Al-Mustaqbal University College
Air Conditioning and Refrigeration Techniques Engineering Department

Strength of Materials

Second Stage

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Simple Strains

NORMAL STRAIN UNDER AXIAL LOADING

Concepts of Strain:

- As already mentioned, wherever a single force (or a system of forces) acts on a body, it undergoes some deformation (Figure 6.1). This deformation per unit length is known as strain. Mathematically strain may be define as deformation per unit length, i.e., strain is:

$$\varepsilon = \frac{\delta l}{l} \quad or \quad \delta l = \varepsilon . l$$

Where δl =Change of length of the body, and

l =Original length of the body

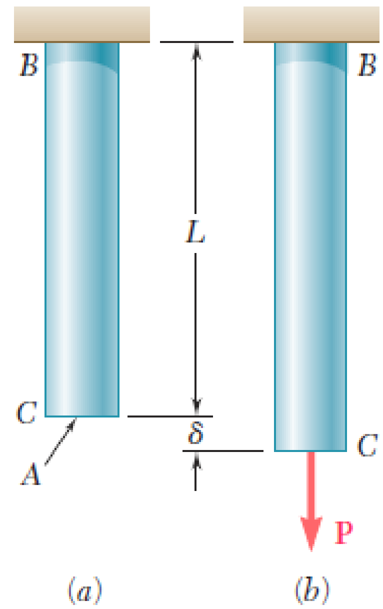


Figure 6.1

- Strain is thus, a measure of the deformation of the material and is a non-dimensional quantity i.e. it has no units. It is simply a ratio of two quantities with the same unit.

Stress – Strain Relationship

- When a material is loaded, the stress is proportional to the strain mathematically, and as shown in Figure (6.2),

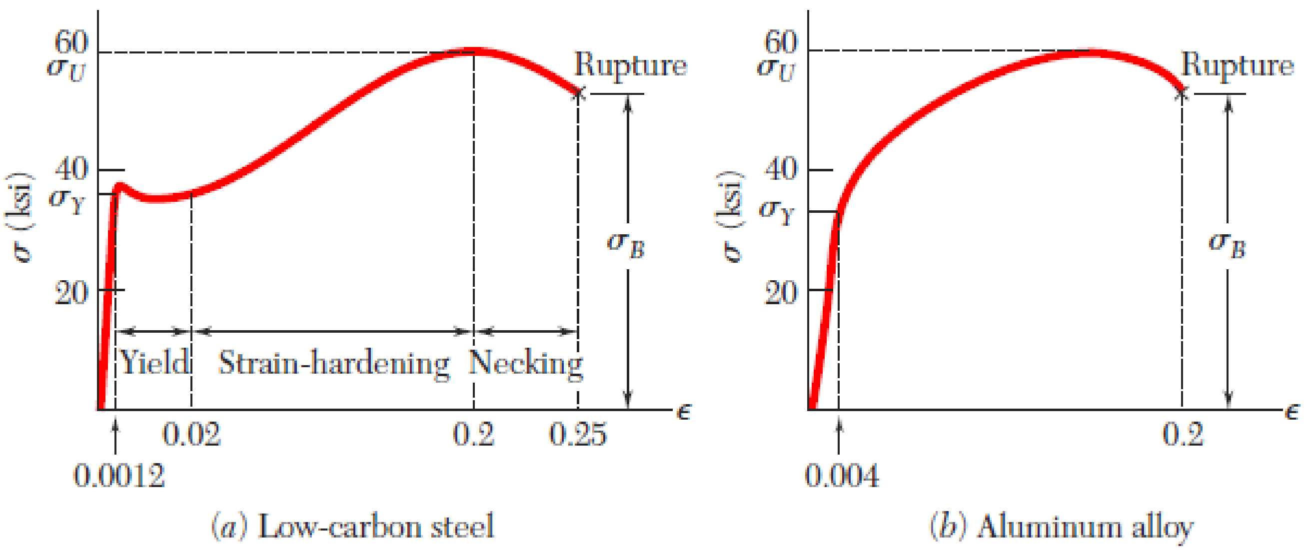


Figure (6.2): Stress-strain diagrams of two typical ductile materials

- The initial portion of the stress – strain diagram for most material used in engineering structures is a straight line. For the initial portion of the diagram, the stress (σ) is directly proportional to the strain (ε). Therefore, for a specimen subjected to a uniaxial load, can write:

$$\frac{\boldsymbol{Stress}}{\boldsymbol{Strain}} = \boldsymbol{E} \quad \rightarrow \quad \boldsymbol{\sigma = E\varepsilon}$$

This relationship is known as ***Hook's Law***.

