Lecture 2 Outlines

1. **Sampling Theorem**
   - ❖ **Impulse Sampling**
   - ❖ **Natural Sampling**
   - ❖ **Differences between Impulse Sampling and Natural Sampling**
2. **Application of Sampling process**

Sampling Theorem

The link between an analog waveform and its sampled version is provided by what is known as the sampling process.

A band limited signal having no spectral components above (f_m Hz) can be determined uniquely by values sampled at uniform intervals of T_s second, where:

$$T_s = \frac{1}{2f_m}$$

Stated another way, the upper limit on T_s can be expressed in terms of the ***sampling rate***, denoted $f_s = \frac{1}{T_s}$

The restriction, stated in terms of sampling rate, is known as the Nyquist criterion. The statement is

$$f_s \geq 2f_m$$

The sampling rate ($f_s = 2f_m$) also called ***Nyquist rate***.

The Nyquist criterion is a theoretically sufficient condition to allow an analog signal to be reconstructed completely from a set of uniformly spaced discrete time samples.

❖ Impulse Sampling

Assume an analog waveform x(t), as shown in Fig. (a), with a Fourier transform, X(f), which is zero outside the interval ($-f_m<f<f_m$), as shown in Fig. (b). The sampling of x(t) can be viewed as the product of x(t) with a train of unit impulses functions, $x_s(t)$, shown in Fig. (c), and defined as follows:

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Let us choose $T_s = \frac{1}{2f_m}$, so that Nyquist rate is just satisfied.

Using shifting property of the impulse function the $x_s(t)$, shown in Fig. (e), can be given by

$$x_s(t) = x(t)x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s)$$

Using frequency convolution property of Fourier transform, the time product $x(t)x_\delta(t)$ transforms to the frequency domain convolution $X(f) \otimes X_\delta(f)$, where $X_\delta(f)$ is the Fourier transform of $x_\delta(t)$ and given by

$$X_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

The convolution with an impulse function simply shifts the original function, as follows:

$$X(f)\otimes\delta(f-nf_s)=X(f-nf_s)$$

The Fourier transform of the sampled waveform, $X_s(f)$, can be given by:

$$X_s(f)=X(f)\otimes X_\delta(f)=X(f)\otimes[\frac{1}{T_s}\sum_{n=-\infty}^{\infty}\delta(f-nf_s)]$$

$$=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X(f-nf_s)$$

Figure below shows the sampling theorem using the frequency convolution property of the Fourier transform (Impulse sampling).

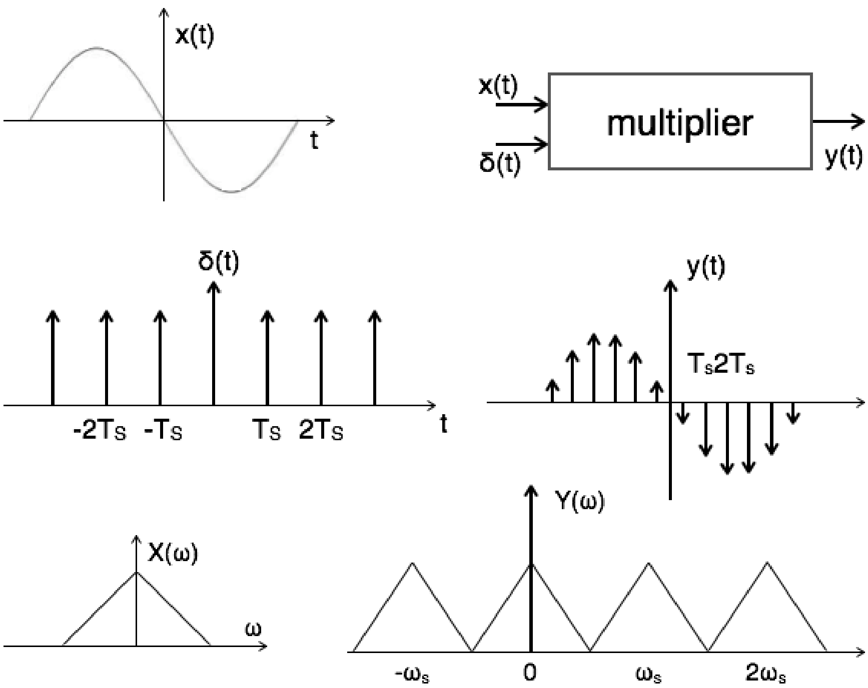

Figure (1): Sampling theorem using impulse method

❖ Natural Sampling

In this way the band limited analog signal x(t), shown in Fig. (2-a), is multiplied by the pulse train or switching waveform $x_p(t)$, shown in Fig. (2-b). Each pulse in $x_p(t)$ has width T and amplitude 1/T.

The resulting sampled data sequence, $x_s(t)$, is shown in Fig. (2-c) and is expressed as

$$x_s(t) = x(t)x_p(t)$$

Figure below shows the sampling theorem using the shifting property of the Fourier transform (Natural sampling).

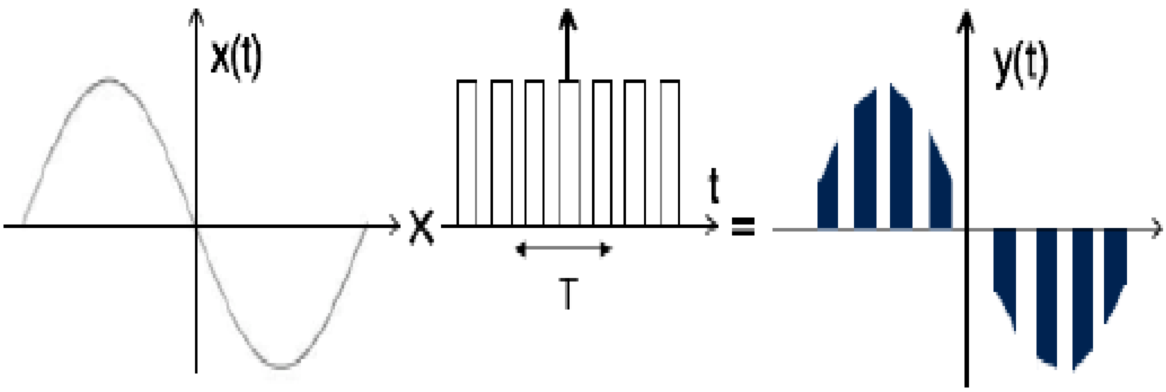

Figure (2): Sampling theorem using the shifting property of the Fourier transform (Natural sampling).

Note: The sampling here is termed natural sampling, since the top of each pulse in the xs(t) sequence retains the shape of its corresponding analog segment during the pulse interval.