +$

$= \overline{A}\,\overline{B}\,(\overline{C}D + C\overline{D}) + \overline{A}B\,(\overline{C}\,\overline{D} + CD)$

$= \overline{A}\,\overline{B}\,(C \oplus D) + \overline{A}B\,(\overline{C \oplus D})$ let $(C \oplus D) = Z$

$= \overline{A}\,\overline{B}Z + \overline{A}B\overline{Z}$

$= \overline{A}\,(\overline{B}Z + B\overline{Z})$

$= \overline{A}\,(B \oplus Z)$ let $(B \oplus Z) = W$

$= \overline{A}W + AB\overline{C}D + ABC\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + A\overline{B}CD$

$= \overline{A}W + AB\,(\overline{C}D + C\overline{D}) + A\overline{B}\,(\overline{C}\,\overline{D} + CD)$

$= \overline{A}W + AB\,(C \oplus D) + A\overline{B}\,\overline{(C \oplus D)}$ let $(C \oplus D) = Z$

$= \overline{A}W + ABZ + A\overline{B}\,\overline{Z}$

$= \overline{A}W + A\,(BZ + \overline{B}\,\overline{Z})$

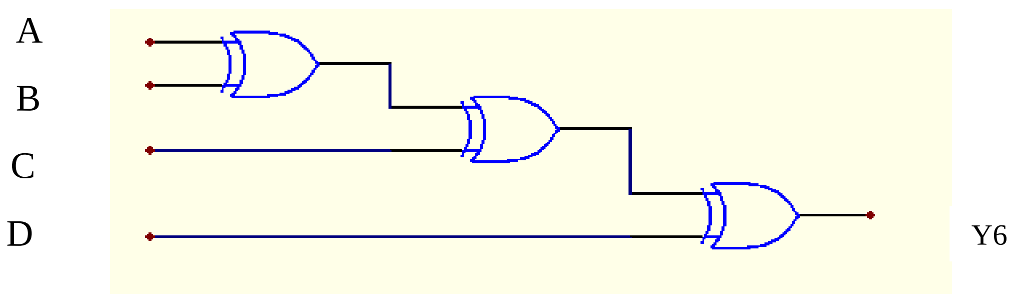
$= \overline{A}W + A\,\overline{(B \oplus Z)}$ let $(B \oplus Z) = W$

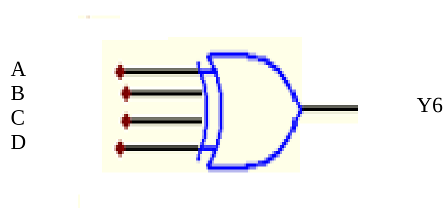
$= \overline{A}W + A\overline{W}$ $W = (B \oplus Z)$

$= A \oplus W$

$= A \oplus B \oplus Z$

$y6 = A \oplus B \oplus C \oplus D$





$Y7 = \sum m\,(0, 3, 5, 6, 9, 10, 12, 15)$

$\overline{y7} = \Pi M\,(1, 2, 4, 7, 8, 11, 13, 14)$

$= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}D + A\overline{B}C\overline{D} + AB\overline{C}\,\overline{D} + ABCD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	0
$\bar{A}B$	0	1	0	1
AB	1	0	1	0
$A\bar{B}$	0	1	0	1

$y7 = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + \bar{A}BC\bar{D} +$

$= \bar{A}\bar{B}\,(\bar{C}\bar{D} + CD) + \bar{A}B\,(\bar{C}D + C\bar{D})$

$= \bar{A}\bar{B}\,(\overline{C \oplus D}) + \bar{A}B\,(C \oplus D)$ let $(C \oplus D) = Z$

$= \bar{A}\bar{B}\bar{Z} + \bar{A}BZ$

$= \bar{A}\,(\bar{B}\bar{Z} + BZ)$ let $(B \oplus Z) = W$

$= \bar{A}\,(\overline{B \oplus Z})$

$= \bar{A}\bar{W} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}D + ABCD + A\bar{B}C\bar{D}$

$= \bar{A}\bar{W} + AB\,(\bar{C}\bar{D}+CD) + A\bar{B}\,(\bar{C}D+C\bar{D})$

$= \bar{A}\bar{W} + AB\,(\overline{C \oplus D}) + A\bar{B}\,(C \oplus D)$ let $(C \oplus D) = Z$

$= \bar{A}\bar{W} + AB\bar{Z} + A\bar{B}Z$

$= \bar{A}\bar{W} + A\,(B\bar{Z} + \bar{B}Z)$

$= \bar{A}\bar{W} + A\,(B \oplus Z)$ let $(B \oplus Z) = W$

$= \bar{A}\bar{W} + AW$

$= \overline{A \oplus W}$

$= \overline{A \oplus B \oplus Z}$ $W = (B \oplus Z)$

$Y7 = \overline{A \oplus B \oplus C \oplus D}$ $Z = (C \oplus D)$