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Strength of Materials

Second Stage

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HOOKE'S LAW FOR PLANE STRESS

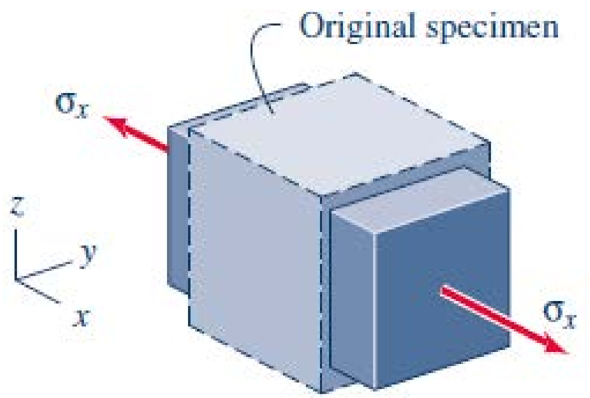
Poisson's Ratio

❑ THE RELATIONSHIP BETWEEN E AND G:

For a ***uniaxial stress state***, $\boldsymbol{\sigma_x}$, (Figure 6.7) the linear relationship between stress and strain, ϵ_x , was given by Hooke's Law:

$$\epsilon_x = \frac{\sigma_x}{E}$$

Poisson's ratio, ***v***, relates the transverse strain, ϵ_y, to ϵ_x:



$$\epsilon_y = \epsilon_z = -v\epsilon_x = -v\frac{\sigma_x}{E}$$

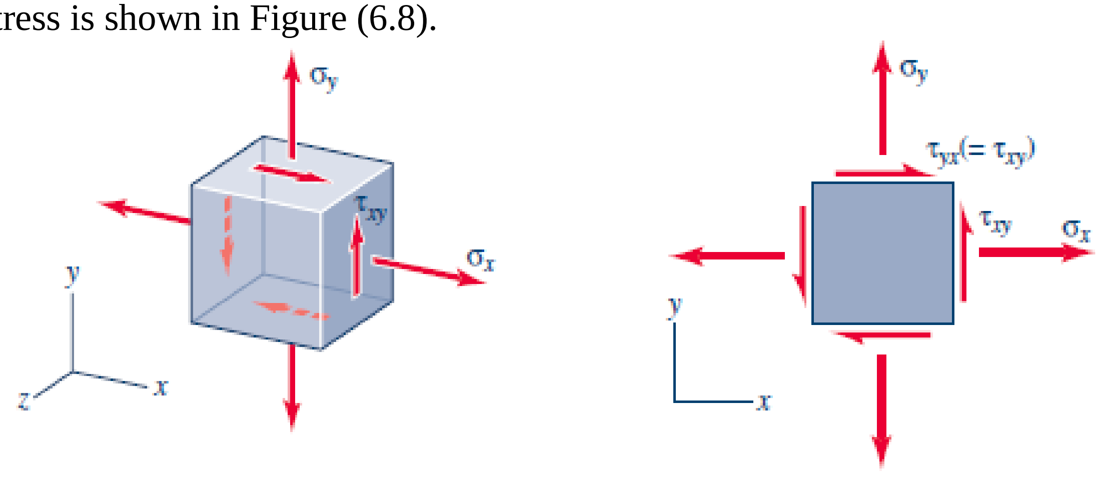
Figure 6.7

For linearly elastic materials, the shear modulus ***G*** and Young's modulus ***E*** are related by the equation:

$$G = \frac{E}{2(1+v)}$$

- ▶ For this equation to apply, the material must not only be linearly elastic, it must also be **isotropic,** and that is, *its material properties like **E** and **v** must be independent of orientation in the body.*

❑ **Plane Stress.** *A body that is subjected to a two-dimensional state of stress with* $\boldsymbol{\sigma_z = \tau_{xz} = \tau_{yz} = 0}$, *is said to be in a state of* **plane stress.** An element in plane stress is shown in Figure (6.8).



