Quantum Mechanics

Third Lecture

Uncertainly principle

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Third Stage

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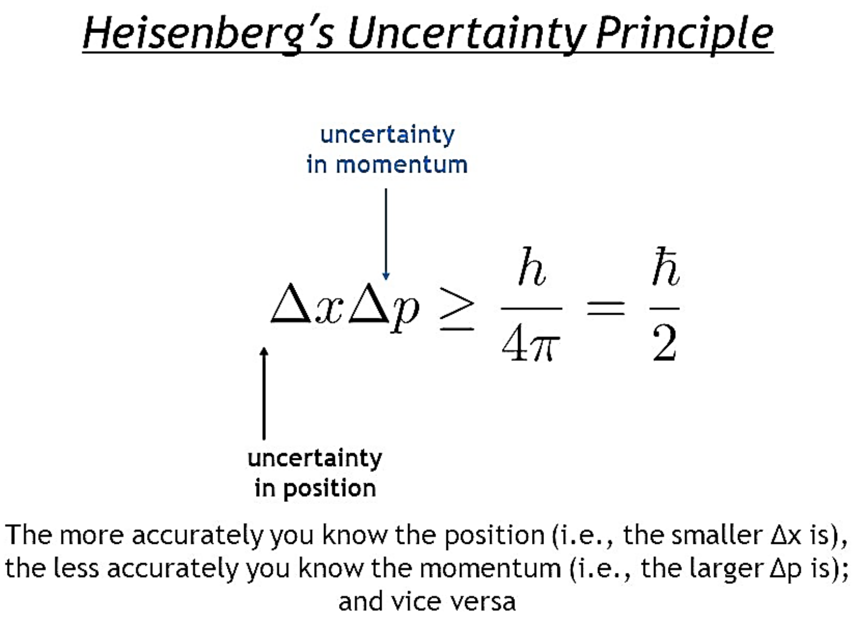
2022- 2023

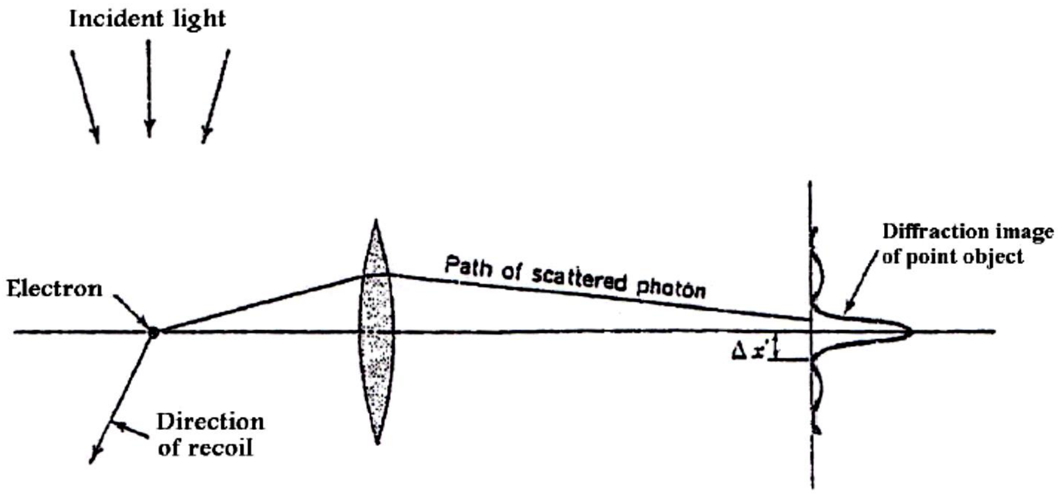
What do you mean by Heisenberg's uncertainty principle?

- ***Heisenberg's uncertainty principle states that it is impossible to measure or calculate exactly, both the position and the momentum of an object. This principle is based on the wave-particle duality of matter***.
- According to the concepts of classical mechanics, there are no limits imposed on the accuracy of measurement. It is the same in quantum mechanics, but only for one variable
- The matter is completely different in the case of measuring two variables at the same time. For example, when measuring the coordinates of a body and its momentum at the same time, we find that when measuring the coordinates, the body receives an external momentum, and this affects the accuracy of measuring its instantaneous momentum, and the opposite is also true, i.e. when measuring its momentum at first, it changes its position, which affects the accuracy of the measurement.
- If the inaccuracy in determining the position of the body is Δx and its momentum is Δp

$$\Delta x \, . \Delta \, p \geq \frac{h}{4\pi}$$



- To clarify this, we take the problem of measuring the position of a small object such as an electron, which can be done using a microscope. In order to make the noise as little as possible, a light of low intensity is used, we assume one photon.
- Weak light with high energy E = hυ falls on electron.
- The photon is scattered after hitting the electron, passing through the lens of the microscope and continuing to move until it reaches a point on the barrier.



- The electron recoil back because it collides with the photon.
- ***What is the magnitude of the uncertainty in the motion of an electron?***
- From the laws of optics, the uncertainty of the position of the electron is given by the relationship:

$$\Delta x \approx \frac{f\lambda}{D}$$

Where: λ is wavelength of incident light, f is Focal Length, D is Diameter of the objective lens of the

- Because of the collision process during the measurement procedure to determine the position, the electron recoil by an unknown amount, because there is no way to know which part of the objective lens the photon has passed through.

