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Strength of Materials

Second Stage

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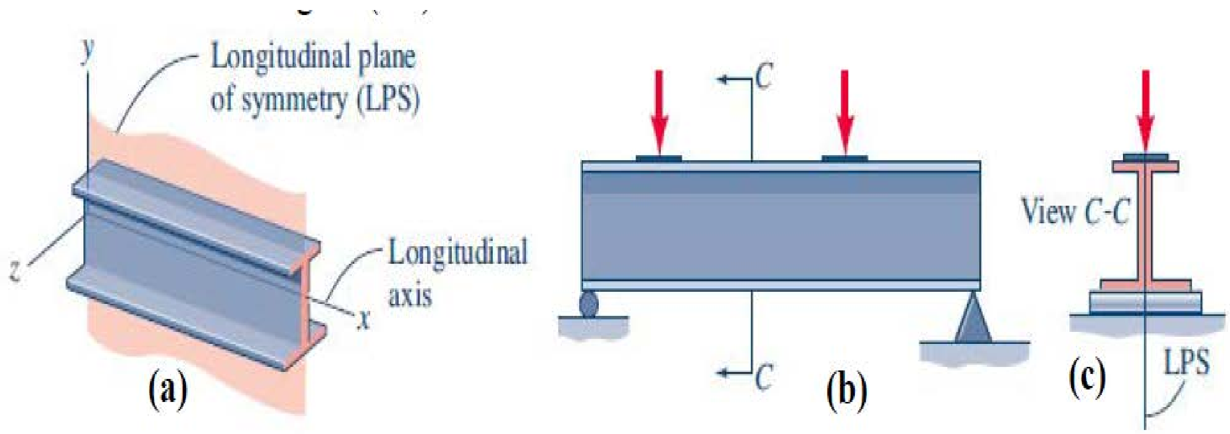
Shearing Stresses in Beams



- Consider a small portion ABDC of a length (dx) of a beam with uniformly distributed load as shown in Figure (5.13).

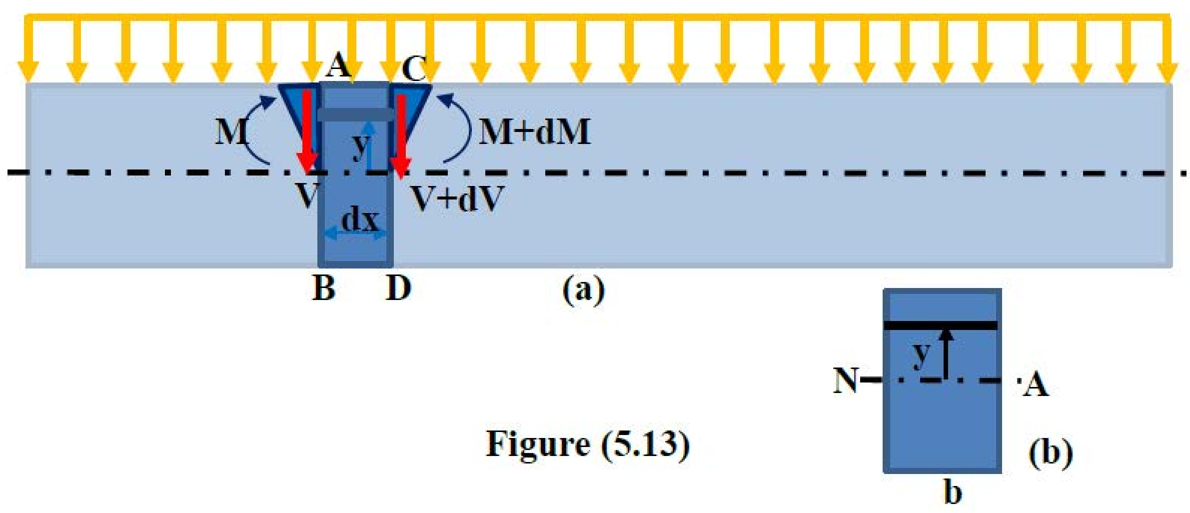


Figure (5.13)