(*a*) Three-dimensional view. (*b*) Two-dimensional view.

Figure (6.8)

If the material of which the body is composed is linearly elastic and isotropic, the effects of stresses $\boldsymbol{\sigma_x}$, $\boldsymbol{\sigma_y}$, and $\boldsymbol{\tau_{xy}}$ can be superposed, giving *Hooke's Law for plane stress:*

$$\epsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y)$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x)$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

<u>Generalized Hooke's Law for Isotropic Materials</u>

Let the body be subjected to stresses σ_x, σ_y, σ_z, τ_{xy}, τ_{xz} and τ_{yz} as shown in Figure (6.9).

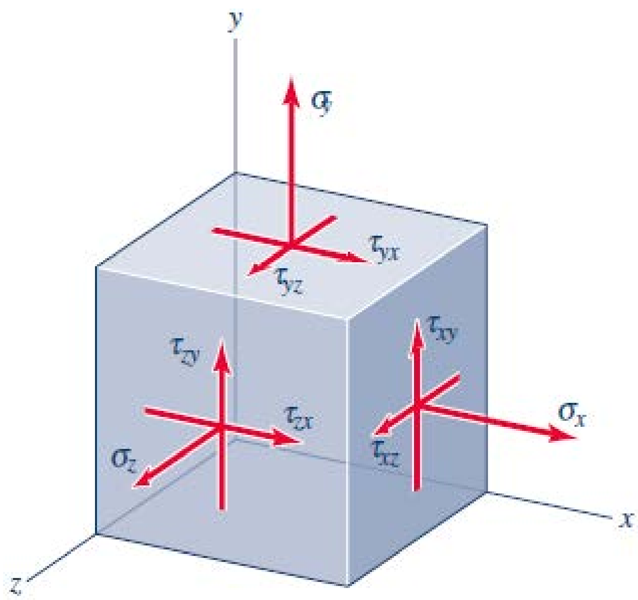


Figure (6.9)

Figure (6.10) illustrate the strains produced separately by the three normal stresses, σ_x, σ_y, and σ_z.

By the superposition principle, the total extensional strains are given by:

$$\epsilon_x = \frac{1}{E}[\sigma_x - v(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - v(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right]$$

