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Al-Mustaqbal University College
Air Conditioning and Refrigeration Techniques Engineering Department

Strength of Materials

Second Stage

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Thermal Deformation

Thermal Stress

The strain due to an increase or decrease in temperature is:

$$\boldsymbol{\varepsilon = \alpha \Delta T}$$

Where $\alpha =$ is a coefficient of linear expansion, and

$\Delta T =$ Change in temperature.

- **$\boldsymbol{\Delta T}$ is positive if increase in temperature and negative if decrease in temperature.**

The deformation due to this change in temperature is:

$$\boldsymbol{\delta l = \varepsilon (l) = \alpha \Delta T l}$$

Then the stress due to this change in temperature is:

$$\boldsymbol{\sigma = \varepsilon (E) = \alpha \Delta T E}$$

<u>Example (6.10):</u> A steel rod with a cross – sectional area of (150 mm^2) is stretched between two fixed points. The tensile load at (20C^o) is (5000N). What will be the stress at (-20C^o)? At what temperature will the stress be zero? Assume ($\alpha = 11.7\ \mu m/m.C^o$) and (E=200 GPa).

Solution:

$$\delta_P = \delta_T \rightarrow \frac{Pl}{EA} = \alpha \Delta T l$$

$$\frac{5000 - P}{200 \times 10^9 (150 \times 10^{-6})} = 11.7 \times 10^{-6} (-20 - 20)$$

$$\rightarrow P = 19040 \ N$$

$$\sigma = \frac{P}{A} = \frac{19040}{150} = 127 \ MPa$$

$$\frac{5000 - 0}{200 \times 10^9 (150 \times 10^{-6})} = 11.7 \times 10^{-6} (-20 - T)$$

$$\rightarrow T = -34.2 \ C^o$$

Example (6.11): At (20 C^o)a rigid slab having a mass of (55 Mg) is placed on two bronze rods and one steel rod as shown in Figure(6.16). At what temperature will the stress in steel rod be zero? For the steel (A=6000 mm^2, E=200 GPa, and $\alpha = 11.7 \, \mu m/m.C^o$), and for bronze (A=6000 mm^2, E=83GPa, and $\alpha = 19 \, \mu m/m.C^o$).

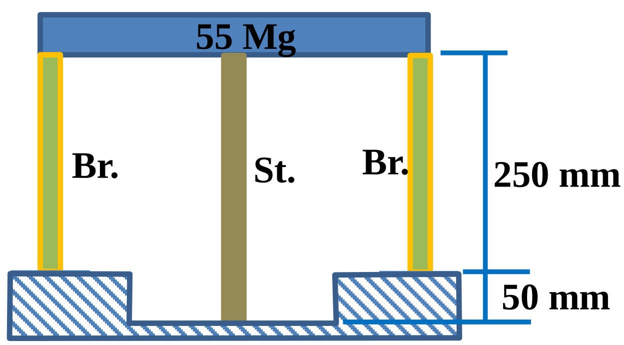
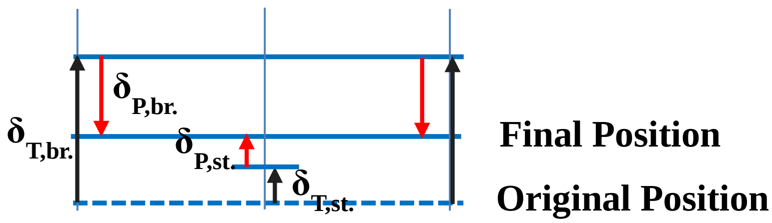


Figure (6.16)

Solution:



From compatibility:

$$\delta_{T,st} + \delta_{P,st} = \delta_{T,br} - \delta_{P,br}$$

$$\alpha \Delta T l_{st} + \frac{P_{st} l_{st}}{(EA)_{st}} = \alpha \Delta T l_{br} - \frac{P_{br} l_{br}}{(EA)_{br}} \quad \ldots \ldots \ldots \ldots \ldots . . (1)$$