- The intensity of bending stress across AB at a distance (y) from N.A. is:

$$\sigma = \frac{My}{I}$$

Similarly,

- The intensity of bending stress across CD at a distance (y) from N.A. is:

$$\sigma + d\sigma = \frac{(M + dM)y}{I}$$

Knowing that the force acting across AB is:

$$V = Stress \times Area = \sigma(dA) = \frac{M}{I}(y)(dA)$$

- Similarly, the force acting across CD is:

$$V + dV = (\sigma + d\sigma)(dA) = \frac{(M + dM)}{I}(y)(dA)$$

- Net unbalanced force on the strip is:

$$dV = \frac{(M + dM)}{I}(y)(dA) - \frac{M}{I}(y)(dA) = \frac{dM}{I}(y)(dA)$$

- The total unbalanced force above N.A may be found out by integrating the last equation between (0 to d/2), get:

$$\int_0^{d/2} \frac{dM}{I}(y)(dA)dy = \frac{dM}{I}\int_0^{d/2} y(dA)dy = \frac{dM}{I}(A)(\bar{y})$$

Where A= Area of the beam above N.A., and

$\bar{y}$=Distance between the center of gravity of the area (A) and N.A.

Knowing that the intensity of shear stress is:

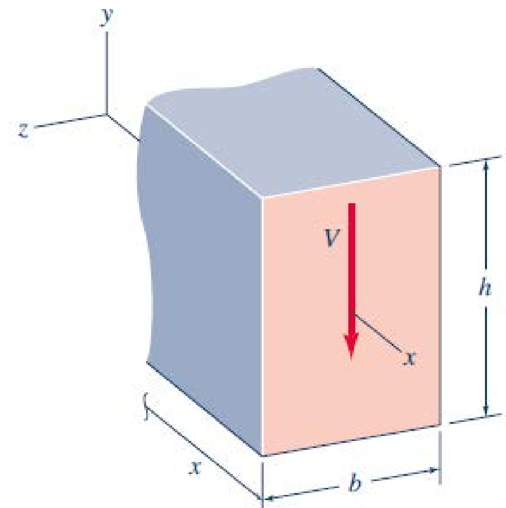
$$\tau = \frac{Total\ Force}{Area} = \frac{\frac{dM}{I}(A)(\bar{y})}{b(dx)} = \frac{dM}{dx} \times \frac{A(\bar{y})}{Ib}$$

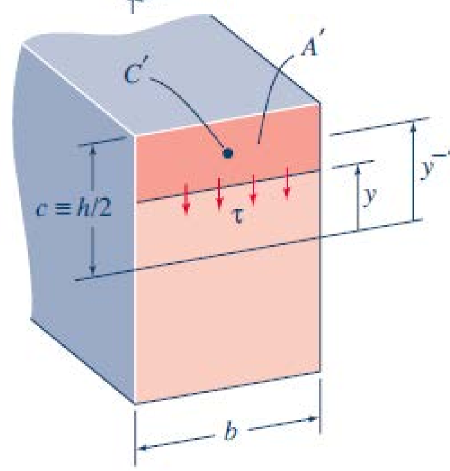
Substituting $\frac{dM}{dx} = V$, and let $A(\bar{y}) = Q$, then:

$$\boldsymbol{\tau = \frac{VQ}{Ib}}$$

Distribution of Shearing Stress over a Section

▶ Rectangular Section:





$$Q_y = b\left(\frac{h}{2} - y\right)\left(\frac{\frac{h}{2} - y}{2} + y\right)$$

$$= b\left(\frac{h}{2} - y\right)\left(\frac{h}{4} - \frac{h}{2} + y\right)$$

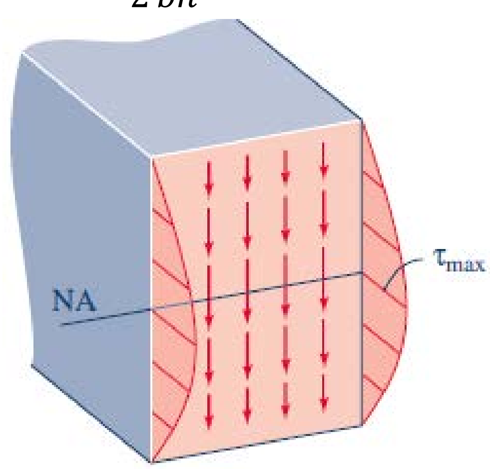
$$= \frac{b}{2}\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)$$

$$\therefore Q_y = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\tau_y = \frac{VQ}{Ib} = \frac{V.\frac{b}{2}\left(\frac{h^2}{4} - y^2\right)}{Ib} = \frac{V}{2I}\left(\frac{h^2}{4} - y^2\right)$$

$$at\ y = 0 \ \rightarrow \ \tau_y = \tau_{max} = \frac{Vh^2}{8I} = \frac{Vh^2}{8\frac{bh^3}{12}} = \frac{12}{8}\frac{V}{bh}$$

$$\therefore \tau_y = \tau_{max} = \frac{3}{2}\frac{V}{bh}$$



Examples

Example (5.8): The T-section shown in Figure (5.14), is the cross section of a beam formed by joining two rectangular pieces of wood together. The beam is subjected to maximum shearing force of (60 kN). Show that the N.A is (34mm) from top and that $I_{N.A}$=10.57x106 mm^4. Using these values, determine the shearing stress:

a) At the N.A, and

b) At the junction between the two pieces of wood.