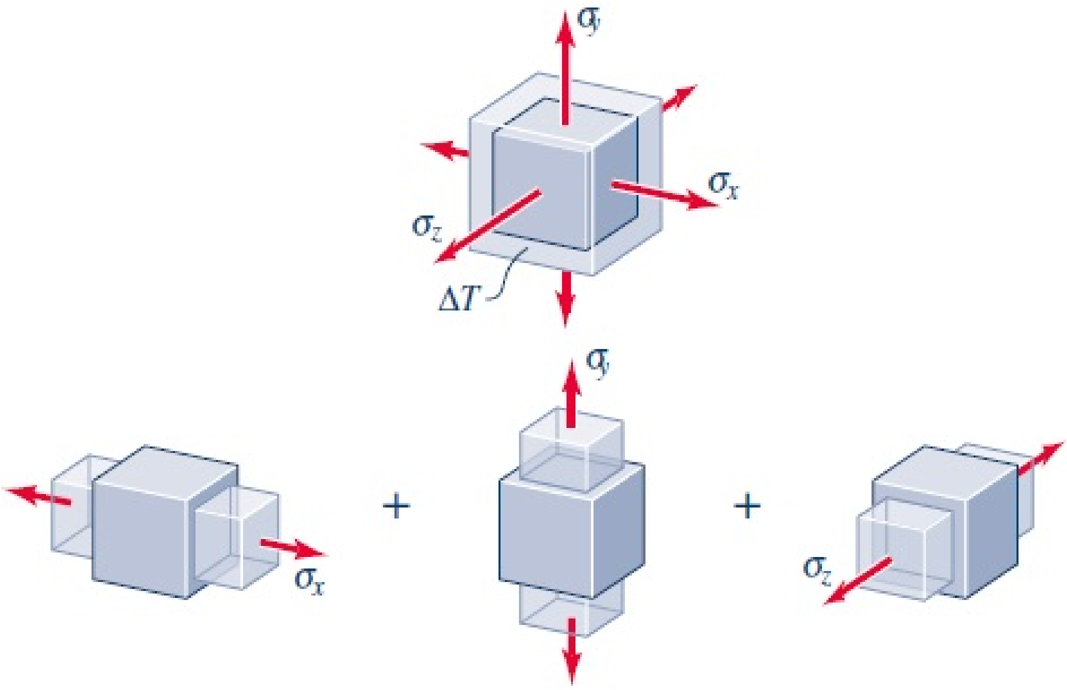


Figure (6.10)

- For an isotropic linearly elastic material, the shear stresses are related to the shear strains by the following equations:

$$\gamma_{xy} = \frac{1}{G}\tau_{xy},\ \ \gamma_{xz} = \frac{1}{G}\tau_{xz},\ \ \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

These shear strains are illustrated in Figure (6.11).

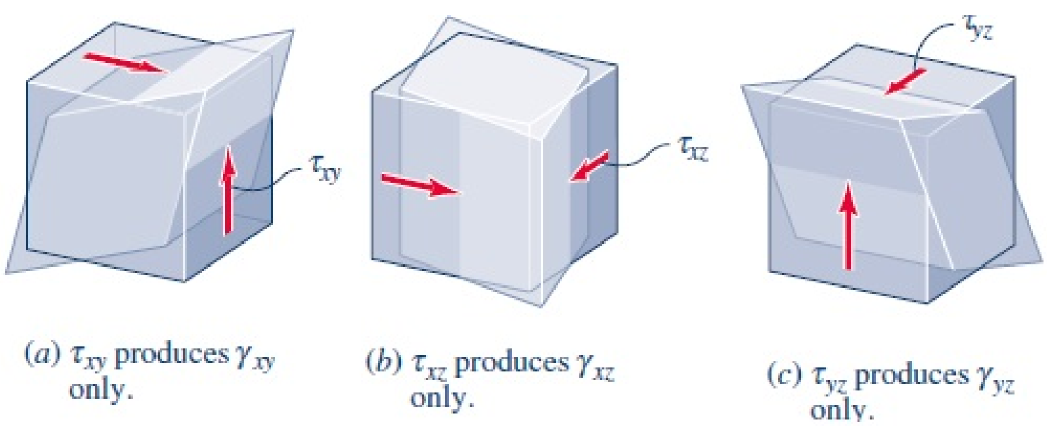


Figure (6.11): Illustration of shear strains.

Solving the above equations for the stresses in terms of the strains, we get:

$$\sigma_x = \frac{E}{(1+v)(1-2v)}\left[(1-v)\epsilon_x + v\left(\epsilon_y + \epsilon_z\right)\right]$$

$$\sigma_y = \frac{E}{(1+v)(1-2v)}\left[(1-v)\epsilon_y + v(\epsilon_x + \epsilon_z)\right]$$

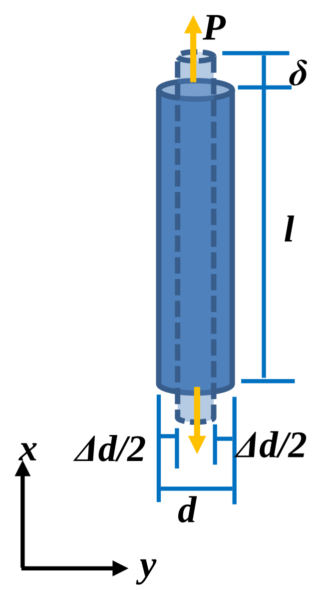
$$\sigma_z = \frac{E}{(1+v)(1-2v)}\left[(1-v)\epsilon_z + v\left(\epsilon_y + \epsilon_x\right)\right]$$

and

$$\tau_{xy} = G\gamma_{xy}\ ,\ \ \tau_{xz} = G\gamma_{xz},\ \ \tau_{zy} = G\gamma_{zy}$$

Examples

Example (6.5): A prismatic bar of circular cross – section is loaded by tensile force (P=85 kN). The bar has length (3 m) and diameter (30 mm). It is made of aluminum with modulus of elasticity (70 GPa) and Poisson's ratio (1/3). Calculate the elongation, the decrease in diameter and the change in volume.



