Fast Fourier Transform (FFT)

Decimation-in-Time FFT

The number of point is assumed as a power of 2 , that is, 2^N , the decimation in time approach is one of breaking of N −point transform in to two $\frac{N}{2}$ −point transforms, then breaking each $\frac{N}{2}$ −point transforms into $\frac{N}{4}$ − point transforms , and continuing this process until two− point transform are obtained.

1) The input data have been shuffled. The input data appear in what is called " bit reversed order" illustrated bellow for $N = 8$.

Position	Binary Equivalent	Bit reversed	Sequence index
$x(6)$	110	011	$x(3)$
$x(2)$	010	010	$x(2)$
$x(1)$	001	100	$x(4)$

It is seen that $x(3)$ is in the sixth position and $x(2)$ in the second position.

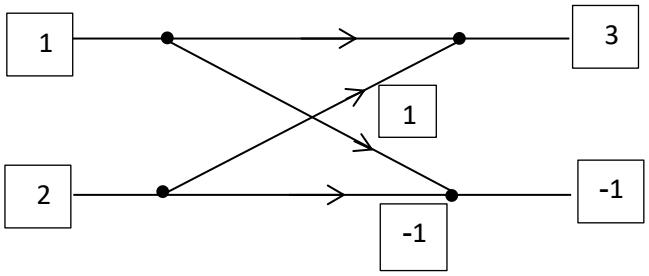
2) The basic computational block in the diagram is called a "butterfly'.

3) The frequency domain values, $X(k)$, are in normal order.

Ex: Determine DFT for the following signal using FFT algorithm:

$x(n) = \{1,2\}$

Solution:



$X(k) = \{3, -1\}$

Ex: Determine FFT for the following signal:

$x(n) = \{1,2,3,4\}$

Solution:

First Step: The position will be reversed

Position	**Binary Equivalent**	**Bit reversed**	**Sequence index**
$x(0)$	00	00	$x(0)$
$x(1)$	01	10	$x(2)$
$x(2)$	10	01	$x(1)$
$x(3)$	11	11	$x(3)$

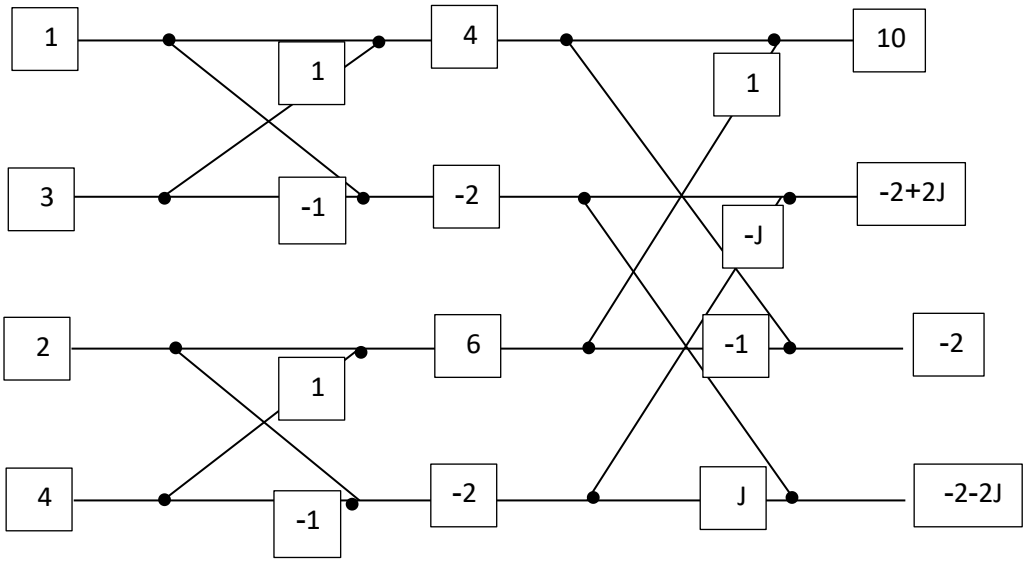
Second Step: Determine values of $W_N^n = e^{-jwnk}$

$W_4^0 = 1$

$W_4^1 = -j$

$W_4^2 = -1$

$W_4^3 = j$



$X(k) = \{10, -2 + 2j, -2, -2 - 2j\}$

Ex: Find FFT for the following sequence:

$x(n) = \{0,1,0,2,3,1,4,1\}$

Solution:

First Step: The position will be reversed

Position	**Binary Equivalent**	**Bit reversed**	**Sequence index**
$x(0)$	000	000	$X(0)$
$x(1)$	001	100	$X(4)$
$x(2)$	010	010	$X(2)$
$x(3)$	011	110	$X(6)$

$x(4)$	100	001	$X(1)$
$x(5)$	101	101	$X(5)$
$x(6)$	110	011	$X(3)$
$x(7)$	111	111	$X(7)$

Second Step: Determine values of $W_N^n = e^{-jwnk}$

$W_8^0 = 1$

$W_8^1 = 0.707 - j0.707$

$W_8^2 = -j$

$W_8^3 = -0.707 - j0.707$

$W_8^4 = -1$

$W_8^5 = -0.707 + j0.707$

$W_8^6 = j$

$W_8^7 = 0.707 + j0.707$

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