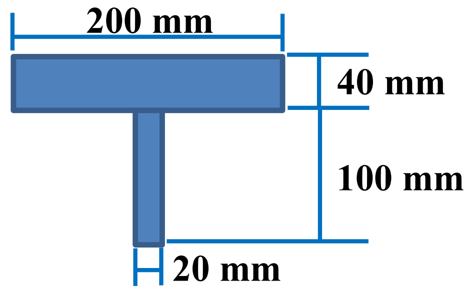


Figure (5.14)

Solution:

$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i} = \frac{200(40)(20) + 100(20)(50+40)}{200(40) + 100(20)} = 34\ mm$$

$$I_{N.A} = \left[\frac{200(40)^3}{12} + 200(40)(34-20)^2\right]$$

$$+ \left[\frac{20(100)^3}{12} + 20(100)(90-34)^2\right] = 10.57 \times 10^6\ mm^4$$

a) $Q = 34(200)\left(\frac{34}{2}\right) = 115600\ mm^3$

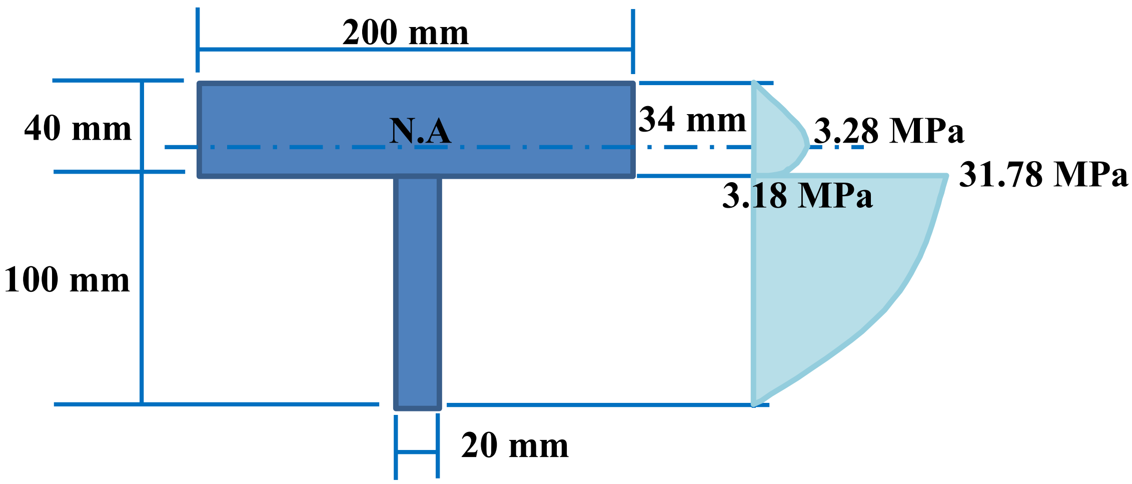
$$\tau = \frac{VQ}{Ib} = \frac{60 \times 10^3(115600)}{10.57 \times 10^6(200)} = 3.28\ MPa$$

b) $Q = 40(200)(34-20) = 112000\ mm^3$

$$\tau = \frac{VQ}{Ib} = \frac{60 \times 10^3(112000)}{10.57 \times 10^6(20)} = 31.78\,MPa$$

Or

$$\tau = \frac{VQ}{Ib} = \frac{60 \times 10^3(112000)}{10.57 \times 10^6(200)} = 3.18\,MPa$$



Example (5.9): Draw the distribution of shear stress of I – beam shown in Figure (5.15) at critical sections for indicated layers.