Solution:

$$\delta = \varepsilon_x . l \quad and \quad \varepsilon_x = \frac{\sigma_x}{E}$$

$$\sigma_x = \frac{P}{A} = \frac{85 \times 10^3}{\frac{\pi}{4}(30)^2} = 120.25\ MPa$$

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{120.25}{70 \times 10^3} = 0.00172$$

$$\delta = \varepsilon_x . l = 0.00172(3 \times 10^3) = 5.16\ mm$$

$$\Delta d = \varepsilon_y . d$$

$$\varepsilon_y = -v\frac{\sigma_x}{E} = -\frac{1}{3}\frac{(120.25)}{(70 \times 10^3)} = -0.00057$$

$$\Delta d = -0.00057(30) = -0.0171\ mm$$

Change in volume (ΔV)=final volume – initial volume

$$\Delta V = \left[(30 - 0.0171)^2 \left(\frac{\pi}{4}\right)(3000 + 5.16)\right] - \left[(30)^2 \left(\frac{\pi}{4}\right)(3000)\right] = 1226.5\ mm^3$$

Example (6.6): Find a single force in x –direction that gives the same change in the direction parallel to x, for shown Figure (6.12). Take $v = {}^{1}/_{3}$ and $E = 70\ GPa$.

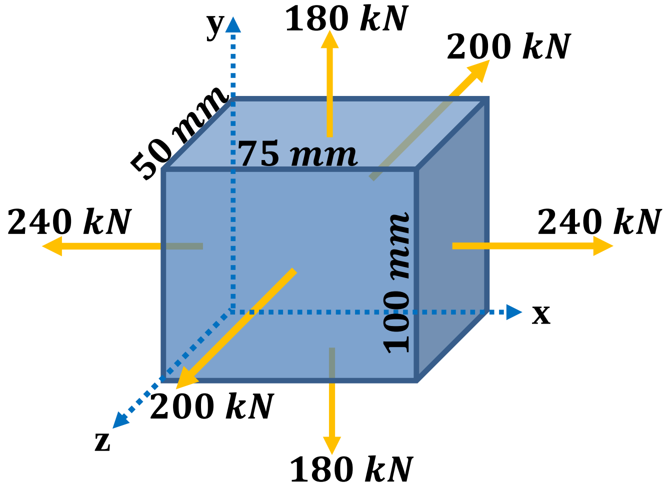


Figure (6.12)

Solution:

$$\varepsilon_x = \frac{\sigma_x}{E} - v\frac{\sigma_y}{E} - v\frac{\sigma_z}{E}$$

$$\sigma_x = \frac{240 \times 10^3}{50(100)} = 48\ MPa$$

$$\sigma_y = \frac{180 \times 10^3}{50(75)} = 48\ MPa$$

$$\sigma_z = \frac{200 \times 10^3}{75(100)} = 26.67\ MPa$$

$$\varepsilon_x = \frac{48}{70 \times 10^3} - \frac{1}{3}\frac{48}{(70 \times 10^3)} - \frac{1}{3}\frac{26.67}{(70 \times 10^3)} = 3.302 \times 10^{-4}$$

