# Lecture#1

# Introduction to Information Theory

## 1-Modeling of Information Transmission System

Information transmission system is a system that represents the main building blocks to convey information from one side (source) to another (destination). When this system represent point to point transmission it is called communication system. On the other hand, the information transmission system may represent multipoint to multipoint transmission in the form of shared channel which represent a network connectivity such as Internet.

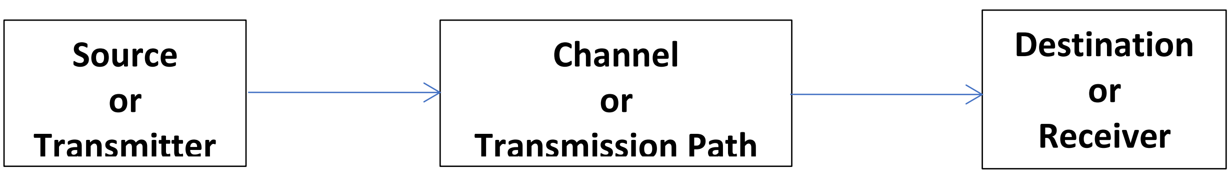
It is important to understand the definition of the following terms;

**Signal:** Signal is the actual entity that is transmitted from transmitter to receiver. It may be discrete (digital) or analog. Usually electrical or electromagnetic signal. (Example: AM, FM, PSK …..)

**Data:** Data denotes the information conveyed in the signal. Data is being carried by the signal. But data may also be stored. Usually binary data, letters, pixel value. Also called the message. (Examples: Text, Music sound, Image, movie, colors)

**Information:** It is the knowledge that is communicated by the data. Receiving information may be discovered by monitoring human impressions or sensory reaction. (Examples: the meaning of the text, song, image)

A simplified information transmission system is given below;




Usually Low Frequency Message

- Text
- Speech
- Music (Audio)
- Image
- Movie (Video)
- Row data
- Computer file

Defined by frequency and the physical media (contamination by noise and distortion)

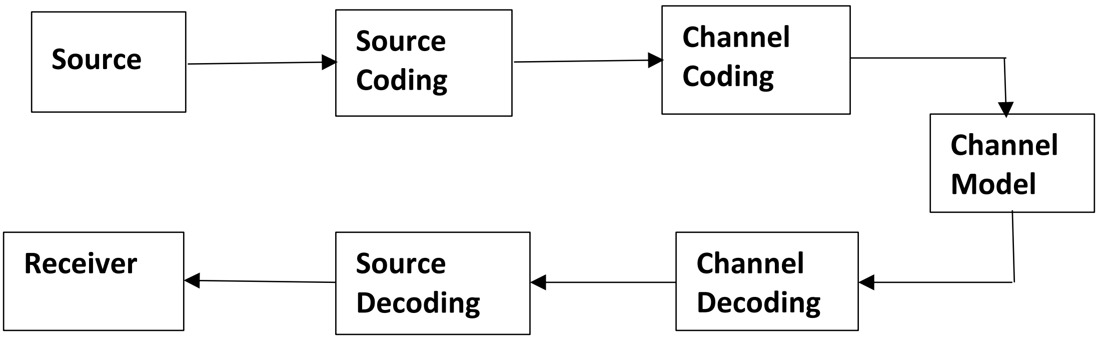
-Wired:
- 2-Wire TL (f < 1 GHz)
- Coaxial Cable ( f < 3 GHz)
- Waveguide (3GHz < f < 24GHz)
- Optical Fiber (f>30 GHz)

-Wireless:
- UHF (300-3000MHz)
- X-Band(8-12 GHz)
- ………etc

Try its best to extract the Message signal

The subject of Information Theory and Coding deals with the mathematical representation of a system that transmit information from one side to another. It also covered the quantitively measure of information and the efficiency of the system main parts. To improve the information transmission source output symbols may be encoded to ease the transmission over the given channel. Error control coding also included in the information transmission system to reduce the error in transmission and hence improve its quality.

The following describe all information transmission system components that will be covered by the subject.