From equilibrium:

$$2P_{br} + P_{st} = W$$

Note that: $W = 55 \times 10^3(9.81) = 539550\ N$

$$2P_{br} + P_{st} = 539550$$

- Wanted the temperature when stress in steel be zero, this led to:

$$\sigma_{st} = 0 \ \rightarrow P_{st} = 0\ ,\ then$$

$$2P_{br} + 0 = 539550 \ \rightarrow P_{br} = 269770\ N$$

Sub. These values in Equ.(1), get:

$$11.7 \times 10^{-6}(T - 20)(300) + 0 = 19 \times 10^{-6}(T - 20)(250) - \frac{269770(250)}{83 \times 10^3(6000)}$$

$$3.51 \times 10^{-3}(T - 20) = 4.75 \times 10^{-3}(T - 20) - 0.1354$$

$$1.24 \times 10^{-3}(T - 20) = 0.1354$$

$$T = 129.22\ C^o$$

<u>Example (6.12):</u> For the assembly shown in Figure (6.17). Find:

a) Change in temperature so that the two bar just to be touched.

b) If change in temperature (100 C°). Find stresses in each bar.

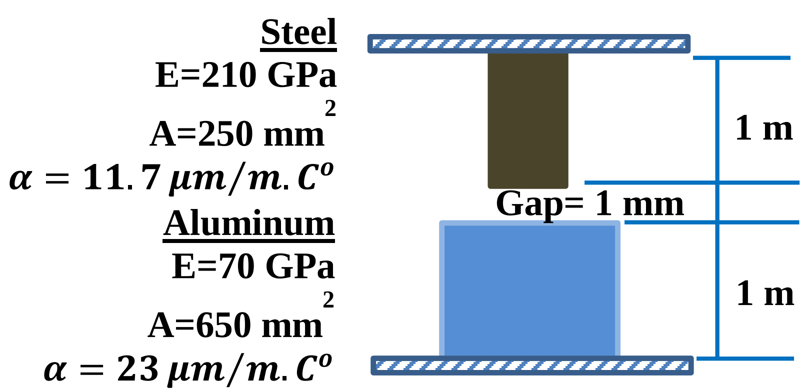


Figure (6.17)

<u>Solution:</u>

a)

$$\delta_{T,s} + \delta_{T,al} = 1$$

$$\alpha\Delta T l_s + \alpha\Delta T l_{al} = 1$$

$$11.7\times10^{-6}\Delta T(1\times10^3) + 23\times10^{-6}\Delta T(1\times10^3) = 1$$

$$0.0117\Delta T + 0.023\Delta T = 1$$

$$\Delta T = 28.82\ C^o$$

b)

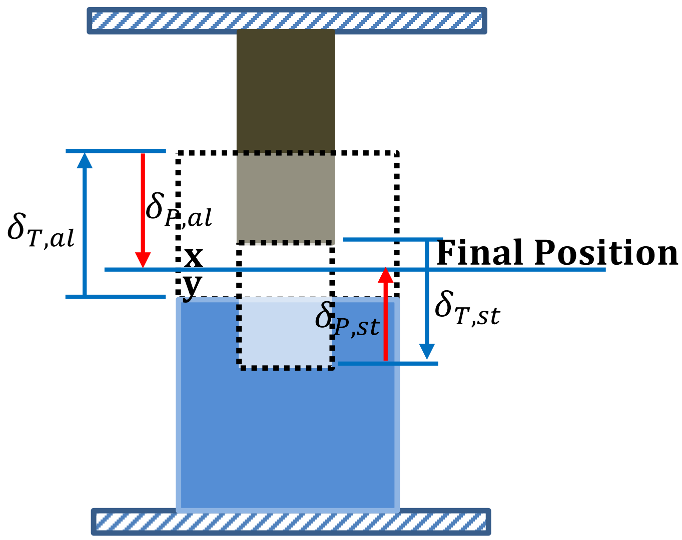
$$x + y = 1 \quad \ldots\ldots\ldots.\ldots\ldots.(1)$$

$$x = \delta_{T,s} - \delta_{P,s} \quad \ldots\ldots\ldots\ldots(2)$$

$$y = \delta_{T,a} - \delta_{P,a} \quad \ldots\ldots\ldots\ldots(3)$$

Sub. Equ.(2) and (3) into Equ.(1), get:

$$(\delta_{T,s} - \delta_{P,s}) + (\delta_{T,a} - \delta_{P,a}) = 1$$



$$\left[11.7\times10^{-6}(100)(1\times10^3) - \frac{P_s(1\times10^3)}{210\times10^3(250)}\right] + \left[23\times10^{-6}(100)(1\times10^3) - \frac{P_a(1\times10^3)}{70\times10^3(650)}\right] = 1$$

$$1.17 - 1.905\times10^{-5}P_s + 2.3 - 2.198\times10^{-5}P_a = 1$$

From equilibrium:

$$P_a = P_s$$

$$1.17 - 1.905\times10^{-5}P_s + 2.3 - 2.198\times10^{-5}P_s = 1$$

$$\rightarrow\ P_s = P_a = 60199.85\ N$$

$$\rightarrow\ \sigma_s = \frac{60199.85}{250} = 240.8\ MPa$$

$$and\ \ \sigma_a = \frac{60199.85}{650} = 92.62\ MPa$$

Problems

Problem 6.1: The 4-mm-diameter cable *BC* is made of a steel with E=200 GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown in Figure (6.18).

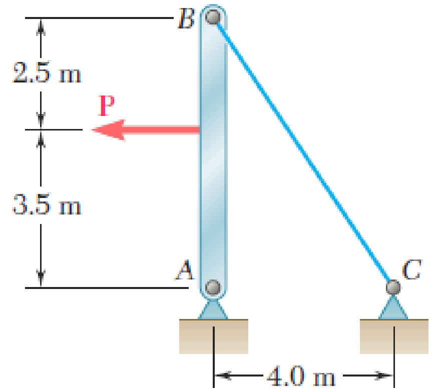


Figure (6.18)

Problem 6.2: The aluminum rod *ABC* (E=10.1x10^6 psi) (Figure 6.19), which consists of two cylindrical portions *AB* and *BC*, is to be replaced with a cylindrical steel rod *DE* (E=29x10^6 psi) of the same overall length. Determine the minimum required diameter ***d*** of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

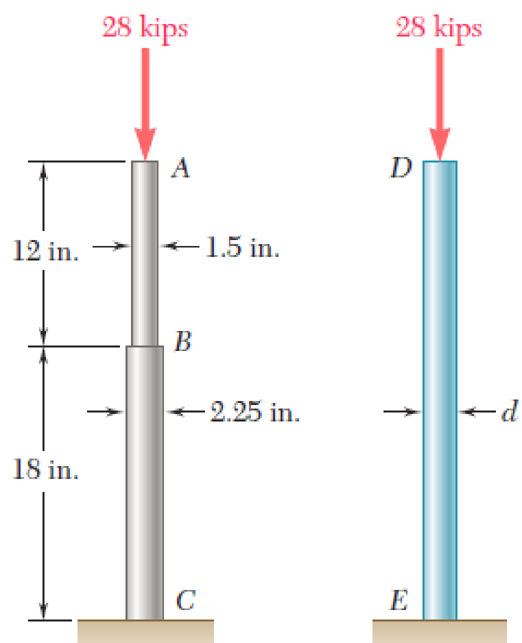


Figure 6.19

Problem 6.3: A rigid, weightless beam *BD* supports a load *P* and is, in turn, supported by two hanger rods, (1) and (2), as shown in Figure (6.20). The rods are initially the same length L=6 ft and are made of the same material. Their rectangular cross sections have original dimensions (w_1=1.5 in., t_1=0.75 in.) and (w_2=2.0 in., t_2=1.0 in.), respectively. (a) At what location, ***b***, must the load ***P*** act if the axial stress in the two bars is to be the same, i.e.,$\sigma_1 = \sigma_2$? (b) What is the magnitude of this tensile stress if a load of ***P*=40** kips is applied at the location determined in Part (a)?