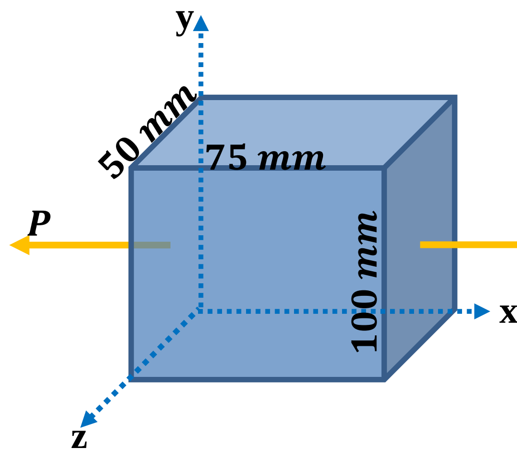
Uniaxial stress state is:

$$\varepsilon_x = \frac{\sigma_x}{E}$$

$$3.302 \times 10^{-4} = \frac{\frac{P}{50(100)}}{70 \times 10^3}$$

$$\rightarrow P = 115.57 \times 10^3\ N$$

$$or \qquad P = 115.57\ kN$$



Example (6.7): A uniform bar of length (***l***), cross – sectional area (***A***), and unit mass (***ρ***) is suspended vertically from one end as shown in Figure (6.13). Show that its total elongation is ($\delta = {}^{\rho g l^2}/_{2E}$). If the total mass of the bar is (***M***), show also that ($\delta = {}^{Mgl}/_{2EA}$).

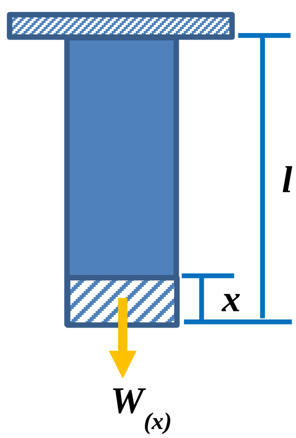


Figure (6.13)

Solution:

$$\rho_{(x)} = W_{(x)} = \gamma . Vol. = \rho(g)(A)(x)$$

$$\delta = \int_0^l \frac{\rho_{(x)}}{EA} dx = \int_0^l \frac{\rho(g)(A)}{EA} dx = \frac{\rho(g)}{E} \left[\frac{x^2}{2}\right]_0^l = \frac{\rho g l^2}{2E}$$

Total Mass= ***M*** , let total weight= ***M.g.l***

$$unit\ weight\ W_{(x)} = \frac{M.g}{l}(x)$$

$$\delta = \int_0^l \frac{\frac{M.g}{l}}{EA}(x)dx = \int_0^l \frac{M.g}{EA(l)}(x)dx$$

$$\delta = \frac{M.g}{EA(l)}\left[\frac{x^2}{2}\right]_0^l = \frac{M.g(l)}{2EA}$$

Example (6.8): The rigid bars shown in Figure (6.14) are separated by a roller at point (C) and pinned at point (A) and (D). A steel rod at point (B) helps support the load of (50 kN). Compute the vertical displacement of the roller at point (C).

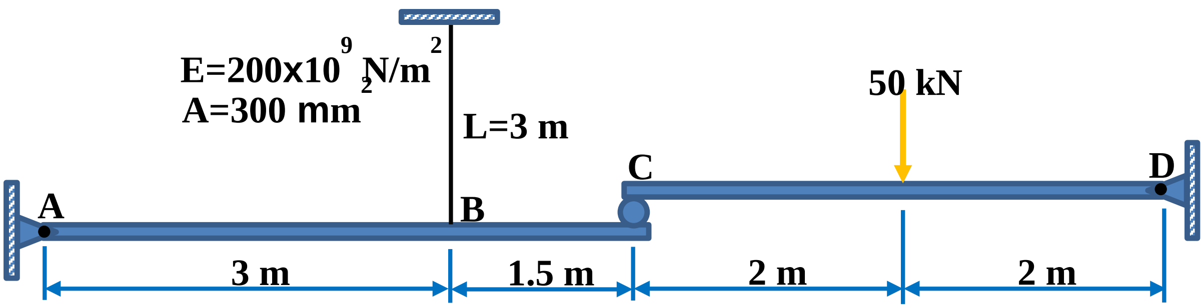
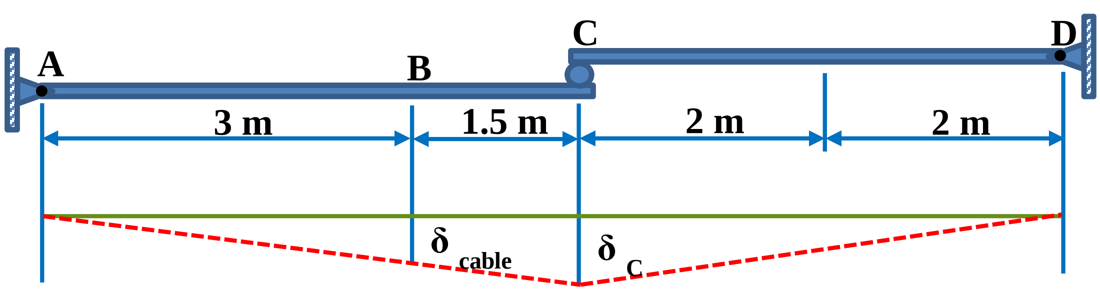