- **Note:** ***Hook's Law*** describes only the initial linear portion of the stress – strain curve for a bar subjected to uniaxial extension.

- The slope of the straight line portion of the stress – strain diagram is called the ***Modulus of Elasticity*** or ***Young's Modulus***.

$$\boldsymbol{E = \frac{\sigma}{\varepsilon}}$$

Deformation of a Body due to Force acting on it

Consider a body subjected to a tensile force as shown in Figure (6.3).

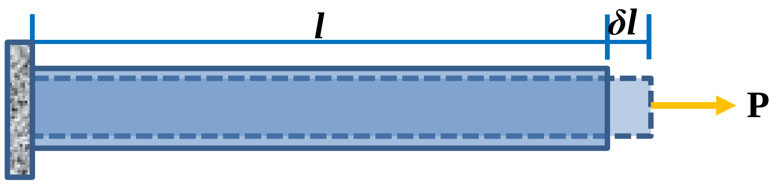


Figure (6.3)

Let P =Load or force acting on the body,

l =Length of the body,

A =Cross – sectional area of the body,

σ =Stress induced in the body,

E =Modulus of elasticity for the material of the body,

ε =Strain, and

δl =Deformation of the body.

Knowing that the stress is:

$$\boldsymbol{\sigma = \frac{P}{A}}$$

And the strain is:

$$\boldsymbol{\varepsilon = \frac{\sigma}{E} = \frac{\frac{P}{A}}{E} = \frac{P}{EA}}$$

Then the deformation is:

$$\boldsymbol{\delta l = \varepsilon(l) = \frac{Pl}{EA}}$$

Examples

Example (6.1): A steel rod (1 m) long and (20 mmx20mm) in cross – section is subjected to a tensile force of (40 kN). Determine the elongation of the rod, if modulus of elasticity for the rod material is (200 GPa).

Solution:

$$\delta l = \frac{Pl}{EA} = \frac{40 \times 10^3 (1 \times 10^3)}{200 \times 10^3 (20 \times 20)} = 0.5\ mm$$

Example (6.2): In an experiment, a steel specimen of (13mm) diameter was found to elongate (0.2 mm) in a (200 mm) gauge length when it was subjected to a tensile force of (26.8 kN). If the specimen was tested within the elastic range, what is the value of Young's modulus for the steel specimen?

Solution:

$$\delta l = \frac{Pl}{EA} \quad \rightarrow \quad 0.2 = \frac{26.8 \times 10^3 (200)}{E\left(\frac{\pi}{4}(13)^2\right)}$$

$$E = 201910.2\ MPa \quad or \quad 201.91\ GPa$$

Example (6.3): Determine the deformation of the steel rod shown in Figure (6.4) under the given loads. E=29 x10^6 psi.

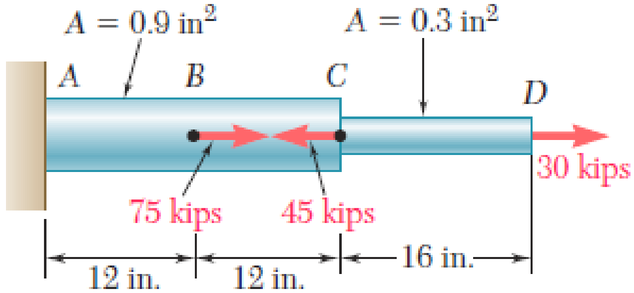


Figure (6.4)

Solution:

- We divide the rod into three component parts shown in Figure (6.5b) and write:

$$L_1 = L_2 = 12\ in.$$

$$L_3 = 16\ in.$$

$$A_1 = A_2 = 0.9\ in^2$$

$$A_3 = 0.3\ in^2$$

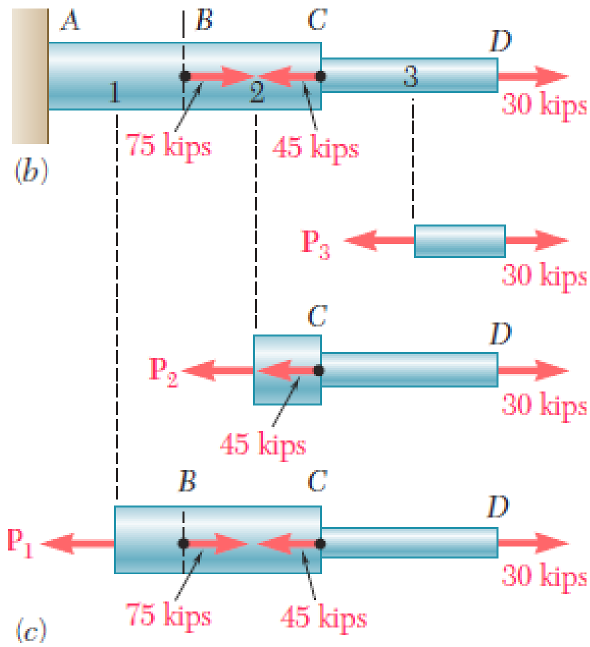


Figure (5.5)

Section in each part as shown in Figure (6.5c), then:

$$\sum F_x = 0$$

$$P_1 = 60\ kips = 60 \times 10^3\ lb$$

$$P_2 = -15\ kips = -15 \times 10^3\ lb$$

$$P_3 = 30\ kips = 30 \times 10^3\ lb$$

The total deformation in steel bar is:

$$\delta L = \sum \frac{P_i L_i}{E_i A_i}$$

$$\delta L = \frac{1}{29 \times 10^6}\left(\frac{60 \times 10^3(12)}{0.9} + \frac{-15 \times 10^3(12)}{0.9} + \frac{30 \times 10^3(16)}{0.3}\right) = \frac{2.2 \times 10^6}{29 \times 10^6}$$
$$= 75.9 \times 10^{-3}\ in.$$

Example (6.4): The rigid bar *BDE* is supported by two links *AB* and *CD*. Link *AB* is made of aluminum (*E*=70 GPa) and has a cross-sectional area of 500 mm^2; link *CD* is made of steel (*E*=200 GPa) and has a cross-sectional area of 600 mm^2. For the 30-kN force shown in Figure (6.6), determine the deflection

- **(*a*)** of *B*,
- **(*b*)** of *D*,
- **(*c*)** of *E*.

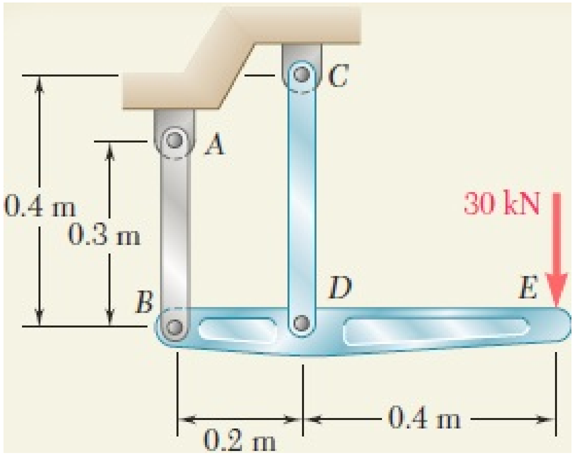
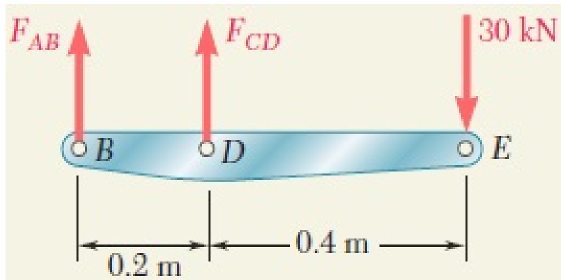


Figure (6.6)