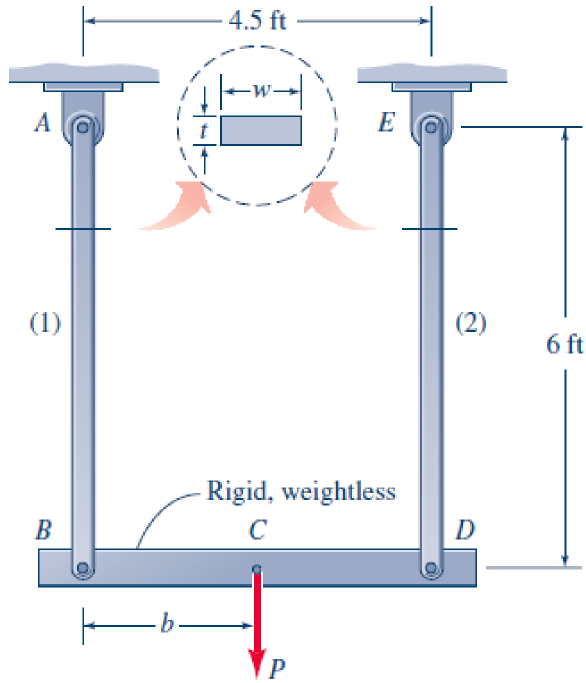


Figure (6.20)

Problem 6.4: Each member of the truss in Figure (6.21) is a solid circular rod with diameter d=10 mm. Determine the axial stress σ_1 in the truss member (1) and the axial stress σ_6 in the truss member (6).

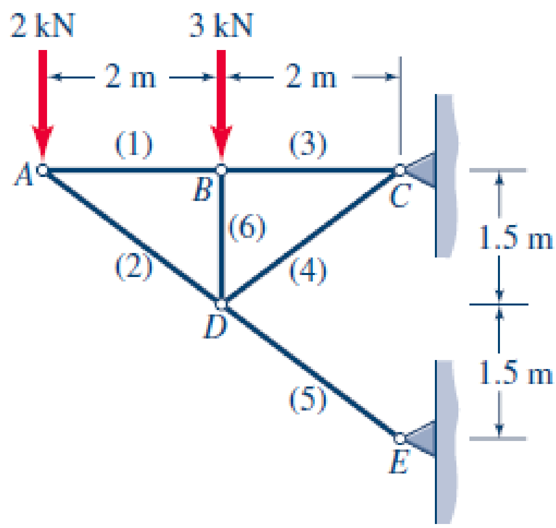


Figure (6.21)

Problem 6.5: The data in Table (6.1) was obtained in a tensile test of a flat-bar steel specimen having the dimensions shown in Figure (6.22).

(a) Plot a curve of engineering stress, σ , versus engineering strain, ϵ , using the given data.

(b) Determine the modulus of elasticity of this material.

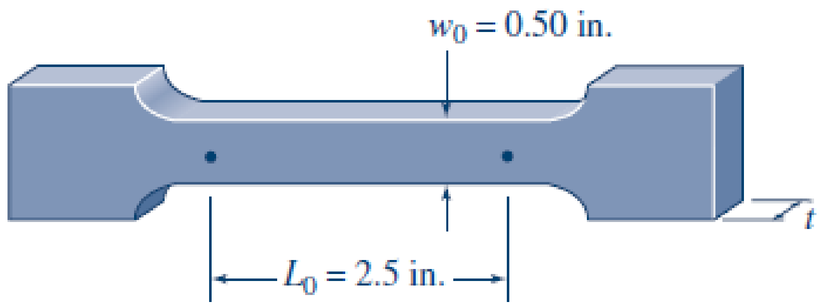


Figure (6.22)

Table (6.1): Tension-test Data; Flat Steel Bar

P (kips)	ΔL (in.)	P (kips)	ΔL (in.)
1.2	0.0008	6.25	0.0060
2.4	0.0016	6.50	0.0075
3.6	0.0024	6.65	0.0100
4.8	0.0032	6.85	0.0125
5.7	0.0040	6.90	0.0150
5.95	0.0050	—	—

Problem 6.6: Under a compressive load of P=24 kips, the length of the concrete cylinder in Figure (6.23) is reduced from 12 in. to 11.9970 in., and the diameter is increased from 6 in. to 6.0003 in. Determine the value of the modulus of elasticity, ***E***, and the value of Poisson's ratio, *v*. Assume linearly elastic deformation.