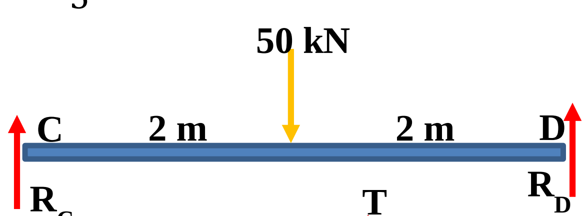
Figure (6.14)

Solution:



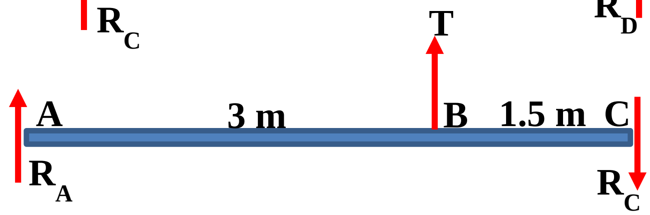
$$\frac{\delta_C}{4.5} = \frac{\delta_{Cable}}{3} \quad \rightarrow \delta_C = \frac{4.5}{3}\delta_{Cable}$$

Bar CD as F.B.D:



$$\circlearrowright \sum M_D = 0 \rightarrow \ R_C(4) = 50(2) \rightarrow R_C = 25\ kN \uparrow$$

Bar ABC as F.B.D:



$$\circlearrowright \sum M_A = 0 \rightarrow \ R_C(4.5) = T(3) \rightarrow T = 37.5\ kN \uparrow$$

$$\delta_{Cable}\frac{Pl}{EA} = \frac{37.5\times10^3}{200\times10^3(300)} = 1.875\,mm$$

$$\delta_C = \frac{4.5}{3}\delta_{Cable} = \frac{4.5}{3}(1.875) = 2.813\,mm$$

<u>Example (6.9):</u> A rod is composed of three segments and carries the axial loads as shown in Figure (6.15). Determine the stress in each material if the walls are rigid.