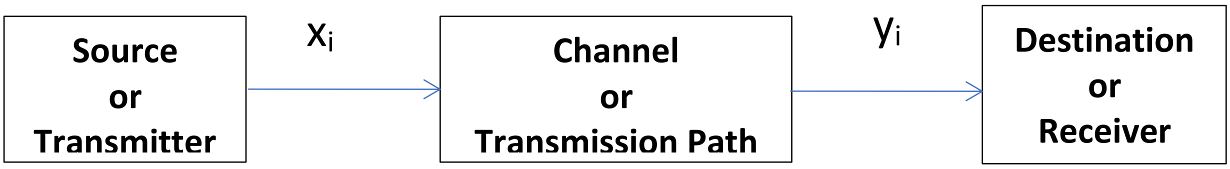



The subject will cover the study of source types (discreet and continues), channel types, and the possible coding required to improve the performance and quality.

Since the information content of a message is function of its probability, the subject will study the whole system in statistical sense.

## 2- Measuring of Information

Consider the block diagram of information transmission system model.




$x_i$ = the transmitted symbol $y_i$ = the received symbol

The source has N symbols { $x_i$ } and the receiver is able to receive M symbols { $y_i$ }.

We shall study in this lecture the information content of { $x_i$ } and the corresponding information received by the receiver.

The information carried by each symbol $x_i$ is called the self-information of $x_i$ denoted by $I_{xi}$. Consider now $x_i$ as an event with associated information of $I_{xi}$ and we need to make some measure of this information. As a common sense for information measure, one may say that the event with high probability has little information and vice versa. Thus, if $p(x_i)$ is the probability of the symbol $x_i$ then one can write:

$I_{x_i} = f\ (p(x_i))$ where $f$(.) is an inverse function.

Consider next the occurrence of two independent events $x_i$ and $x_n$ at the same time. Then the self-information of both events will add together. Then using:

$I_{x_n} = f\ (p(x_n))$

The total information is given by:

$$I_T = I_{x_i} + I_{x_n} = f\left(p(x_i)\right) + f\ (p(x_n))$$

Since $x_i$ and $x_n$ are independent then: $p(x_i, x_n) = p(x_i).\ p(x_n)$

Also $I_T = f\ (p(x_i).(p(x_n)) = f\left(p(x_i)\right) + f\ (p(x_n))$

Only one function that convert multiplication into addition in mathematics, which is the Logarithmic function thus $f$(.) is logarithm. Now we have logarithmic and invers function then, the measure of self-information is given by:

$$I_{x_i} = -Log_b(p(x_i))$$

The -ve sign is used so that $I_{x_i}$ becomes a +ve quantity (since 0< $p(x_i) < 1$ and its logarithm is -ve always).

The value of $b$ defines the base of the logarithm and also defines the unit of $I_{x_i}$ .

| **Value of b** | 10 | e | 2 |
|---|---|---|---|
| **The unit of $I_{x_i}$** | Hartley unit | Natural unit (or Nat.) | Bit (or Binary Digit) |

We shall use the Bit unit very often. Since there is no logarithmic operator with base-2 in the calculator then we use the following to convert into bit units:

$$Log_2(A) = \frac{Log_{10}(A)}{Log_{10}(2)} = \frac{ln(A)}{Ln(2)}$$

## Example-1

A bag lock consists of 3 decimal digits, find the information required to open such lock in Hartley units and in Bits.

Solution:

The number of possible 3-decimal digits combinations =N= 10x10x10= 1000

Since only one combination will open the lock then p(x) = 1/N =1/1000 = 0.001

$I_x = -Log_{10}(p(x)) = -Log_{10}(0.001) = -Log_{10}(10^{-3}) = 3.Log_{10}(10) = 3$ Hartely

To convert the above into Bits unit;

$$I_x = \frac{3}{Log_{10}(2)} = \frac{3}{0.301} = \frac{3}{Log_{10}(2)} = 9.967\ Bits$$

# 3- Set and Set Operations

## Set

- **Definition**: A **set** is a (unordered) collection of objects. These objects are sometimes called **elements** or **members** of the set. (Cantor's naive definition)

