Ministry of Higher Education and Scientific Research

Al-Mustaqbal University College Department of Chemical Engineering and petroleum Industrials

Mathematics II

2^{nd} Stage

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1. **Double integral**

The definite integral can be extended to functions of more than one variable. Consider, for example, a function of two variables z = f (x, y). The double integral of function f (x, y) is denoted by

$$\iint F(x,y)$$

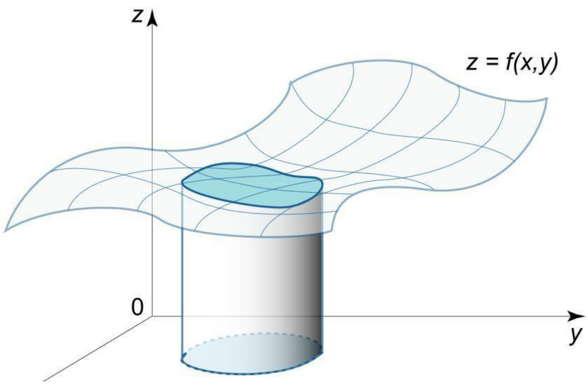


Figure 1

Where R is the region of integration in the xy-plane.

The definite integral $\int_a^b f(x)dx$ of a function of one variable $f(x) \geq 0$ is the area under the curve f(x) from x=a to x=b, then the double integral is equal to the volume under the surface z=f (x, y) and above the xy-plane in the region of integration R (Figure 1).

a- **Properties of double integral**

If $f(x, y)$ and $g(x, y)$ are continuous on the bounded region R, then the following properties hold.

1. *Constant Multiple:* $\iint_R cf(x, y)\, dA = c \iint_R f(x, y)\, dA$ (any number c)

2. *Sum and Difference:*

$$\iint_R (f(x, y) \pm g(x, y))\, dA = \iint_R f(x, y)\, dA \pm \iint_R g(x, y)\, dA$$

3. *Domination:*

(a) $\iint_R f(x, y)\, dA \geq 0$ if $f(x, y) \geq 0$ on R

(b) $\iint_R f(x, y)\, dA \geq \iint_R g(x, y)\, dA$ if $f(x, y) \geq g(x, y)$ on R

4. *Additivity:* $\iint_R f(x, y)\, dA = \iint_{R_1} f(x, y)\, dA + \iint_{R_2} f(x, y)\, dA$

if R is the union of two nonoverlapping regions R_1 and R_2

b- **Cartesian form**

Double integral of f(x,y) over the region R is denoted by:

$$\iint_R F(x,y)dA = \iint_R F(x,y)\,dx\,dy = \int_c^d \int_{x1}^{x2} F(x,y)\,dx\,dy$$

or

$$\iint_R F(x,y)dA = \iint_R F(x,y)\,dy\,dx = \int_a^b \int_{y1}^{y2} F(x,y)\,dy\,dx$$

C- <u>Finding Limits of Integration in Cartesian form</u>

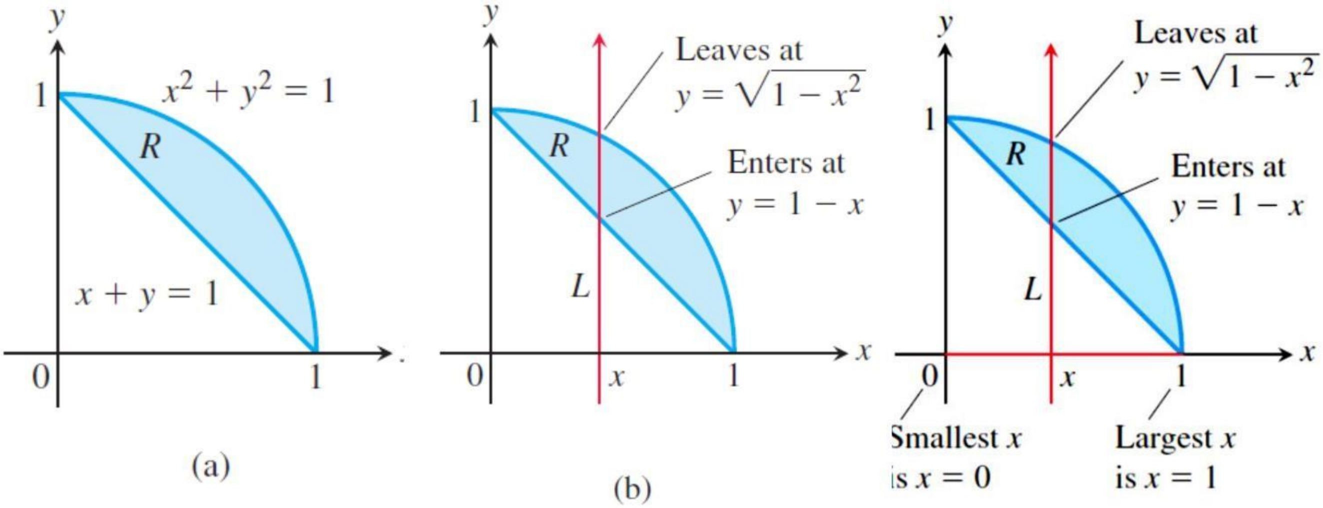
Using Vertical Cross-Sections

When faced with evaluating ∬ ( ) , integrating first with respect to *y* and then with respect to *x*, do the following three steps: 1- Sketch. Sketch the region of integration and label the bounding curves. (Figure 3 a).

2- Find the y-limits of integration. Imagine a vertical line L cutting through R in the direction of increasing y. Mark the y-values where L enters and leaves. These are the y-limits of integration and are usually functions of x (instead of constants) (Figure 3 b).

3- Find the x-limits of integration. Choose x-limits that include all the vertical lines through R. The integral shown here (see Figure 3 c) is

$$\iint_R f(x,y)\,dA = \int_{x=0}^{x=1}\int_{y=1-x}^{y=\sqrt{1-x^2}} f(x,y)\,dy\,dx.$$



Using Horizontal Cross-Sections

To evaluate the same double integral as an iterated integral with the order of integration reversed, use horizontal lines instead of vertical lines in Steps 2 and 3 (see Figure 4). The integra

$$\iint_R f(x,y)\,dA = \int_0^1\int_{1-y}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy.$$