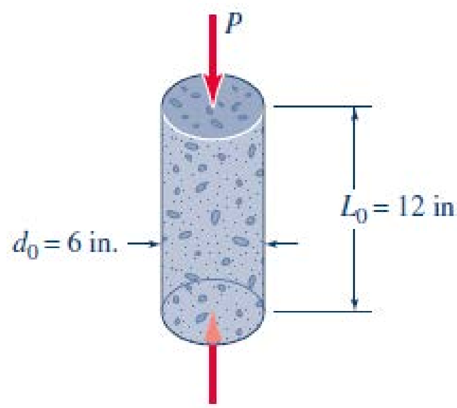


Figure (6.23)

Problem 6.7: An angle bracket, whose thickness is ***t*=12.7 mm**, is attached to the flange of a column by two 15-mm-diameter bolts, as shown in Figure (6.24). A floor joist that frames into the column exerts a uniform downward pressure of ***p*=2** MPa on the top face of the angle bracket. The dimensions of the loaded face are ***L*=152 mm** and ***b*=76 mm**. Determine the average shear stress, $\tau_{av.}$, in the bolts. (Neglect the friction between the angle bracket and the column.)

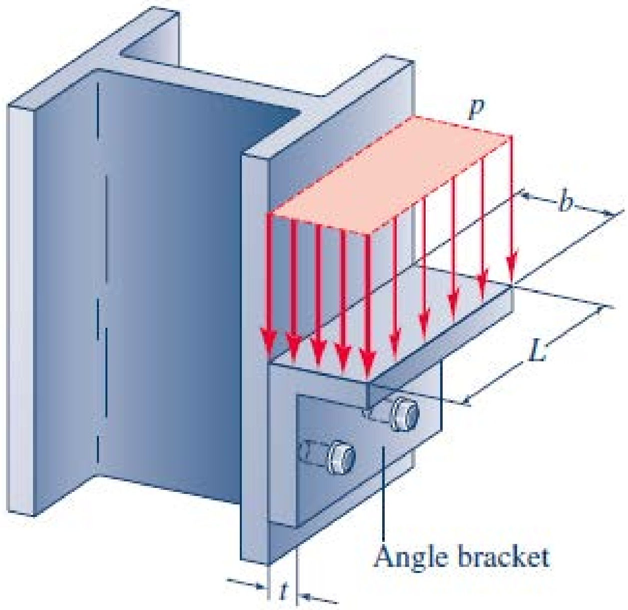


Figure (6.24)

Problem 6.7: A thin, rectangular plate shown in Figure (6.25) is subjected to a uniform biaxial state of stress (σ_x, σ_y). All other components of stress are zero. The initial dimensions of the plate are L_x=4 in. and L_y=2 in., but after the loading is applied, the dimensions are $L_1^* = 4.00176\ in.$, and $L_2^* = 2.00344\ in.$ If it is known that $\sigma_x = 10\ ksi$ and E=10x10^3 ksi, (a) what is the value of Poisson's ratio? (b) What is the value of σ_y?

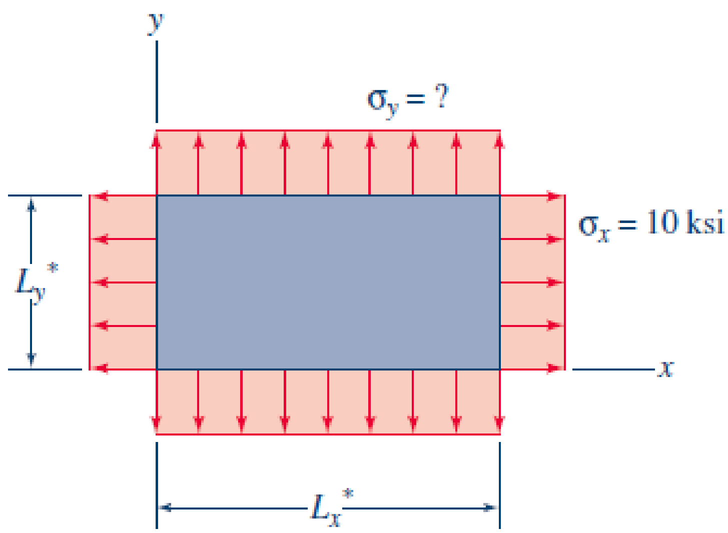


Figure (6.25)