Quantum Mechanics

Second Lecture

The Wave-Particle Duality of Electromagnetic Radiation

Dr. Nasma Adnan

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Department of Medical Physics

Al-Mustaqbal University-College

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1. Interpretation the duality of radiation

- Electromagnetic radiation consists of particles called ***photons***.
- The phenomena of ***interference*** and ***diffraction*** can be explained only by assuming the wave characteristic of radiation.
- Interference and diffraction experiments need ***wave interpretation***, and they are experiments in which they investigate ***how and where light travels***.
- While experimental such as photoelectric phenomenon, the Compton effect, the generation of minimum X-rays, the generation and annihilation of the pair need a ***particle explanation*** to investigate the interaction of electromagnetic radiation with matter.
- Therefore, we assume that the electromagnetic radiation consists mainly of ***photons that are walking by a straight line.***
- Based on this interpretation, the light waves can be considered: ***Waves guide the photons the intensity of the wave at any point is proportional to the probability of the photon being present at that point.***
- The idea of a particle and a wave can be combined by ***de Broglie's equation***

2. Derivation of the de Broglie equation

$$\because \quad p = mv$$

$$E^2 = p^2c^2 + m_\circ^2 c^4$$

For photon $\quad m_\circ = 0$ The rest mass of a photon

$$\therefore \quad E^2 = p^2c^2 + 0$$

$$E = pc$$

But $\quad E = h\upsilon = \dfrac{hc}{\lambda}$

$$pc = \frac{hc}{\lambda}$$

$$\boldsymbol{p = \frac{h}{\lambda}}$$

p → Momentum of photon

h → Planck constant

λ → Wavelength of photon

Example (1): ***Calculate the de Broglie wavelength for an electron traveling at a speed of (v = 2 x 10^6 m\s) and mass of electron (m_e = 9.1x10^{-31} kgm), then compare it with a particle traveling at a speed of 10 m\s and mass 1 gm?***

Solution:

1) For Electron

$$p = \frac{h}{\lambda} \longrightarrow \lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\lambda = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times2\times10^{6}}$$

$$\lambda = 3.6\times10^{-10} = 3.6\ \mathring{A}$$

2) For particle, when m =1 gm

$$\lambda = \frac{6.6\times10^{-34}}{1\times10^{-3}\times10}$$

$$\lambda = 6.6\times10^{-32} = 6.6\times10^{-22}\ \mathring{A}$$

∴ Particles that have a large mass do not show wave characteristics, so the electron has clear wave characteristics.

Example (2): ***What is the de Broglie wavelength of an electron with an energy of 50 eV?***

Solution:

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} = \frac{\sqrt{2(mc^2)E}}{c}$$

$$\lambda = \frac{hc}{\sqrt{2(mc^2)E}} = \frac{1240eVnm}{\sqrt{2(0.511x10^6eV)(50eV)}}$$

$$= 0.17nm$$

H.W(1): ***What is the de Broglie wavelength of a tennis ball that has a mass of 70 g and a speed of 25 m/s?***

H.W(2): ***Calculate the deBroglie wavelength of a particle whose momentum (impulse) is 1 keV/speed of light?***

H.W(3): ***Calculate the wavelength of the neutron, knowing that its kinetic energy at the moment of its release from the nucleus is 10 MeV?***

3. Wave Aspect of Matter (function of de Broglie waves)

- In 1924, the scientist de Broglie suggested that light consists of photons that are driven by a wave.
- Wave behavior is not limited to photons, but includes all physical particles (electrons, protons, neutrons, ...
- De Broglie assumed the following relationships:

$$E = h\upsilon$$

$$p = \frac{h\upsilon}{c} = \frac{h}{\lambda} \qquad E = h\upsilon = \hbar\omega$$