- Therefore, we expect the momentum uncertainty to be in the x direction equal to:

$$\Delta p \approx p_x \frac{D}{2f}$$

$$\approx \frac{h\upsilon}{c}\frac{D}{2f} = \frac{h}{\lambda}\frac{D}{2f}$$

- And by multiplying the two values,

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}$$

- For energy and time, the uncertainty principle is given by:

$$\Delta E \cdot \Delta t \approx \hbar$$

Example (1): ***Prove The Impossibility of Finding the Electron Inside Nucleus?***

Solution:

Since the radius of nucleus is about 10^{-14} m this means that $\Delta x = 10^{-14}$ m

$$\because \quad \Delta x \, \Delta p \geq \hbar$$

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{1.05\times10^{-34}}{10^{-14}} = 1.05\times10^{-20} \ \ \text{kg m s}^{-1}$$

$$E = \sqrt{p^2c^2 + m_\circ^2 c^4}$$

$$p^2c^2 >> m_\circ^2 c^4$$

$$\therefore \quad E = p\,c$$

$$\therefore \quad \Delta E = \Delta p\,c$$

$$1.05\times10^{-20} \times 3\times10^{8} \approx 20\text{MeV}$$

$\therefore$ ***Since the average binding energy of particle inside nucleus is 8MeV and 20 MeV is grate than thus the electron can not be found inside the nucleus.***

Example (2): ***If the position of the electron is measured within an accuracy of + 0.002 nm, calculate the uncertainty in the momentum of the electron. Suppose the momentum of the electron is h / 4pm × 0.05 nm. Is there any problem in defining this value?***

Solution:

a) $\Delta x = 2\times10^{-12}$ m

$$\Delta x\,.\Delta\,p \geq \frac{h}{4\pi} \longrightarrow \Delta\,p \geq \frac{h}{4\pi\Delta x}$$

$$\Delta\,p = \frac{6.626\,x\,10^{-34}}{4(3.14)(2x10^{-12})} = 2.64 \times 10^{-23}\ \text{Kg m s}^{-1}$$

b) Momentum $p = mv = \frac{h\,(0.05\,nm)}{4\,pm} = \frac{(6.626\,x\,10^{-34})(5\,x\,10^{-11})}{(4x10^{-12})} = 28 \times 10^{-33}$

Error in momentum measurement is 10^{10} times larger than the actual momentum. The given momentum will not be acceptable.

Example (3): ***Position of a chloride ion on a material can be determined to a maximum error of 1μm. If the mass of the chloride ion is 5.86 × 10-26Kg, what will be the error in its velocity measurement?***

Solution:

- ✓ $\Delta x = 10^{-6}$ m

$$\Delta x\,.\Delta\,p \geq \frac{h}{4\pi} \longrightarrow \Delta x\,.\Delta\,mv \geq \frac{h}{4\pi} \longrightarrow \Delta\,v \geq \frac{h}{4\pi m\Delta x}$$

$$\Delta v = \frac{6.626\,x\,10^{-34}}{4(3.14)(5.86x10^{-26})(10^{-6})}$$

$$\Delta v = 9 \times 10^{-4}\ \text{m s}^{-1}$$

Example (3): ***The lifetime of an excited state of an atom is 3×10^{-3}s. What is the minimum uncertainty in its energy in eV?***

- ✓ Time and energy are conjugate pairs with Js unit. The product of measurement error is given by Heisenberg's principle.

$$\Delta t \, . \Delta \, E \geq \frac{h}{4\pi} \quad \Longrightarrow \quad \Delta E \geq \frac{h}{4\pi\Delta t}$$

$$\Delta E = \frac{6.626 \, x \, 10^{-34}}{4(3.14)(3x10^{-3})} = 0.176 \text{ x } 10^{-31} \text{ J}$$

- ✓ $\because$ 1 Joule = 6.242×10^{18} eV,

$\therefore$ $\Delta E = 0.176 \text{ x } 10^{-31}$ J x 6.242×10^{18} eV = $1.09 \text{ x } 10^{-13}$ eV

H.W(1): ***A wet ball weighing 10.1gm has a water of 0.1g on it. The ball is moving with a constant velocity with an uncertainty of momentum of 10^{-6} kg m/s. What will be the uncertainty in the measurement of the position of the ball, water and electron in the water molecule?***

H.W(2): ***Determine the minimum uncertainties in the positions of the electron if their speeds are known with a precision of 3.0×10^{-3}m/s.***

H.W(3): ***choose the correct answer***

The uncertainty principle was enunciated by

Options :

- A Heisenberg
- B Einstein
- C Rutherford
- D Pauli

The uncertainty principle applies to

Options :

- A gases
- B macroscopic particles
- C microscopic particles
- D none of these

Simultaneous determination of exact position and momentum of an electron is

Options :

- A Impossible
- B Possible
- C Sometimes possible sometimes impossible
- D None of the above

The uncertainty principle can be easily understandable with the help of

Options :

- A Electron effect
- B Compton's effect
- C Dalton's effect
- D Rhombic effect

The position and velocity of a small particle like electron cannot be simultaneously determined.? This statement is

Options :

- A Aufbau's principle
- B Heisenberg uncertainty principle
- C Principle of de Broglie's wave nature of electron
- D Pauli's exclusion principle