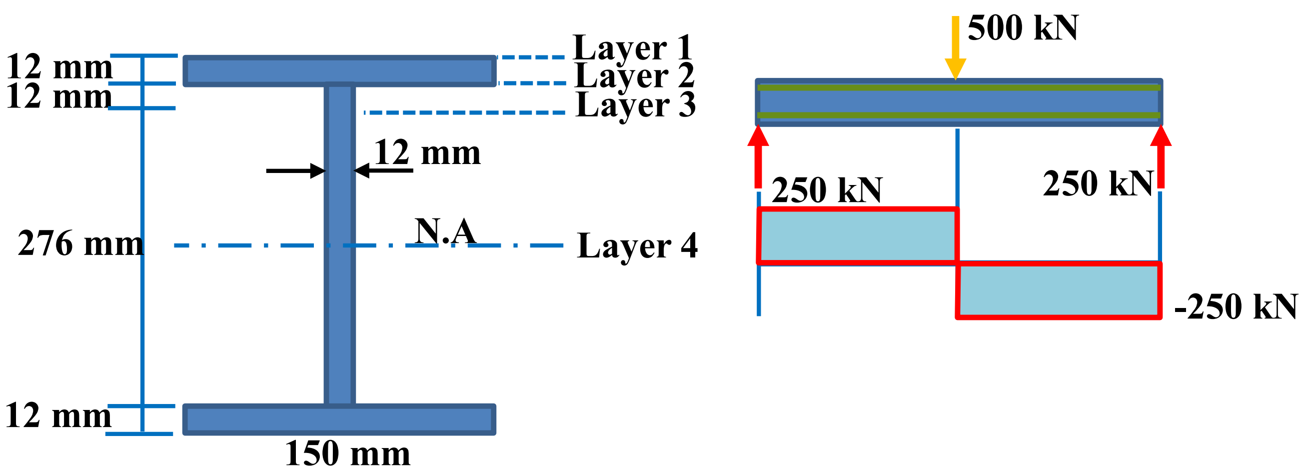


Figure (5.15)

Solution:

$$V_{max} = 250\ kN$$

$$I_{N.A} = \frac{150(300)^3}{12} - 2\left[\frac{69(276)^3}{12}\right] = 95.7 \times 10^6 mm^4$$

▶ **Layer 1:**

$$Q = 0\ \ and\ \ b = 150mm\ \ \rightarrow \tau = 0$$

▶ **Layer 2:**

$$Q = 150(12)\left(\frac{276}{2} + \frac{12}{2}\right) = 259200\ mm^3$$

$$\tau = \frac{250 \times 10^6(259200)}{95.7 \times 10^6(150)} = 4.5\ MPa$$

Or

$$\tau = \frac{250 \times 10^6(259200)}{95.7 \times 10^6(12)} = 56.4\ MPa$$

▶ **Layer 3:**

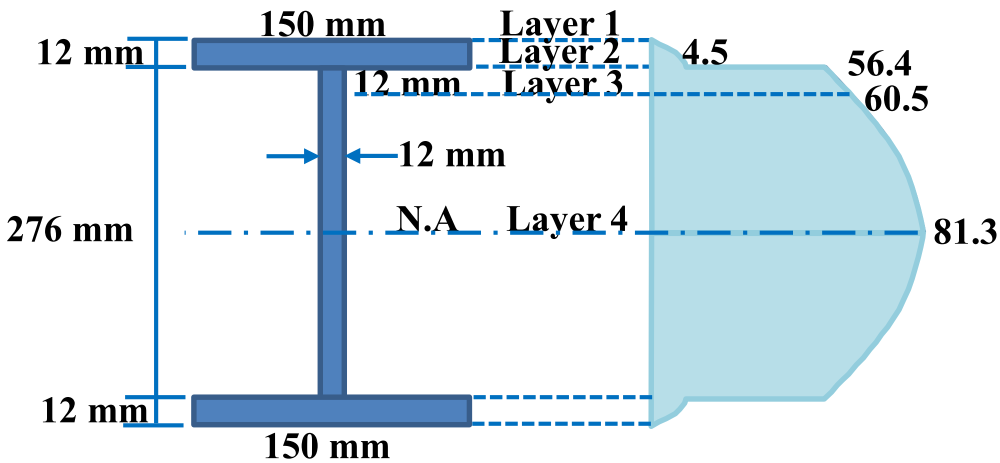
$$Q = 150(12)\left(\frac{276}{2} + \frac{12}{2}\right) + 12(12)\left(\frac{276}{2} + \frac{12}{2}\right) = 278208\ mm^3$$

$$\tau = \frac{250 \times 10^6(278208)}{95.7 \times 10^6(12)} = 60.5\ MPa$$

▶ **Layer 4:**

$$Q = 150(12)\left(\frac{276}{2} + \frac{12}{2}\right) + (\frac{276}{2})(12)(\frac{276}{4}) = 373464\ mm^3$$

$$\tau = \frac{250 \times 10^6(373464)}{95.7 \times 10^6(12)} = 81.3\ MPa$$



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