<u>**Solution:**</u>

Bar BDE as F.B.D:



$$\circlearrowleft \sum M_B = 0$$

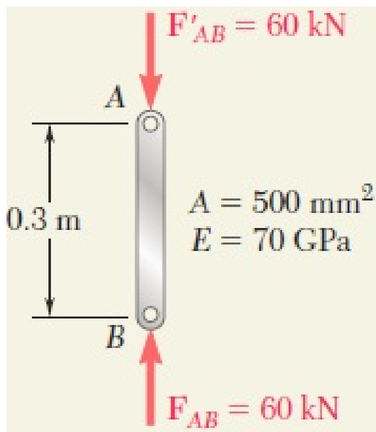
$$F_{CD}(0.2) - 30(0.6) = 0$$

$$F_{CD} = +90\ kN\ (Tens.)$$

$$\uparrow \sum F_y = 0 \rightarrow F_{AB} + F_{CD} - 30 = 0$$

$$\rightarrow F_{AB} = 30 - 90 = -60\ (Comp.)$$

<u>**(a) Deflection of B:**</u>



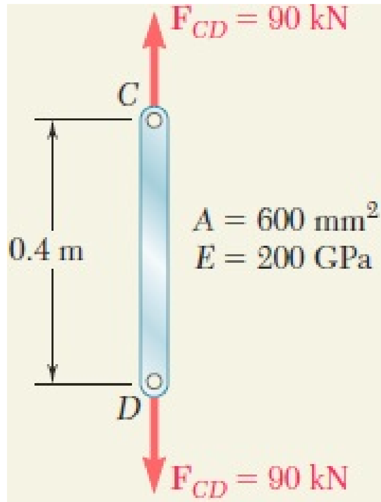
$$\delta_B = \frac{Pl}{EA} = \frac{(-60 \times 10^3)(0.3)}{(70 \times 10^9)(500 \times 10^{-6})}$$

$$\delta_B = -514 \times 10^{-6}\ m\ \ or\ \delta_B = -0.514\ mm$$

The negative sign indicates a contraction of member *AB*, and, thus, an upward deflection of end *B*:

$$\delta_B = 0.514\ mm\ \uparrow$$

(b) Deflection of D:



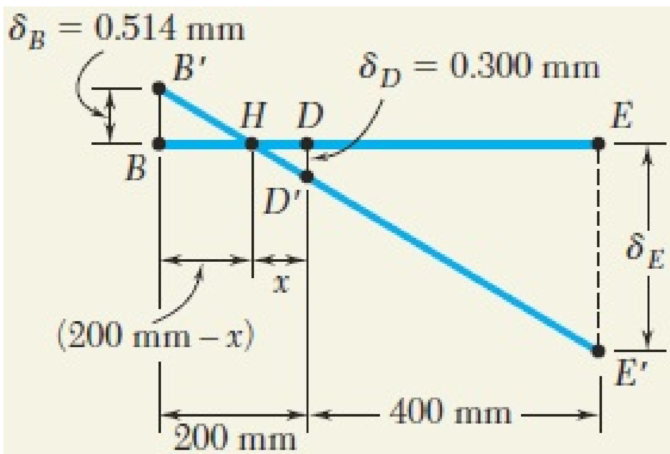
$$\delta_D = \frac{Pl}{EA} = \frac{(90 \times 10^3)(0.4)}{(200 \times 10^9)(600 \times 10^{-6})}$$

$$\delta_D = 300 \times 10^{-6}\ m \ \ or$$

$$\delta_D = 0.300\ mm \downarrow$$

(c) Deflection of E:

We denote by *B*' and *D*' the displaced positions of points *B* and *D*. Since the bar *BDE* is rigid, points *B*', *D*', and *E*' lie in a straight line and we write:



$$\frac{BB'}{DD'} = \frac{BH}{HD} \rightarrow \frac{0.514}{0.300} = \frac{200 - x}{x} \rightarrow x = 73.7\ mm$$

$$\frac{EE'}{DD'} = \frac{HE}{HD} \rightarrow$$

$$\frac{\delta_E}{73.7} = \frac{400 + 73.7}{73.7}$$

$$\rightarrow \delta_E = 1.928\ mm\ \downarrow$$