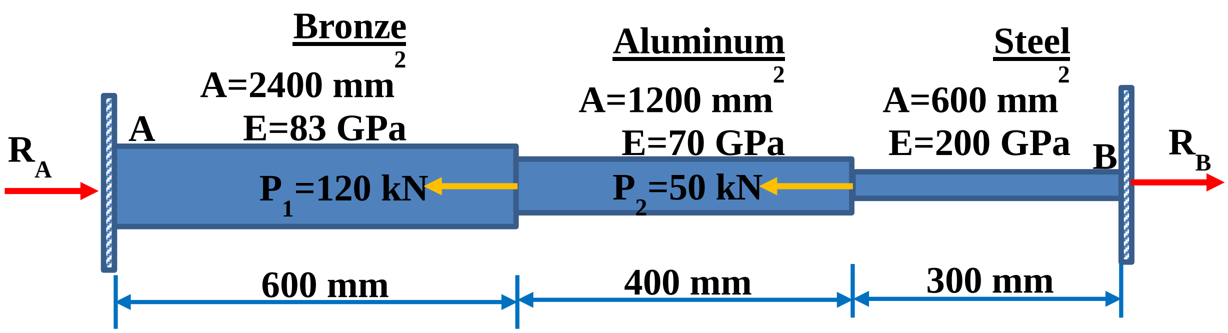
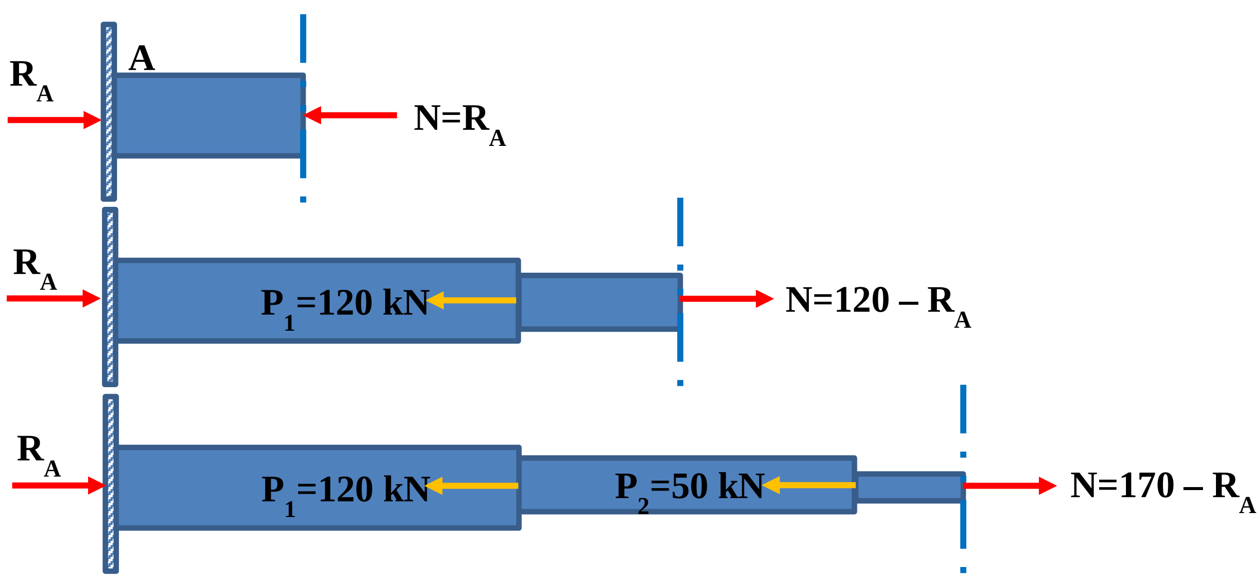


Figure (6.15)

<u>Solution:</u>



From equilibrium:

$$R_A + R_B = P_1 + P_2 \quad \rightarrow R_A + R_B = 170 \quad \ldots\ldots\ldots\ldots(1)$$

From compatibility:

$$\boxed{\delta_{Br.} + \delta_{Al.} + \delta_{St.} = 0}$$

$$\frac{R_A \times 10^3(600)}{83 \times 10^3(2400)} + \frac{(120 - R_A) \times 10^3(400)}{70 \times 10^3(1200)} + \frac{(170 - R_A) \times 10^3(300)}{83 \times 10^3(600)} = 0$$

$$\rightarrow R_A = 96.99 \cong 97\ kN$$

Sub. in Equ.(1),get:

$$R_A + R_B = 170 \ \rightarrow R_B = 170 - 97 = 73\ kN$$

$$\sigma_{Br.} = \frac{P}{A} = \frac{97 \times 10^3}{2400} = 40.42\ MPa$$

$$\sigma_{Al.} = \frac{P}{A} = \frac{(120 - 97) \times 10^3}{1200} = 19.17\ MPa$$

$$\sigma_{St.} = \frac{P}{A} = \frac{73 \times 10^3}{600} = 121.67\ MPa$$