



Discrete Time Fourier Transform (DTFT)

EX: Determine the frequency response of the following system

$$h(n) = \delta(n) + \delta(n - 2)$$

Solution:

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} [\delta(n) + \delta(n - 2)]e^{-j\omega n}$$

$$H(e^{j\omega}) = e^0 + 1 \cdot e^{-2j\omega} = 1 + e^{-2j\omega}$$

2.Frequency response to sinusoidal signal

EX: Find the frequency response of linear shift invariant characterized by unit sample response $h(n) = a^n u(n)$, $|a| \leq 1$,

Solution:

$$\begin{aligned} H(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} a^n u(n)e^{-j\omega n} \\ &= \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \sum_{n=0}^{\infty} (ae^{-j\omega})^n \\ H(e^{j\omega}) &= \frac{1}{1 - ae^{-j\omega}} \end{aligned}$$



Encountered Series

$\sum_{n=0}^{N-1} a^n = \frac{1 - a^N}{1 - a}$ $\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$ $\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$	$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad a < 1$ $\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2} \quad a < 1$ $\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$
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3. Frequency response of rectangular window

EX: A discrete time system has $h(n)$ given by

$$h(n) = \frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)$$

1. Find frequency Response;
2. Find the total response to $x(n) = u(n)$ assuming the system is initially at reset.

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} [\frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)]e^{-j\omega n}$$

$$H(e^{j\omega}) = \frac{1}{2}e^0 + 1 \cdot e^{-j\omega} + \frac{1}{2}e^{-2j\omega} = e^{-j\omega}(\frac{1}{2}e^{j\omega} + 1 + \frac{1}{2}e^{j\omega})$$

$$H(e^{j\omega}) = e^{-j\omega}(1 + \cos\omega)$$

The total response can be written in terms of convolution

$$y(n) = x(n) * h(n)$$

$$= u(n) * [\frac{1}{2}\delta(n) + \delta(n-1) + \frac{1}{2}\delta(n-2)]$$



$$= 0.5u(n) + u(n - 1) + 0.5u(n - 2)$$

$$y(n) = \begin{cases} 0.5 & n = 0 \\ 1.5 & n = 1 \\ 2 & n \geq 2 \end{cases}$$

Ex: Determine the DTFT of the sequence

$$x(n) = -a^n u(-n - 1)$$

Solution:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} -a^n u(-n - 1)e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{-1} -a^n e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=1}^{\infty} -a^{-n} e^{+j\omega n}$$

$$-n - 1 \geq 0$$

$$-n \geq 1$$

$$n \geq -1$$