$$\lambda = \frac{h}{p}$$ ***The relationship between wavelength and momentum***

- The electron orbitals in the hydrogen atom, according to Bohr's theory, can be defined as (***the closed orbital must be equal to an integer multiplied by λ***) prove it:
  - ✓ suppose that the circumference of the circular Bohr path is given by the relation:

$$2\pi r = n\lambda$$

$$2\pi r = n\frac{h}{p} = \frac{h}{mv}$$

$$\therefore\ p_e = mvr = n\frac{h}{2\pi} = n\hbar$$ **Bohr's first hypothesis**

❖ **Q: What is the nature of photons? speed? its energy?**

1. The photon is "quantum," or fundamental unit, of light and other electromagnetic radiation.

2. The speed of a photon is the same as the speed of light ($c = 3 \times 10^8$ m\s).

3. Photon energy ($E = h\upsilon$) where h is Planck's constant ($h= 6.63 \times 10^{-34}$ J.s) and υ is photon frequency.

4. A photon has momentum.

5. A photon has no mass

❖ **Q: Why does a photon have no mass?**

- ✓ according to Einstein's relativity photons do not have mass simply because they travel at the speed of light this is also backed up by the theory of quantum
- ✓ from energy-momentum relation:

$$E = \sqrt{p^2c^2 + m^2c^4}$$

- ✓ This equation states that the total energy (E) of a particle is given in terms of its momentum (p) and its mass (m).
- ✓ If the particle is at rest, it has zero momentum (p=0) and we get for the energy: $E = mc^2$
- ✓ One of the consequences of Einstein's theory of special relativity is that particles which move at the speed of light (c), can't have any mass. Or another way to say it is that particles with mass can't move at the speed of light.

$$E = \frac{m\,c^2}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow m = \frac{E}{c^2}\sqrt{1-\frac{v^2}{c^2}}$$

- ✓ Now, if we set the velocity v equal to the speed of light c, this gives the mass equal to zero:

$$m = \frac{E}{c^2}\sqrt{1-\frac{c^2}{c^2}} \quad = m = \frac{E}{c^2}\sqrt{1-1} \quad = 0$$

Example (3): ***An electron has a velocity v = 0.999 c.***

a. Calculate the kinetic energy in MeV of the electron.

b. Compare this with the classical value for kinetic energy at this velocity.

(The mass of an electron is 9.11 x 10^{-31}kg).

Solution:(a)

- ✓ From equation of the relativistic kinetic energy

$$E_{rel} = \frac{m\,c^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)mc^2$$

$$= \left(\frac{1}{\sqrt{1-\frac{(0.990\,c)^2}{c^2}}} - 1\right)(9.11x10^{-31})(3x10^8)^2$$

$$= (7.0888 - 1)(9.11x10^{-31}kg)(3x10^8 m\backslash s)^2$$

$$= 4.9922 \text{ x } 10^{-13} \text{ J}$$

- ✓ Convert units:

$$E_{rel} = (4.9922 \text{ x } 10^{-13}\text{ J})\left(\frac{1MeV}{1.6\ X\ 10^{-13}J}\right)$$

$$= 3.12 \text{ MeV}$$

Solution:(b)

- ✓ From equation of the classical kinetic energy:

$$E_{class} = \frac{1}{2}\,m\,v^2 = \frac{1}{2}\,(9.11x10^{-31}kg)\,(0.990\,c)^2$$

$$= \frac{1}{2}(9.11x10^{-31}kg)\,(0.990\,)^2\,(3x10^8 m\backslash s)^2$$

E_{class} = 4.0179 $\times 10^{-14}$ J.

- ✓ Convert units:

E_{class} = (4.0179 $\times 10^{-14}$ J) $(\frac{1MeV}{1.6\ X\ 10^{-13}J})$

= 0.251 MeV

- ✓ because the velocity is 99.0% of the speed of light, the classical kinetic energy differs significantly from the correct relativistic value. Note also that the classical value is much smaller than the relativistic value. In fact, in this case. This illustrates how difficult it is to get a mass moving close to the speed of light.