- **Examples:**
  - **Vowels in the English alphabet**

    V = { a, e, i, o, u }
  - **First seven prime numbers.**

    X = { 2, 3, 5, 7, 11, 13, 17 }

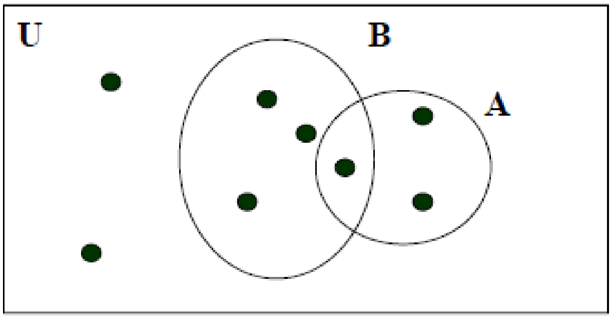
## Special sets

- **Special sets:**
  - **The universal set** is denoted by **U:** the set of all objects under the consideration.
  - **The empty set** is denoted as ∅ **or { }.**

# Set operations

**Definition**: Let A and B be sets. The **union of A and B**, denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

- Alternate: $A \cup B = \{ x \mid x \in A \vee x \in B \}$.

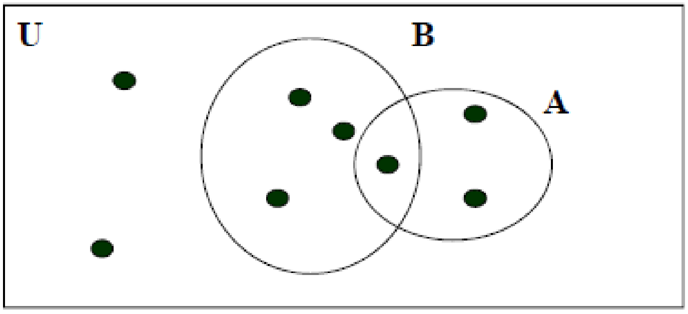



- **Example:**
- $A = \{1,2,3,6\}$ $B = \{2,4,6,9\}$
- $A \cup B = \{1,2,3,4,6,9\}$

# Set operations

**Definition**: Let A and B be sets. The **intersection of A and B**, denoted by $A \cap B$, is the set that contains those elements that are in both A and B.

- Alternate: $A \cap B = \{ x \mid x \in A \wedge x \in B \}$.



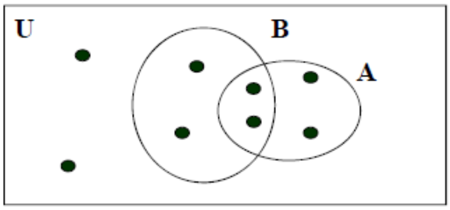

Example:

- $A = \{1,2,3,6\}$ $B = \{2, 4, 6, 9\}$
- $A \cap B = \{2, 6\}$

## Set difference

**Definition**: Let A and B be sets. The **difference of A and B**, denoted by **A - B**, is the set containing those elements that are in A but not in B. The difference of A and B is also called the complement of B with respect to A.

- Alternate: $A - B = \{ x \mid x \in A \wedge x \notin B \}$.



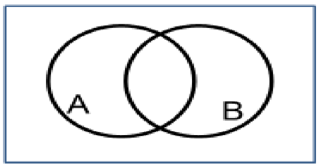

**Example:** A= {1,2,3,5,7} B = {1,5,6,8}

- A - B ={2,3,7}

## De Morgan's Laws

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$




Proof technique:
To show C = D show
$x \in C \rightarrow x \in D$ and
$x \in D \rightarrow x \in C$

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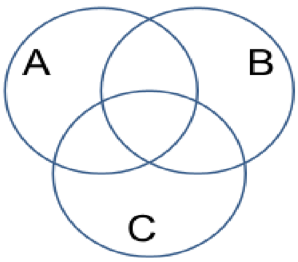
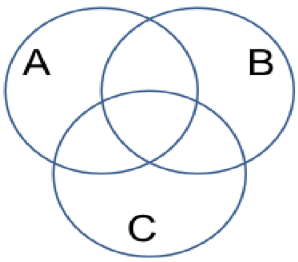
## Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

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